ქართულ - ამერიკული უნივერსიტეტი
GEORGIAN AMERICAN UNIVERSITY (GAU)

შემთხვევითი პროცესებისა და მათემატიკური სტატისტიკის გამოყენებანი ფინანსურ ეკონომიკასა და სოციალურ მეცნიერებებში II

თბილისის მეცნიერებისა და ინოვაციების 2017 წლის ფესტივალი

კონფერენციის მასალები

ბიზნესის სკოლა

ბიზნეს კვლევების სამეცნიერო ცენტრი

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Applications of Stochastic Processes and Mathematical Statistics to Financial Economics and Social Sciences II

თბილისის მეცნიერებისა და ინოვაციების 2017 წლის ფესტივალი

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კონფერენციის მასალები
Conference Materials

ბიზნესის სკოლა

Business School

ბიზნეს კვლევების სამეცნიერო ცენტრი

Business Research Scientific Center

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Specificity of Staff Motivation & Values-Based Framework Within Educational Institutions

R. Michael Cowgill
PhD Candidate of Business Administration, Management
President and Co-founder
Georgian American University (GAU),
10 Merab Aleksidze Street, Tbilisi 0160, Georgia

ABSTRACT

This initial paper begins to define the challenges in creating a relevant (and usable) menu of performance-based motivational techniques by describing the main organizational categories of educational institutions and the various types of motivational behavior commonly observed in their staff. Further, it addresses what additional research is necessary for the final dissertation.

***Keywords**: motivation, performance-based, educational institutions, tenure*

Some values-based framework issues are relevant to other organizations and some are completely unique within the educational sphere. This dichotomy also applies to what motivates staff in such educational institutions. Having been involved in hundreds of organizations, I can easily say that the staff (including lecturers) in the educational sector have a more diverse and varied set of behavioral norms that challenge the development of a set of motivational techniques that cover the range necessary to be usable.

<u>Educational Institutional Frameworks</u>

Educational Institutions have both unique and common characteristics related to their values-based frameworks, depending on their legal status as follows:

- Private – operate as businesses, fully subject to market conditions regarding supply/demand; tuition pricing; quality of educational product; competition. My contention is that the definition of clients is an easy one – those organizations that hire the students and graduates. Financials can be more complicated where there is a reliance on endowments and research grants for economic viability. Usually, private institutions make program decisions based on demand from the hiring organizations.
- Public – in addition to market conditions, they are extensions of government and reliant on government subsidies/funding; political factions. Depending on the governmental

oversight and limited budget conditions, some public institutions may be held to a higher standard regarding financial viability. Public institutions also have a different role to play in providing programs that may or may not be in great demand – that is providing "education for education sake".

- Not-for-Profit – similar to other NGO's where, although not saddled with the links to government, there are issues when donor funding is required and often blurs the lines when defining who are the real clients – students, donors, or hiring organizations. Program decisions are often more complicated and again, subject to influence from donors when necessary.

All the categories of educational institutions above struggle with decisions as to optimum size and as mentioned – what programs to include in their curricula.

<u>What motivates staff in educational institutions?</u>

- Financial – similar to every other organizational staff member. There are numerous studies which define this characteristic, including "how much is enough" and methods, timing and forms of payment. Equal pay for equal work is also relevant here.
- Benefits – both formal and informal - including vacation, holidays, sick leave, maternity/paternity leave, insurance, pension/retirement, sabbaticals, training, access to courses, discounts for family members, etc. Is a menu approach to benefits better for motivating staff? Similar to the financial characteristic, there is a plethora of research on this subject.
- Status & title, including "trappings" – including public exposure, facilities, office space, equipment, etc.
- Organizational & managerial – being involved in a complex organizational structure.
- Leadership role – being and "being considered" as a leader within the organization, with the students, and outside the institution
- Imparting knowledge and developing minds and character of students – a combination of assuming one has something which is of value to the students and a truly altruistic concept of wanting to see students grow in knowledge and maturity.
- Improving society in general and a sense of "giving back" – where there is a realization that education is a key aspect in the growth and success of society. Also, many involved in education feel a responsibility to give something back to the same society that gave them whatever measure of success they feel.
- Being associated with a younger generation – there is definitely a motivating factor of being associated with a dynamic younger generation.
- Learning from students – with some similarities to being associated with a younger generation, it can be very motivating to actually learn from the students, especially when there are international and adult students' programs.
- Social interaction – this motivation for social interaction can be manifested with other staff and/or with students.

Generally, the motivating factors listed above are considered positive. I have chosen to avoid any factors that could be construed as negative, although they do exist, as they would not clearly be related to enhancing performance of the institutions.

History of Autonomy and Tenure

There are some issues that also challenge the link between motivation and performance in educational institutions.

One is that education in general (including the institutions and lecturers – regardless of legal status) has historically been considered to have a semblance of autonomy. That concept of being autonomous is decreasing and educational institutions are held more accountable to all stakeholders. As an example, Deans are not only responsible for ensuring high-quality curricula with qualified lecturers – the Deans must also liaise with hiring organizations to better understand their program needs and also work with the university marketing departments to increase student enrollment in their specific faculties (schools).

The other issue is that of tenure with certain university professors where there is a guarantee of continued employment after a defined number of years of service. It is easy to see where this concept could have a negative impact on motivational techniques.

Defining the Need for Additional Research

The next step in this dissertation development is to conduct research as follows:

- Review existing research on financial and benefits to determine their applicability to the educational sphere.
- Conduct additional research within a relevant educational institution to determine the relevancy of other motivational techniques on behavior that will enhance their performance within their institutional frameworks.

SYMBOL WITHIN CULTURAL ENGINEERING PROCESS IN ORGANIZATIONS

Dina Aslamazishvili
Candidate of Philosophical Sciences, Professor,
Management Direction Head, Business School,
Georgian American University,
0160, Georgia, Tbilisi, Aleksidze Street 10

ABSTRACT

The paper covers the issues related to organizational culture engineering process and the role of the symbol in it. There are discussed the concepts of spiritual and symbolic production in organization, and presented some ideas related to the symbolic sphere of culture, symbolic forms and areas where symbol realized itself through this forms.

Keywords: cultural engineering, organizational culture symbolism, symbolic forms, spiritual.

> Pay heed, my brothers, to every hour where your spirit wants to speak in symbols: there lies the origin of your virtue [1]
>
> F. Nietzsche

"*Culture*" in casual Latin language meant land cultivation; later human education and learning. Later Marcus Tullius Cicero and Plutarkh expanded culture into a complex context by adding "animi" (from "animus" – spirit). So "cultura animi' – cultivation of the spirit, got meanings like self-cultivation, self-creation, and care for the spiritual growth of others [2].

Nowadays the processes of spiritual production (including those in organizations) create framework for developing mind, based on the historically accumulated social experience, specifically symbolizing experience. Culture saves and transmits this experience from generation to generation. There are mastered the symbolizing practice, generated new programs of activity, behavior and communication. Creating the world of symbols is the priority of a human in the culture-creating activities. Social philosophy underlines the key difference of *homo sapiens*, - it is mind, - i.e. ability for heuristic symbolic modelling of the world through abstract, logical and verbal thinking. Much information circulating in culture is made tangible by symbolic forms: signs, images, metaphors. Symbol is a 'border' phenomenon, symbolic sphere connects and divides the form and the content, creating meaning, and integrating the world of explicit and implicit, nature and culture, one culture state with another. Symbol presents the unique connection, it is a connection itself, - connection of meanings. One meaning is not a must connected with another one, however the chain of associations in signs, metaphors and images builds the chain of meanings connected and leading to the one transcendent prime-sense of symbol.

Culture consists of meanings connections, related to spiritual, symbolic processes. They are not tangible in their meaning, but can be found in tangible forms in organizational culture (like artifacts, stories, behavioral patterns, leadership styles, etc.). Organizational culture satisfies the human need in spiritual, it gives the special space to express the meanings of real. It is a space of reflection and the indivisible need of a person to symbolize the world around.

Spiritual has many explications, - Greek philosophers, religion and nowadays humanity sciences. For example, by M.S. Kagan spiritual includes not only the spiritual product, but as well its generating process. The philosopher defines four aspects in spiritual explication: in the process of learning about reality, in its transformation, its values realization, and communication of people in their common activities. It can be presented as the spiritual sphere of society. The same four aspects can be found in organizations, representing the spiritual sphere of organization, where all spiritual processes take place (like spiritual needs genesis, spiritual activity launching, spiritual consumption).

Spiritual and symbolic processes are treated by many experts as in connection. For E. Cassirer in his philosophy of symbolic forms, the main general notion becomes not 'cognition', but ‘spirit’, associated with the ‘spiritual culture’. Cassirer underlines that ‘culture’ understanding is indivisible with the main forms and directions of spiritual creativity [3]. Spiritual is arranged in symbol, “symbolic form” (Cassirer). Symbolic forms (language, myth, religion, art, scientific knowledge) philosopher defines as the spiritual culture dimensions.

In organizations these spiritual culture dimensions are presented in the social glue of organizational culture. Organizational culture functions as a symbolic system, as it seizes intangible nature of inner-organizational connections and as well the meaning of organization in relation to the external environment. Spiritual processes setting and change constitute the cultural engineering processes, where symbol with its prime-sense keeps the central position, and with its transformational potential provides the background for cultural engineering. Organizational culture is the product of the human symbolic activity, and organizational culture engineering is the fluctuating process of the system of metaphors, images and signs. The system is rocked and meanings are transferred and transformed.

Cultural engineering as a vague term, more a concept in transitions, is understood like cultural management with the help of practical strategies design, more applied to cultural institutions. In our interpretation cultural engineering is the process of understanding and developing spiritual foundations in organizations with the help of symbolic forms (like metaphors, signs and images) design and management. So, cultural engineering related to the work with meanings in organizations. Managers compose and decompose the meanings in their departments, teams and at the organizational level. They do that consciously or unconsciously, which is the specificity of culture as organizational phenomenon. Symbol with its prime-sense creates the intangible and transcendent center for organizational culture existence. Symbol with its transitional forms (signs, images and metaphors) creates the transformational ability (change in organizational culture). There forms create the dimensions of symbolic in organizational culture as the following: communicative (sign), semantic (metaphor), and psychological (image).

In the communicative area symbol is realized by the sign (sign system). Symbolizing results function as sign systems with the language codes and multiple interpretations. This is what we call the language element of organizational culture, like culture 'talks', expressing itself (in culture it can be literature, architecture, music, dance etc.; in organizational culture it is jargon, architecture and design of the offices and workplaces, employees creativity, missions, states goals, etc.).

Semantic area realizes symbol by metaphors. Symbolizing results in this area function as metaphoric meanings and senses, used in interpretations. Semantic area deals with meanings and their transition mechanics. In organizational culture we can meet stories, different interpretations of the existing rules and goals, and some mechanics (like gamification) to help employees deal with metaphorical meanings.

In the psychological area of the symbolic sphere of culture symbol is realized through image as its symbolic form. In the psychological area symbolizing results function as images, which create the subjective picture of the world, including the subject, other people, environment, and time perceptions. In organizational culture this area is presented by the perception and the behaviors people tend to demonstrate based on their perceptions, attitudes, and assumptions. Psychological area is the introverted one, closed in its inner world, dealing with inner spiritual attitudes, moral principles and values. In organizational culture it is related to ethical codes, values and inner image of organization.

Cultural engineering process is a spiritual development of organization, including change in organizational culture and in symbolic fields (symbols functioning with the prime-sense keeping). Symbol through its symbolic forms creates the tension in meanings, and by this makes cultural engineering process related to the spiritual and cultural transitions. Effective management of organizational culture means effective understanding and design of symbolic forms in order to keep the symbolic transitional energy active.

References:

1. Ницше, Ф. Сочинения [Текст] / Ф. Ницше / пер. с нем. Ю.М. Антоновского и др. – Калининград : ФГУИПП «Янтар. Сказ», 2002.

2. Teaching and Learning Culture: Negotiating the Context. Edited by Mads Jakob Kirkebaek, Xiang-Yun Du, Annie Aarup Jensen, Sense Publishers, 2013, p.13

3. Кассирер, Э. Философия символических форм [Текст]. В 3 т. Т.1. Язык / Э. Кассирер / пер. с нем. – М.; СПб. : Университетская книга, 2001, р. 17.

LEARNING AS THE MODERN CHALLENGE OF ORGANIZATIONS

Tinatin Zeragia

Assistant Professor, Lecturer, Business School,

Georgian American University,

0160, Georgia, Tbilisi, Aleksidze Street 10

ABSTRACT

Management, this is the field where we can't consider any specific issue or problem to be fully researched or decided. There is no situation which has the one and only right decision or explanation. This is the field which needs continuous research and update. Important challenge that is discussed in this paper is learning. Learning is widely studied by psychology. Modern researches shows us that psychology is tightly connected to the management, as long as the main task of management is human and getting the best results from them. The paper discusses learning from the organizational point of view. Organizations, as independent entities as well can learn and become more competitive on the market, attract best employees, retain and develop them, create innovative and competitive product/service and achieve goals by continuous improvement and innovation implementation.

„Tell me and I forget,
teach me and I may remember,
involve me and I learn."

Benjamin Franklin

Development of management as independent field of study and research has long history, beginning from the late 19th century. During the 20th century many theories have been worked out by leading scientists to distinguish the factors which affect effective workplace environment and productivity.

More than a century, scientists work on developing and finding better understandings, better ways of managing. While they think and contemplate, environmental issues change around them and what was considered to be true for specific moment can become totally wrong after several years. These circumstances leave no room for scientists to be sure that this or that issue is completely researched and analyzed; there is no moment to relax and think that solutions have been made. There are no exact answers that can be resistant to time. This field cannot be considered as scientifically exact, every decision can be right in a specific moment and every decision can be wrong in another specific moment.

The field of management is full of dilemmas that need to be researched and analyzed. Our paper will discuss only one of them, namely, learning. Learning is the tool to change human behavior. When we learn something it affects our way of thinking and way of doing things that is so much important for successful transformation of ourselves and our behavior. Managers always face the situation when they need to change behavior of their employees, change their way of doing job, change the way they think and perceive their jobs and organizations, change their attitudes towards internal or external environment. These tasks are impossible to achieve without both teaching and learning as the whole process.

Learning makes us different, it makes us think more. We learn when we study at school, when we attend University or other courses; we learn when we work and gather experience; we learn when we observe others, see their success or failure; we learn on our mistakes and failures.

Learning makes thing achievable, more tangible and accessible for us. We invest in our learning as well as learning of our children and we strongly believe we will have important return. Mostly this happens.

We learn and we teach. Knowledge needs to be transferred and shared to have its effect. We invest not only financial resources in this process but as well effort. We use effort to teach our children, friends, employees, peers, managers or students, everyone who we need to change their way of thinking or doing things. To make them better, make better ourselves and our general environment.

We learn to learn. We need to know how to be able to learn. We are all different, that means we need different approaches and methods to achieve desired results. We need to learn how to teach others, what their opportunities are and what the best approach is to make others learn and change their selves.

Learning is widely studied by psychology. This is connected to the key psychological factors. Learning shapes our thought and language, our motivations and emotions, our personalities and attitudes. Great contributors in researching this issue are scientists like J.B. Watson, I. Pavlov and B. F. Skinner with their important ideas and experiments.

Contemporary business and management uses the principles of learning to improve their results and effectiveness as well as motivation and commitment of its employees. From one point of view people are learning, from another organizations are learning as an independent entity able to learn and develop. Learning in organizations is based on some key factors that decide what changes will be caused by this experience. The key elements or the major factors that affect learning are motivation, practice, environment, and mental group. Motivation – the encouragement, the support one gets to complete a task, to achieve a goal is known as motivation. It is a very important aspect of learning as it acts gives us a positive energy to complete a task. Example – the coach motivated the players to win the match.

Practice – we all know that "practice makes us perfect". In order to be a perfectionist or at least complete the task, it is very important to practice what we have learnt. Example – we can be a programmer only when we execute the codes we have written.

Environment – we learn from our surroundings, we learn from the people around us. They are of two types of environment – internal and external. Example – a child when at home learns from the family which is an internal environment, but when sent to school it is an external environment.

Mental group – It describes our thinking by the group of people we chose to hang out with. In simple words, we make a group of those people with whom we connect. It can be for a social cause where people with the same mentality work in the same direction. Example – a group of readers, travelers, etc. (Tutorials Point Simply Easy Learning, 2017).

Organizations can use diverse tools to increase learning degree inside their employees. Nowadays there are lots of methods like individual or group trainings, lectures, degree program financing, on-job trainings, coaching, mentoring and so forth. Saving company expenses on learning and development of human resources is not considered to be the right decision any more. Environment on labor market has proven the importance of skilled and educated employees. This is as well seen in results, that companies achieve using well-trained people, who in addition have increased motivation by the fact that organization cares on their future growth and development (Professor Arun Kumar, arunk.com).

As popularized by Senge (1990), a learning organization is "an organization that has woven a continuous and enhanced capacity to learn, adapt and change into its culture. Its values, policies, practices, systems and structures support and accelerate learning for all employees" (Nevis et al., 1995).

Another interesting term connected to learning and organizations is organizational learning. This is the process whereby an organization becomes a learning organization. It requires that an organization be prepared to learn from both failures and successes; rather than being a blaming organization, it becomes one that celebrates and learns. Organizational learning is often used synonymously with learning organization. While the distinction may not be significant, organizational learning is the process an organization uses to become a learning organization (Gary N. Mclean, 2006).

The most important thing to focus after this discussion is that, we live in a changing world, we live in the era of organizations, we need to adapt and we need to make others adaptable. Managers need to think this way to make their organizations competitive and effective and employees need to think this way to make themselves competitive and attractive on labor market, achieve their career and personal goals. Learning is one of the ways to achieve these goals. We need to learn and organizations need to learn as well.

References:

Gary N. Mclean, Organizational development, Berrett-Koehler Publishers, Inc. 2006, p. 271

Nevis et al., 1995, p. 73

Senge, P. The Fifth Discipline: The Art and Practice of the Learning Organisation. New York: Doubleday Currency, 1995.

Professor Arun Kumar, Shuchita Technologies (P) Ltd., available at: http://www.arunk.com/pdf/study%20material/Unit-6.pdf

Tutorials Point Simply Easy Learning, 2017, available at: https://www.tutorialspoint.com/organizational_behavior/organizational_behavior_learning.htm

ქართულ–ამერიკული უნივერსიტეტი
ბიზნესის სკოლა
სადოქტორო პროგრამა

ბექა გოგიჩაშვილი

ნანული ლაზრიევა

RAROC-ის მოდელის მოდიფიცირება და მისი გამოყენება საკედიტო რისკის მართვაში

შესავალი

ნაშრომში შემოთავაზებულია RAROC-ის მოდელის მოდიფიცირებული ვარიანტი, რომელიც განსხვავებით ტრადიციული მიდგომისგან აღარ გულისხმობს შემოსავლიანობის მრუდის მუდმივობას და მის პარალელურ გადაადგილებას.

RAROC-ის მოდელი ერთეული სესხის პირობებში

RAROC არის რისკის მიხედვით შეწონილი კაპიტალის რენტაბელობის მოდელი (იხ. [1][4][8]). მისი ტრადიციული მიდგომა მდგომარეობს შემდეგში, რომ ერთეული სესხიდან მიღებული ერთი წლის წმინდა მოგება შევაფარდოთ მის მოსალოდნელ დანაკარგთან.

$$RAROC_i = \frac{LN_i \times (BR + m_i + OF_i - R_D)}{-D^M{}_i \times LN_i \times \Delta R/(1+R)} = \frac{LN_i \times (APR_i + OF_i - R_D)}{\Delta LN_i} \qquad (1.1)$$

სადაც, LN_i -i-ური სესხის მოცულობა; BR - კაპიტალის შეწონილი ღირებულება, m_i - რისკის პრემია, $(BR + m_i) \equiv APR_i$ i-ური სესხის წლიური ნომინალური საპროცენტო განაკვეთი; OF_i- საკომისიოების ერთობლიობა; R_D - მოზიდული კაპიტალის ღირებულება; $D^M{}_i$ - i-ური სესხის მაკოლეის დურაცია; R -საბაზროს საპროცენტო განაკვეთი; ΔR -საპროცენტო განაკვეთის ცვლილება; LN_i-i-ური სესხის მოცულობა ნომინალში; ΔLN_i- i-ური სესხის ცვლილება ნომინალში, სადაც $BR > 0, m_i \geq 0, OF_i \geq 0$ ხოლო $R_D > 0,\ \Delta R/(1+R) \neq 0$.

გადაწყვეტილების მიღების წესი შემდეგია: $RAROC_i > R_D$ სესხი გაიცემა; $RAROC_i \leq R_D$ სესხი არ გაიცემა.(იხ. [2]).

RAROC-ის ერთ – ერთ პარამეტრს წარმოადგენს მაკოლეის დურაცია (იხ. [5][6]), რომელიც მოიცემა ფორმულით:

$$D^M{}_i = \frac{\sum_{t=1}^{N} \frac{(CF_t)_i \times t}{(1+R)^t}}{\sum_{t=1}^{N} \frac{(CF_t)_i}{(1+R)^t}} = \frac{\sum_{t=1}^{N} (PV_t)_i \times t}{\sum_{t=1}^{N} (PV_t)_i} \qquad (1.2)$$

$D^M{}_i$- i -ური სესხის მაკოლეის დურაცია წლებში
$(CF_t)_i$- სესხზე მიღებული ფულადი ნაკადები t პერიოდის ბოლოს
N- ფულადი ნაკადის მიღების ბოლო პერიოდი
R-წლიური საბაზრო საპროცენტო განაკვეთი ან მიმდინარე საპროცენტო განაკვეთი ბაზარზე
$(PV_t)_i$-ფულადი ნაკადების დღევანდელი მნიშვნელობა t პერიოდის ბოლოს.

მთავარი დაშვება რაზეც მაკოლეის დურაციული მოდელი არის დაფუძნებული ისაა, რომ შემოსავლიანობის მრუდი არ იცვლება ანუ მუდმივია პერიოდის განმავლობაში და საპროცენტო განაკვეთის ცვლილების დროს შემოსავლიანობის მრუდი გადაადგილდება პარალელურად.

RAROC მოდელის მოდიფიცირება

დავუშვათ რომ შემოსავლიანობის მრუდი აღმავალი ან დაღმავალია და ამავე დროსი ის არ გადაადგილდება პარალელურად, მაშინ მაკოლეის დურაცია RAROC-ის მოდელში აღარ გამოდგება რისკის შესაფასებლად. ამისათვის შემოვიღოთ დურაციის სხვაგვარი განმარტება, რომელიც უფრო ზუსტად ასახავს საპროცენტო განაკვეთის მგრძნობიარობას რისკის შეფასებაში.

თუ გავიხსენებთ, რომ

$$CF_i = CF_i(R_i, \cdots, R_N) = \sum_{t=1}^{N} \frac{(CF_t)_i}{(1+R_t)^t} \qquad (2.1)$$

მაშინ მრავალი ცვლადის ფუნქციისთვის ტეილორის ფორმულის გამოყენებით, დაშვებაში რომ ΔR_t, $1 \le t \le N$ საკმარისად მცირეა (როგორც წესი $R_t \le 0.01$) გვექნება (იხ. [3][7]):

$$\frac{1}{LN_i}[LN_i(R_1+\Delta R_1, R_2+\Delta R_2, \cdots, R_N+\Delta R_N) - LN_i(R_1, R_2, \cdots, R_N)] = \frac{1}{LN_i}\sum_{t=1}^{N}\frac{\partial LN_i}{\partial R_t}\Delta R_t \qquad (2.2)$$

სადაც,

$$\frac{\partial LN_i}{\partial R_t} = -\frac{t}{(1+R_t)}\frac{CF_t}{(1+R_t)^t}$$

თუ დამატებით დავუშვებთ, რომ ფარდობა $\frac{\Delta R_t}{(1+R_t)}$ მუდმივია და, მაგალითად, $\frac{\Delta R_t}{(1+R_t)} = \frac{\Delta R_1}{(1+R_1)}$ ყოველი t-სთვის, მაშინ (2.2) ტოლობა მიიღებს შემდეგ სახეს

$$\frac{1}{LN_i}\sum_{t=1}^{N}\frac{CF_t}{(1+R_t)^t}\frac{\Delta R_t}{(1+R_t)} = D^*\frac{\Delta R_1}{(1+R_1)} \qquad (2.3)$$

სადაც,

$$D_i^{\ *} = \frac{\sum_{t=1}^{N}\frac{(CF_t)_i \times t}{(1+R_i)^t}}{\sum_{t=1}^{N}\frac{(CF_t)_i}{(1+R_i)^t}} = \frac{\sum_{t=1}^{N}(PV_t)_i \times t}{\sum_{t=1}^{N}(PV_t)_i}$$

აქედან გამომდინარე სამართლიანია შემდეგი პირობა: თუ შემოსავლიანობის მრუდი აღმავალია $D^* > D^M$ და პირიქით, შემოსავლიანობის მრუდი დაღმავალია $D^* < D^M$, ხოლო მუდმივობის პირობებში $D^* = D^M$.

აქედან $RAROC^*$ მოდელის მოდიფიცირებული ვარიანტი ასეთი სახით წარმოდგება

$$RAROC^*{}_i = \frac{LN_i \times (BR + m_i + OF_i - R_D)}{-D^*{}_i \times LN_i \times \Delta R_i/(1+R_i)} = \frac{LN_i \times (APR_i + OF_i - R_D)}{\Delta LN^*{}_i}$$

გადაწყვეტილების მიღების წესი კი უცვლელი დარჩება $RAROC_i^{\ *} > R_D$ სესხი გაიცემა; $RAROC_i^{\ *} \leq R_D$ სესხი არ გაიცემა.

გამოყენებული ლიტერატურა

[1] E. Zaik, "RAROC at Bank of America, From Theory to Practice," 1996

[2] Anthony Saunders, Marcia Millon Cornette, Financial Institution Management,
Risk Management Approach, McGraw-Hill Irwin, 2008.

[3] J. M. &. Co, "CreditMetrics," 2007.

[4] Anthony Saunders, Linda Allen, Credit Risk Measurement,
New Approaches to Value at Risk and other paradigms, John Wiley & Sons, Inc., 2010.

[5] F. J. Fabozzi, Fixed Income Mathematics Analytical and Statistical Techniques, McGraw-Hill, 2006.

[6] S. Benninga, Financial Modeling, The Mit Press, 2000.

[7] F.J Fabozzi, Modigliani Jones , Foundations of financial Market and Institutions, 2010.

[8] Anthony Saunders, Cornet, Financial Institution Management, 2003.

Georgian-American University
Business School
PhD Program

Investment Projects Robust Valuation

Levan Gachechiladze
Tamaz Uzunashvili

Introduction

This paper describes investment project's robust valuation methodology that is necessary for valuing assets in incomplete markets. Incomplete markets imply that asset prices and derivative instruments on these assets are not directly observable on the market. Even more, in incomplete markets it becomes impossible to fully hedge the assets using replicating portfolio principle discussed in [8] and [11].

As long as we are interested in valuation of assets in incomplete markets, we cannot model asset prices using simplified methods as discussed in [3] and [4]. Instead, this paper considers a general one-period model and its specific cases to value investment projects using robust mean-variance hedging. Robust mean-variance hedging of contingent claims in continuous time models is discussed in [1].

Finally, this paper presents particular investment project undertaken in incomplete market environment and its robust valuation together with solved robust mean-variance hedging problem. As an illustrative example, an investment in gold widgets producing factory is evaluated.

Robust Mean-Variance Hedging of Contingent Claims in One-period Model

Let's examine two assets. One asset is tradable and the other is non-tradable. At the same time, let's be given a derivative instrument $H = G(\eta^1)$ on non-tradable asset. Let S^t be the price of tradable asset at time t, and η^t be the price of non-tradable asset. Let's consider two points in time, i.e. $t = 0$ and $t = 1$. At $t = 0$, we assume (S^0, η^0) are known, while (S^1, η^1) take some values from set S, where $S \subset \mathbf{R}^2$. We denote by $\mathcal{P}(S)$ probabiity measure given on S. We consider it's particular subset $\mathcal{P}^0$, which takes (S^1, η^1) values of joint distribution at $t = 1$. Derivative instrument $H = G(\eta^1)$ is given by increasing function $G(y)$. We consider robust mean-variance hedging problem given as follows:

$$\min_{(x_0\pi)\in R^2} \max_{P\in P^0} E^P \left(G(\eta^1) - x_0 - \pi(S^1 - S^0)\right)^2 \text{ (1).}$$

Namely, its specific case when $n = m = 2$, where

S is $\{(S_k^1, \eta_l^1), k = 1, \dots, n; l = 1, \dots, m\}$, and $\mathcal{P}^0 = \mathcal{P}(S)$ which in turn is analogous to $\{(P_{kl}): \sum_{l=1}^m \sum_{k=1}^n P_{kl} = 1\}$, where P_{kl} is the probability of $P(S^1 = S_k^1, \eta^1 = \eta_l^1)$. For $n = m = 2$, S is given as $\{(S_H, \eta_H), (S_H, \eta_T), (S_T, \eta_H), (S_T, \eta_T)\}$, and $P^0 = \left\{\mathbb{P} = (P_{HH}, P_{HT}, P_{TH}, P_{TT}): \sum_{\substack{i=H,T \\ j=H,T}} P_{ij} = 1, P_{ij} \geq 0\right\}$.

Problem (1) is convex by arguments (x_0, π), and linear, thus concave by $\mathbb{P}$ argument. As a result, using Von Neumann's theorem we conclude that for problem (1) there is a saddle point. Thus,

$$\min_{(x_0\pi)\in R^2} \max_{\mathbb{P}\in P^0} E^{\mathbb{P}}\left(G(\eta^1) - x_0 - \pi(S^1 - S^0)\right)^2 = \max_{\mathbb{P}\in P^0} \min_{(x_0\pi)\in R^2} E^R\left(G(\eta^1) - x_0 - \pi(S^1 - S^0)\right)^2.$$

Below are given 2 propositions with solutions to problem (1) when at least one asset's distribution is known.

Proposition 1 When *P (q)* distribution is given, robust mean-variance hedging problem solution is

$$\pi^* = 0, \qquad x_0^* = E^q\big(G(\eta_1)\big) = \Delta G q + G_T$$

, and robust mean-variance is equal to $R_{minmax} = var^q\big(G(\eta_1)\big) = (\Delta G)^2 q(1-q)$.

Proposition 2 When *P (p)* distribution is given, robust mean-variance hedging problem solution is

$$\pi^* = 0, \;\; x_0^* = \frac{1}{2}\Delta G + G_T = \frac{1}{2}G_H + \frac{1}{2}G_T,$$

, and robust mean-variance is equal to $R_{minmax} = \frac{1}{4}(\Delta G)^2$.

Robust Valuation of Investments in Incomplete Markets

To illustrate an application of robust mean-variance hedging problem, let us examine the valuation of investment project that involves use of derivative instruments in incomplete markets.

Let's consider an investment in a company that makes jewelry using gold. As long as gold jewelry prices are not available on the organized exchange markets, their prices are not readily observable. So, the non-tradable asset's price, η^0, will denote the price of jewelry that the company makes, and the tradable asset's price, S^0, will represent the price of gold. Let's assume that one unit of gold is required to produce one unit of jewelry. Let's also introduce a derivative instrument, G, namely, put option on a non-tradable asset, η^t, which would insure the selling price of jewelry, η^t, at the put option's strike price, denoted by *K*. Thus, the payoff from put option is $G(\eta^t) = (K - \eta^t)^+$ and, by setting $r = 0$, present value of investment becomes:

$$\eta^0 - S^0 - put\ price + E\eta^t - ES^t + (K - E\eta^t)^+.$$

Because put option is on a non-tradable asset, its price is not observed on the market. Using explicit solutions for hedging parameter, x_0^* and hedging error, R_{minmax} coefficients investment project's present value becomes:

$$\eta^0 - S^0 - x_0^* - R_{minmax} + E\eta^t - ES^t + (K - E\eta^t)^+$$

, where $x_0^* + R_{minmax}$ is the put option's robust mean-variance price. Using results from previous section, investment project's present value is given below under both propositions.

Under Proposition 1 When *P (q)* probability distribution of non-tradable asset is known:

$$\eta^0 - S^0 - \Delta G q - G_T - \sqrt{(\Delta G)^2 q(1-q)} + E\eta^t - ES^t + (K - E\eta^t)^+.$$

Under Proposition 2 When $P(p)$ probability distribution of tradable asset is known:

$$\eta^0 - S^0 - \frac{1}{2}G_H - \frac{1}{2}G_T - \sqrt{\frac{1}{4}(\Delta G)^2 + E\eta^t - ES^t + (K - E\eta^t)^+}.$$

Furthermore, if we allow for buying a call option on tradable asset, S^t, while holding put option on non-tradable asset, η^t, we can strictly define the worst case scenario of executing both options as the minimum value of our investment project. Setting the strike price of call option equal to today's price of gold, S^0, the minimum value of the investment can be defined as:

$$\eta^0 - S^0 + K - S^0 - x_0^* - R_{minmax} - \tilde{E}(S^t - S^0)^+$$

, where $\eta^0 - S^0$ is revenue received by selling gold jewelry at time $t = 0$, K is strike price of put option which is the selling price of jewelry when put option is exercised, S^0 is minimum price paid for gold when the call option is exercised, and $\left(x_0^* + R_{minmax}; \tilde{E}(S^1 - S^0)^+\right)$ represent put and call option prices, respectively.

Conclusion

The paper exactly described robust mean-variance hedging methodology for general one-period model for the specific case. The results give values for derivative instruments having non-tradable assets as their underlying asset, which essentially is described by a hedging parameter, x_0^* and hedging error, R_{minmax}. As a result, using the methods defined in the paper it is possible to determine investment projects robust value as well as its minimum value guaranteed by derivative instruments employed on investment project's assets.

References

[1] Robust Mean-Variance Hedging And Pricing of Contingent Claims In A One Period Model (2011) / *R. Tevzadze and T. UzunaShvili* / International Journal of Theoretical and Applied Finance / World Scientific Publishing Company

[2] Real Options Analysis: tools and techniques for valuing strategic investments and decisions / *Johnathan Mun* /2nd edition, published by John Wiley & Sins, Inc., 2006

[3] Investment Projects Valuation Using Real Options and Game Theory (2013) / *Levan Gachechiladze and Irakli Chelidze* / Academic-analytical Journal "Economics and Banking" 2013 Vol. I, N3

[4] Cox et al. (1979) / *Coxx, Ross, Rubinstein*

[5] Robustness (2008) / *Lars Peter Hansen, Thomas J. Sargent* / Published by Princeton University Press

[6] Hedgin By Sequential Regression: An Introduction to the Mathematics of Option Trading (1989) / *H. FÄollmer and M. Schweizer* / ETH ZÄurich

[7] Real Option Valuation of the Project with Robust Mean-variance Hedging (2013) / *T. Uzunashvili* / PHD Thesis, Georgian-American University

[8] Investment under Uncertainty (1994) / *Avinash K. Dixit and Robert S. Pindyck* / Princeton University Press

[9] Monte Carlo Methods in Financial Engineering/Paul Glasserman/published by Springer Science + Business Media, Inc., 2004

[10] Derivatives Market / Robert L. McDonald / published by Addison-Wesley, 2006

[11] Strategic Investment: real options and games / Han T.J. Smit and Lenos Trigeorgis / published by Princeton University Press, 2004

An extension of the mixed Novikov-Kazamaki condition

B. Chikvinidze

[1)] Institute of Cybernetics of Georgian Technical University,
E-mail: beso.chiqvinidze@gmail.com

Abstract Given a continuous local martingale M, the associated stochastic exponential $\mathcal{E}(M) = \exp\{M - \frac{1}{2}\langle M\rangle\}$ is a local martingale, but not necessarily a true martingale. To know whether $\mathcal{E}(\mathcal{M})$ is a true martingale is important for many applications, e.g., if Girsanov's theorem is applied to perform a change of measure. We give a several generalizations of Kazamaki's results and finally construct a counterexample which does not satisfy the mixed Novikov-Kazamaki condition, but satisfies our conditions.

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Lower function.

1 Introduction

Let us given a basic probability space $(\Omega, \mathcal{F}, P)$ and continuous filtration $(\mathcal{F}_t)_{0\leq t\leq\infty}$, which means that every local martingale is continuous, and let $\mathcal{F}_\infty$ be the smallest $\sigma-$Algebra containing all $\mathcal{F}_t$ for $t < \infty$. Let $M = (M_t)_{t\geq 0}$ be an adapted process such that M is a local martingale on the stochastic interval $[[0; T]]$ where T is a stopping time which might be equal to ∞. Denote by $\mathcal{E}(M)$ the stochastic exponential of the local martingale M:

$$\mathcal{E}_t(M) = \exp\{M_t - \frac{1}{2}\langle M\rangle_t\}.$$

Let $(B_t)_{t\geq 0}$ be a standard Brownian motion. Recall that continuous function $\varphi : R_+ \longrightarrow R_+$ is said to be a *lower function* if

$$P\Big\{\omega \ : \ \exists\, t(\omega),\ \forall\, t > t(\omega) \Rightarrow B_t < \varphi(t)\Big\} = 0.$$

Now we formulate the main result of this paper:

Theorem Let a_s be a predictable, M-integrable process and let φ be a lower function such that the following conditions hold:
(i) $|a_s - 1| \geq \varepsilon$ for some $\varepsilon > 0$;
(ii) $\varphi(x)$ can be represented as a sum of non decreasing and bounded functions $\varphi(x) = f(x) + g(x)$;

$$(iii) \quad D = \sup_{0\leq\tau\leq T} E\exp\left\{\int_0^\tau a_s dM_s + \int_0^\tau \Big(\frac{1}{2} - a_s\Big) d\langle M\rangle_s - \varepsilon\varphi(\langle M\rangle_\tau)\right\} < \infty$$

where *sup* is taken over all stopping times. Then the stochastic exponentials $\mathcal{E}(\int a dM)$ and $\mathcal{E}(M)$ are uniformly integrable martingales.

This Theorem is a generalization of Kazamaki's result (Kazamaki [5], p.19, Theorem 1.12).

Notice that Novikov's [7], Kazamaki's [5] and mixed Novikov-Kazamaki's [5] criteria correspond to the case when $\varphi \equiv 0$ *and* $a_s \equiv 0, a_s \equiv \frac{1}{2}$ *and* $a_s \equiv a \neq 1$ *is a constant, respectively.*

In third section we construct a local martingale M on $[0;\infty]$ as a counterexample of mixed Novikov-Kazamaki condition with lower functions. For this local martingale M mixed Novikov-Kazamaki condition fails for any real number $a \neq 1$ and for any lower function, but there exists predictable process a_s such that conditions of Theorem are satisfied (In case when $\varphi \equiv 0$).

It is well-known, exponential martingales play an essential role in various questions concerning the absolutely continuity of probability laws of stochastic processes. In 1960, I. V. Girsanov [3] showed that if $\langle M\rangle_\infty$ is bounded, then $\mathcal{E}(M)$ is a uniformly integrable martingale. In 1972, this assertion was proved by I. I. Gihman and A. V. Skorohod [2] when $e^{(1+\delta)\langle M\rangle_\infty} \in L_1$ for some $\delta > 0$ and then by R. S. Liptser and A. N. Shiryayev [6] when $e^{(\frac{1}{2}+\delta)\langle M\rangle_\infty} \in L_1$ for some $\delta > 0$. After that, A. A. Novikov [7] showed that $\mathcal{E}(M)$ is a uniformly integrable martingale if $e^{\frac{1}{2}\langle M\rangle_\infty} \in L_1$ and that the constant $\frac{1}{2}$ can not be improved. In 1979 Kazamaki [4] proved that $\sup_\tau e^{\frac{1}{2}M_\tau} < \infty$ is sufficient for uniform integrability of $\mathcal{E}(M)$. Then in 1994 Kazamaki [5] generalized his assertion introducing mixed Novikov-Kazamaki condition using constant $a \neq 1$ and lower functions (Kazamaki [5], p.19, Theorem 1.12). In 2013 J. Ruf [8] generalized mixed Novikov-Kazamaki criterion introducing general condition using general function of local martingale and its quadratic variation. In 2017 B. Chikvinidze [1] generalized mixed Novikov-Kazamaki criterion using predictable process a_s instead of the constant a, but without lower functions and in case when $\langle M\rangle_\infty < \infty$ P $a.s.$ The proof was based on technique of backward stochastic differential equations. In this paper we declined the condition $\langle M\rangle_\infty < \infty$ P $a.s.$ and used lower functions in addition.

2 Counterexample

Let us given a basic probability space $\big(\Omega, \mathcal{F}, P\big)$ with two independent Brownian motions $(B_t)_{t\geq 0}$ and $(W_t)_{t\geq 0}$. Let $(\mathcal{F}_t)_{t\geq 0}$ be filtration generated by B and W. Now we choose any $t_0 > 0$ and an event $A \in \mathcal{F}_{t_0}$ such that $0 < P(A) < 1$. Consider processes $\overline{B}_t = B_t - B_{t\wedge t_0}$ and $\overline{W}_t = W_t - W_{t\wedge t_0}$. It is evident that $\overline{B}_t$ and $\overline{W}_t$ are independent Brownian motions starting from t_0 and they both will be independent from events A and A^c.

Define stopping time $\tau = \inf\big\{t \geq t_0\ :\ \overline{B}_t \leq t - (1 + t_0)\big\}$.

It is known from Kazamaki ([5], p. 18. Example 1.10) that for the martingale $\overline{B}^\tau$ holds Novikov's [7] condition:

$$Ee^{\frac{1}{2}\langle\overline{B}^\tau\rangle_\infty} \leq e < \infty \tag{1}$$

which implies that $\mathcal{E}(\overline{B}^\tau)$ is uniformly integrable martingale. But with this Kazamaki ([5], p. 23. Example 1.12) showed that for any $\alpha > 1$, $\mathcal{E}(\alpha\overline{B}^\tau)$ is not an uniformly integrable martingale.

Now define another stopping time $\sigma = \inf\big\{t \geq t_0\ :\ \overline{W}_t \geq t + (1 - t_0)\big\}$.

It is known from Kazamaki ([5], p. 18. Example 1.11) that $\mathcal{E}(\overline{W}^{\sigma})$ in uniformly integrable martingale, because $\sup_\theta Ee^{\alpha\overline{W}^{\sigma}_\theta+(\frac{1}{2}-\alpha)\langle\overline{W}^{\sigma}\rangle_\theta} < \infty$ holds true when $\alpha = 2$:

$$\sup_\theta Ee^{2\overline{W}^{\tau}_\theta-\frac{3}{2}\langle\overline{W}^{\tau}\rangle_\theta} \le e < \infty. \tag{2}$$

But for any $\alpha < 1$, $\mathcal{E}(\alpha\overline{W}^{\sigma})$ is not a uniformly integrable martingale (Kazamaki [5], p. 23. Example 1.13).

Now consider the martingale $M_t = 1_A\overline{B}^{\tau}_t + 1_{A^c}\overline{W}^{\sigma}_t$. We will show that for any $\alpha \neq 1$ $E\mathcal{E}_\infty(\alpha M) < 1$:

$$E\mathcal{E}_\infty(\alpha M) = Ee^{\alpha M_\infty-\frac{\alpha^2}{2}\langle M\rangle_\infty} = Ee^{1_A(\alpha\overline{B}^{\tau}_\infty-\frac{\alpha^2}{2}\langle\overline{B}^{\tau}\rangle_\infty)+1_{A^c}(\alpha\overline{W}^{\sigma}_\infty-\frac{\alpha^2}{2}\langle\overline{W}^{\sigma}\rangle_\infty)} =$$

$$= E\big(1_A\mathcal{E}_\infty(\alpha\overline{B}^{\tau}) + 1_{A^c}\mathcal{E}_\infty(\alpha\overline{W}^{\sigma})\big) = P(A)E\mathcal{E}_\infty(\alpha\overline{B}^{\tau}) + P(A^c)E\mathcal{E}_\infty(\alpha\overline{W}^{\sigma}).$$

As we have mentioned above if $\alpha > 1$ then $E_\infty(\alpha\overline{B}^{\tau}) < 1$ and if $\alpha < 1$ then $E\mathcal{E}_\infty(\alpha\overline{W}^{\sigma}) < 1$, so in both cases we obtain that $E\mathcal{E}_\infty(\alpha M) < 1$. This implies that for any $\alpha \neq 1$ and any lower function φ the mixed Novikov-Kazamaki condition fails:

$$\sup_\theta Ee^{\alpha M_\theta+(\frac{1}{2}-\alpha)\langle M\rangle_\theta-|1-\alpha|\varphi(\langle M\rangle_\theta)} = \infty.$$

Now taking the process $a_s = 2\cdot 1_{A^c\times[t_0;T]}$ we show that

$$\sup_\theta E\exp\Big\{\int_0^\theta a_s dM_s + \int_0^\theta (\frac{1}{2}-a_s)d\langle M\rangle_s\Big\} < \infty.$$

Using (1) and (2) we obtain:

$$E\exp\Big\{\int_0^\theta a_s dM_s+\int_0^\theta \Big(\frac{1}{2}-a_s\Big)d\langle M\rangle_s\Big\} = E\exp\big\{2\cdot 1_{A^c}M_\theta+\frac{1}{2}\langle M\rangle_\theta-2\cdot 1_{A^c}\langle M\rangle_\theta\big\} =$$

$$= E\exp\big\{2\cdot 1_{A^c}\overline{W}^{\sigma}_\theta + \frac{1}{2}1_A\langle\overline{B}^{\tau}\rangle_\theta + \frac{1}{2}1_{A^c}\langle\overline{W}^{\sigma}\rangle_\theta - 2\cdot 1_{A^c}\langle\overline{W}^{\sigma}\rangle_\theta\big\} =$$

$$= E\exp\big\{1_A\frac{1}{2}\langle\overline{B}^{\tau}\rangle_\theta + 1_{A^c}(2\overline{W}^{\sigma}_\theta - \frac{3}{2}\langle\overline{W}^{\sigma}\rangle_\theta)\big\} =$$

$$= P(A)Ee^{\frac{1}{2}\langle\overline{B}^{\tau}\rangle_{\theta}} + P(A^c)Ee^{2\overline{W}^{\sigma}_{\theta} - \frac{3}{2}\langle\overline{W}^{\sigma}\rangle_{\theta}} \leq$$
$$\leq P(A)Ee^{\frac{1}{2}\langle\overline{B}^{\tau}\rangle_{\infty}} + P(A^c)\sup_{\theta} Ee^{2\overline{W}^{\sigma}_{\theta} - \frac{3}{2}\langle\overline{W}^{\sigma}\rangle_{\theta}} \leq$$
$$\leq P(A)e + P(A^c)e = e.$$

Finally because $|a_s - 1| \geq 1$ it is obvious that the process a_s satisfies conditions of Theorem.

References

[1] B. Chikvinidze. *A new sufficient condition for uniform integrability of stochastic exponentials*, Accepted to Stochastics-An International Journal of Probability and Stochastic Processes.

[2] I. I. Gihman and A. V. Skorohod. *Stochastic Differential Equations*, Berlin Heidelberg New York, Springer-Verlag 1972.

[3] I. V. Girsanov. *On transforming a certain class of stochastic processes by absolutely continuous substitution of measures*, Theor. Prob. Appl. 5 (1960), 285-301.

[4] N. Kazamaki. *A sufficient condition for the uniform integrability of exponential martingales*, Math Rep. Toyama Univ. 2 (1979), 1-11.

[5] N. Kazamaki. *Continuous Exponential Martingales and BMO* , Vol. 1579 of *Lecture Notes in Mathematics* , Springer, Berlin-Heidelberg, 1994.

[6] R. Sh. Liptser and A. V. Shiryayev. *On absolute continuity of measures associated with processes of diffusion type with respect to the Wiener measure*, Izv. A. N. USSR, Ser. Math. 36 (1972), 847-889 (In Russian).

[7] A. A. Novikov. *On an identity for stochastic integrals* , Theor. Prob. Appl. 17 (1972), 717-720.

[8] J. Ruf. *A new proof for the conditions of Novikov and Kazamaki* , Stoch. Process. Appl. 123, 404-421 (2013)

Connections between a system of Forward-Backward SDEs and Backward Stochastic PDEs related to the utility maximization problem

M. Mania and R. Tevzadze

We consider a financial market model, where the dynamics of asset prices is described by the continuous semimartingale S defined on the complete probability space $(\Omega, \mathcal{F}, P)$ with continuous filtration $F = (F_t, t \in [0,T])$, where $\mathcal{F} = F_T$ and $T < \infty$.

Denote by $\mathcal{M}^e$ the set of probability measures Q equivalent to P such that S is a local martingale under Q. Throughout the paper we assume that the filtration F is continuous and $\mathcal{M}^e \neq \emptyset$.

The continuity of F and the existence of an equivalent martingale measure imply that S admits the decomposition

$$S_t = M_t + \int_0^t \lambda_s \, d\langle M\rangle_s, \quad \int_0^t \lambda_s^2 \, d\langle M\rangle_s < \infty, \quad t \in [0,T],$$

where M is a continuous local martingale and λ - a predictable process.

Let $U = U(x) : R \to R$ be a utility function taking finite values at all points of R such that U is continuously differentiable, increasing, strictly concave and satisfies the Inada conditions $U'(\infty) = 0, \quad U'(-\infty) = \infty$. For the utility function U we denote by $\widetilde{U}$ its convex conjugate

$$\widetilde{U}(y) = \sup_x (U(x) - xy), \quad y > 0.$$

The wealth process, determined by a self-financing trading strategy π and initial capital x, is defined as a stochastic integral

$$X_t^{x,\pi} = x + \int_0^t \pi_u dS_u, \quad 0 \le t \le T.$$

We consider the utility maximization problem, i.e. the problem of finding a trading strategy $(\pi_t, t \in [0,T])$ such that the expected utility of terminal wealth $X_T^{x,\pi}$ becomes maximal. The value function $V(x)$ associated to the problem is defined by

$$V(x) = \sup_{\pi \in \Pi_x} E\left[U\left(x + \int_0^T \pi_u \, dS_u\right)\right], \tag{1}$$

where Π_x is some class of admissible strategies. We assume that Π_x is a class of predictable, $S-$integrable process π such that an optimal strategy of the problem (1) in this class exists (e.g., one can take the class $\mathcal{H}_1(x)$ from [5]).

Let us introduce a dynamic value function of the problem (1) defined as

$$V(t,x) = esssup_{\pi \in \Pi_x} E\left(U\left(x + \int_t^T \pi_u \, dS_u\right) \Bigm| F_t\right). \tag{2}$$

It is well known that for any $x \in R$ the process $(V(t,x), t \in [0,T])$ is a supermartingale admitting an RCLL modification. Therefore, using the Galchouk–Kunita–Watanabe decomposition, the value function is represented as

$$V(t,x) = V(0,x) - A(t,x) + \int_0^t \varphi(s,x) \, dM_s + L(t,x),$$

where for any $x \in R$ the process $A(t,x)$ is increasing and $L(t,x)$ is a local martingale orthogonal to M.

Definition 1. We shall say that $(V(t,x), t \in [0,T])$ is a regular family of semimartingales if $V(t,x)$ is two-times continuously differentiable at x P-a.s. for any $t \in [0,T]$, for any $x \in R$ the processes $V(t,x)$ and $V'(t,x)$ are special semimartingales with bounded variation parts absolutely continuous with respect to an increasing process $(K_t, t \in [0,T])$.

It was shown in [3] that if the value function $V(t,x)$ is a regular family of semimartingales, then it solves the following Backward Stochastic Partial Differential Equation (BSPDE)

$$V(t,x) = V(0,x) + \frac{1}{2} \int_0^t \frac{(\varphi'(s,x) + \lambda(s) V'(s,x))^2}{V''(s,x)} \, d\langle M \rangle_s +$$

$$+\int_0^t \varphi(s,x)\,dM_s + L(t,x), \quad V(T,x) = U(x) \tag{3}$$

and the optimal wealth satisfies the SDE

$$X_t(x) = x - \int_0^t \frac{\varphi'(s,X_s(x)) + \lambda(s)V'(s,X_s(x))}{V''(s,X_s(x))} dS_s. \tag{4}$$

Note that the BSPDE (3), (4) is of the same form for utility functions defined on half real line and also for random utility functions $U(\omega,x)$.

Assume now that the utility function is of the form $U(x+H)$, where $U(x), x \in \mathbb{R}$ is a nonrandom utility and H is a F_T-measurable random variable. In the paper [2] a new approach was developed, where a characterization of optimal strategies in terms of a system of Forward-Backward Stochastic Differential Equations (FBSDE) in the Brownian framework was given. The key observation was an existence of a stochastic process Y with $Y_T = H$ such that $U'(X_t + Y_t)$ is a martingale. The same approach was used in [4], where similar results were obtained in semimartingale setting with continuous filtration rejecting also some technical conditions imposed in [2]. The FBSDE for the pair (X,Y) (where X is the optimal wealth and Y the process mentioned above) is of the form

$$Y_t = Y_0 + \int_0^t \big[\lambda_s^2 \frac{U'(X_s+Y_s)}{U''(X_s+Y_s)} - \frac{1}{2}\lambda_s^2 \frac{U'''(X_s+Y_s)U'(X_s+Y_s)^2}{U''(X_s+Y_s)^3} + \tag{5}$$

$$+Z_s\lambda_s\big] d\langle M\rangle_s - \frac{1}{2}\int_0^t \frac{U'''(X_s+Y_s)}{U''(X_s+Y_s)} d\langle N\rangle_s + \int_0^t Z_s dM_s + N_t, \;\; Y_T = H.$$

$$X_t = x - \int_0^t \frac{\lambda_s U'(X_s+Y_s) + Z_s U''(X_s+Y_s)}{U''(X_s+Y_s)} dS_s, \tag{6}$$

where N is a local martingale orthogonal to M.

Definition 2 ([1]). The function $u(t,x)$ is called a decoupling field of the FBSDE (5), (6) if $u(T,x) = H$ and for any $x \in R, s, \tau \in R_+$ such that $0 \le s < \tau \le T$ the FBSDE

$$Y_t = u(s,x) + \tag{7}$$

$$\int_s^t \big(\lambda_r^2 \frac{U'(X_r+Y_r)}{U''(X_r+Y_r)} - \frac{1}{2}\lambda_r^2 \frac{U'''(X_r+Y_r)U'(X_r+Y_r)^2}{U''(X_r+Y_r)^3} + Z_r\lambda_r\big) d\langle M\rangle_r -$$

$$\frac{1}{2}\int_s^t \frac{U'''(X_r+Y_r)}{U''(X_r+Y_r)}d\langle N\rangle_r + \int_s^t Z_r dM_r + N_t - N_s, \;\; Y_\tau = u(\tau, X_\tau),$$

$$X_t = x - \int_s^t \big(\lambda_r \frac{U'(X_r+Y_r)}{U''(X_r+Y_r)} + Z_r\big)dS_r, \tag{8}$$

has a solution (Y, Z, N, X) satisfying

$$Y_t = u(t, X_t), \quad t \in [s, \tau]. \tag{9}$$

We shall say that $u(t, x)$ is a regular decoupling field if it is a regular family of semimartingales (in the sense of Definition 1).

If we differentiate equation BSPDE (3) at x (assuming that all derivatives involved exist), we obtain the BSPDE

$$V'(t,x) = V'(0,x) + \int_0^t \Big[\frac{(V''(s,x)\lambda_s + \varphi''(s,x))(V'(s,x)\lambda_s + \varphi'(s,x))}{V''((s,x))} -$$

$$-\frac{1}{2}V'''(s,x)\frac{(V'(s,x)\lambda_s + \varphi'(s,x))^2}{V''(s,x)^2}\Big]\, d\langle M\rangle_s +$$

$$+\int_0^t \varphi'(s,x)\, dM_s + L'(t,x), \quad V'(T,x) = U'(x+H), \tag{10}$$

where the optimal wealth satisfies the same SDE (4).

The FBSDE (5), (6) is equivalent, in some sense, to BSPDE (10),(4) and the following statement establishes a relation between these equations.

Theorem 1. Let $U(x)$ be three-times continuously differentiable.

a) Let $(V'(t,x), \varphi'(t,x), L'(t,x), X_t)$ be a solution of BSPDE (10),(4). Then the quadruple (Y_t, Z_t, N_t, X_t), where

$$Y_t = -\widetilde{U}'(V'(t, X_t)) - X_t,$$

$$Z_t = \lambda_t \widetilde{U}'(V'(t, X_t)) + \frac{\varphi'(t, X_t) + \lambda_t V'(t, X_t)}{V''(t, X_t)},$$

$$N_t = -\int_0^t \widetilde{U}''(V'(s, X_s))d\big(\int_0^s L'(dr, X_r)\big),$$

will satisfy the FBSDE (5), (6). Moreover, the function $u(t,x) = -\widetilde{U}'(V'(t,x)) - x$ will be the decoupling field of this FBSDE.

b) Let $u(t,x)$ be a regular decoupling field of FBSDE (5), (6) and let $(U'(X_t + Y_t), s \le t \le T)$ be a true martingale for every $s \in [0, T]$. Then

$(V'(t,x), \varphi'(t,x), L'(t,x), X)$ will be a solution of BSPDE (10),(4) and following relations hold

$$V'(t,x) = U'(x + u(t,x)), \quad \text{hence} \quad V'(t,X_t) = U'(X_t + Y_t),$$

$$\varphi'(t,X_t) = (Z_t + \lambda_s \frac{U'(X_t + Y_t)}{U''(X_t + Y_t)})V''(t,X_t) - \lambda_t U'(X_t + Y_t),$$

$$\int_0^t L'(ds, X_s) = \int_0^t U''(X_s + Y_s) dN_s,$$

where $\int_0^t L'(ds, X_s)$ is a stochastic line integral with respect to the family $(V'(t,x), x \in R)$ along the process X.

References

[1] A. Fromm and P. Imkeller, Existence, Uniqueness and Regularity of Decoupling Fields to Multidimensional Fully coupled FBSDEs, arXiv:1310.0499v2 , (2013).

[2] U.Horst, Y.Hu, P.Imkeller, A.Reveillac and J.Zhang,Forward-backward systems for expected utility maximization, Stochastic Processes and their Applications, 124, N. 5, (2014), pp. 1813–1848.

[3] M. Mania and R. Tevzadze, Backward stochastic partial differential equations related to utility maximization and hedging, Journal of Mathematical Sciences, Vol. 153, No. 3, (2008), pp. 292–376.

[4] M. Santacroce and B. Trivellato, Forward backward semimartingale systems for utility maximization, SIAM Journal on Control and Optimization, 52(6), (2014), pp. 3517–3537.

[5] W. Schachermayer, A Super-Martingale Property of the Optimal Portfolio Process. Finance and Stochastics, Vol. 7 , No. 4, (2003), pp. 433–456.

Recursive estimation of one-dimensional parameter of Compound Poisson process*

Nanuli Lazrieva[a,b], Temur Toronjadze[a,b]

[a] *Business School, Georgian–American University, 8 M. Aleksidze Str., Tbilisi 0160, Georgia*
[b] *A. Razmadze Mathematical Institute of I. Javakhishvili Tbilisi State University, 6 Tamarashvili Str., Tbilisi 0177, Georgia*

Abstract

Recursive estimation procedure of a one-dimensional parameter of Levy measure of Compound Poisson process is introduced and its asymptotic properties are investigated.

The object of our investigation is a parameter filtered statistical model

$$\mathcal{E} = \big(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\geq 0}, (P_\theta, \theta \in R)\big) \tag{1}$$

associated with one-dimensional $\mathbb{F}$-adapted RCLL process $X = (X_t)_{t\geq 0}$ in the following way: for each $\theta \in R$, P_θ is assumed to be the unique measure on $(\Omega, \mathcal{F})$ such that under P_θ, $X = (X_t)_{t\geq 0}$ is a semimartingale with the triplet of predictable characteristics $(B_\theta, 0, \nu_\theta)$, where $B_\theta(t) = \lambda t \int_R x I_{(|x|\leq 1)} \nu(\theta, dx)$, $\nu_\theta(dt, dx) = \lambda\, dt\, \nu(\theta; dx)$, where $\lambda > 0$, $\nu(\theta, \cdot)$ is probability measure on $(R, \mathcal{B}(R))$ with $\int_R x^2 \nu(\theta, dx) < \infty$.

It is obvious that under P_θ, $X = (X_t)_{t\geq 0}$ is a Compound Poisson process that can be written in the following form:

$$X_t = \sum_{i=1}^{N_t} \xi_i,$$

where $N = (N_t)_{t\geq 0}$ is a Standard Poisson process with intensity $\lambda > 0$, and $\xi = (\xi_n)_{n\geq 1}$ is the sequence of i.i.d. random variables with the probability distribution $\nu(\theta, \cdot)$ (see, e.g., [1]).

Our aim is to construct recursive estimation procedure for unknown parameter $\theta \in R$.

Suppose that for each pair (θ, θ') the measures $\nu(\theta, \cdot)$ and $\nu(\theta', \cdot)$ are equivalent. Fix some $\widetilde{\theta} \in R$ and denote $P_{\widetilde{\theta}} := P$, $\nu_{\widetilde{\theta}} := \nu$, $\nu(\widetilde{\theta}; x) = \nu(\cdot)$. Then

$$P_\theta \overset{loc}{\sim} P, \quad \theta \in R,$$

and the local density process $\rho_t(\theta) = \frac{dP_{\theta,t}}{dP_t}$ can be represented as the Dolean esponential

$$\rho_t(\theta) = \mathcal{E}_t(M(\theta)),$$

where $M(\theta) = (Y(\theta) - 1) * (\mu - \nu)$, $Y(\theta, x) = \frac{d\nu(\theta, x)}{d\nu(x)}$.

* *Email addresses*: `lazrieva@gmail.com` (Nanuli Lazrieva), `toronj333@yahoo.com` (Temur Toronjadze)

Further, assume that the density $Y(\theta, x)$ is continuously differentiable in θ for each $x \in R$, and differentiability under integral signs is possible. Note that under assumptions listed above the model (1) is regular in the sense given in [2].

It is not hard to observe that

$$L_t(\theta) := \frac{\partial}{\partial \theta} \ln \rho_t(\theta) = \frac{\dot{Y}(\theta)}{Y(\theta)} * (\mu - \nu_\theta) = \Phi(\theta) * (\mu - \nu_\theta) \quad \Big(\Phi(\theta) = \frac{\dot{Y}(\theta)}{Y(\theta)}\Big). \tag{2}$$

Hence, the maximum likelihood equation is

$$L_t(\theta) = \Phi(\theta) * (\mu - \nu_\theta)_t = 0. \tag{3}$$

Remark 1. The problem of solvability of Eq. (3) in more general setting is studied in [3].

Eq. (3) can be rewritten in the equivalent form

$$\sum_{n=1}^{N_t} \frac{\dot{Y}(\theta, \xi_n)}{Y(\theta, \xi_n)} = 0 \quad \Longrightarrow \quad \widehat{\theta}_t - \text{MLE}. \tag{3$'$}$$

So, for each $t > 0$, we need to solve Eq. (3) or (3$'$) which is not easy task (in general).

Instead in [2] we proposed the recursive procedure to obtain the process $\theta = (\theta_t)_{t \geq 0}$ (recursive estimate) with the same asymptotic properties as MLE $\widehat{\theta}_t$, as $t \to \infty$, P_θ-a.s.

To develop this procedure first of all assume that

$$I(\theta) := \int\limits_R \Phi^2(x, \theta) \nu(\theta, dx) < \infty.$$

Then $L(\theta) = (L_t(\theta), t \geq 0) \in M^2_{loc}(P_\theta)$ and the Fisher information process is

$$I_t(\theta) = \langle L(\theta), L(\theta) \rangle_t = \lambda t I(\theta).$$

Denote $\gamma_t(\theta) = I_t^{-1}(\theta)$. The recursive estimation procedure, SDE (3.4) of [4] in the case under consideration is of the following form:

$$\begin{aligned} \theta_t = \theta_0 &+ \int\limits_0^t \int\limits_R \gamma_s(\theta_{s-}) \Phi(\theta_{s-}, x) \Big(1 - \frac{Y(\theta_{s-}, x)}{Y(\theta, x)}\Big) \nu_\theta(ds, dx) \\ &+ \int\limits_0^t \int\limits_R \gamma_s(\theta_{s-}) \Phi(\theta_{s-}, x) (\mu - \nu_\theta)(ds, dx). \end{aligned} \tag{4}$$

Remark 2. Although Eq. (4) is equivalent to the following equation

$$\theta_t = \theta_0 + \sum_{n=1}^{N_t} \frac{\Phi(\theta_{n-1}, \xi_n)}{n I(\theta)}\,, \tag{4$'$}$$

we prefer the form of Eq. (4) to investigate asymptotic properties of θ_t, as $t \to \infty$, P_θ-a.s., based on results concerning asymptotic behaviour of solutions of Robbins–Monro (RM) type SDE.

The Robbins–Monro type SDE

$$z_t = z_0 + \int_0^t H_s(z_{s-})dK_s + \int_0^t M(ds, z_{s-}), \tag{5}$$

where $H_s(0) = 0$, $H_s(u)u < 0$, $u \neq 0$, was introduced in [5]. In [5], [6], the asymptotic behaviour of $z = (z_t)_{t\geq 0}$, as $t \to \infty$, P_θ-a.s. was investigated.

Assume that for each $x \in R$ the function $Y(\theta, x)$ is strongly monotone in θ. Denote $z_t = \theta_t - \theta$, then Eq. (4) becomes

$$\begin{aligned} z_t = z_0 &+ \int_0^t \int_R \gamma_s(\theta + z_{s-})\Phi(\theta + z_{s-}, x)\left(1 - \frac{Y(\theta + z_{s-}, x)}{Y(\theta, x)}\right)\nu_\theta(ds, dx) \\ &+ \int_0^t \int_R \gamma_s(\theta + z_{s-})\Phi(\theta + z_{s-}, x)(\mu - \nu_\theta)(ds, dx) \end{aligned} \tag{5'}$$

and is of the form (5) with $K_t = \lambda t$,

$$H_t(u) = \int_R \gamma_t(\theta + u)\Phi(\theta + u, x)\left(1 - \frac{Y(\theta + u, x)}{Y(\theta, x)}\right)\nu(\theta, dx), \tag{6}$$

$$M(u) = [M_t(u), t \geq 0] = \left[\int_0^t \int_R \gamma_s(\theta + u)\Phi(\theta + u, x)(\mu - \nu_\theta)(ds, dx),\ t \geq 0\right]. \tag{7}$$

Denote

$$h_t(u) = \frac{d\langle M(u), M(u)\rangle_t}{dK_t} = \gamma_t^2(\theta + u)\int_R \Phi^2(\theta + u, x)\nu(\theta, dx) = \gamma_t^2(\theta + u)I(\theta + u).$$

Hence, $(5')$ is the special case of the RM type SDE (5) with objects $H(u) = (H_t(u))_{t\geq 0}$ and $M(u) = (M(t, u))_{t\geq 0}$ specified by Eqs. (6) and (7), respectively.

Therefore one can use the results about asymptotic behaviour of solution $z = (z_t)_{t\geq 0}$ of general SDE (5) to establilsh asymptotic behaviour of solution of SDE $(5')$, as $t \to \infty$.

Namely, one can use Theorem 3.1 of [5] to derive sufficient conditions for the convergence: $z_t \to 0$, as $t \to \infty$, P_θ-a.s., for all $\theta \in R$ (recall that from now $z = (z_t)_{t\geq 0}$ is the solution of SDE $(5')$). Further, sufficient conditions for the convergence: for all δ, $0 < \delta < \frac{1}{2}$, $I_t^\delta z_t \to 0$ (rate of convergence), as $t \to \infty$, P_θ-a.s., can be obtained from Theorem 2.1 of [6] and, finally, to establish the asymptotic distribution of $I_t^{1/2}(\theta)z_t$, as $t \to \infty$ (under measure P_θ), one can use Theorem 3.1 of [6].

As an illustration, in the present work we restrict ourselves by results concerning the convergence: $z_t \to 0$, as $t \to \infty$, P_θ-a.s., rate of convergence, to avoid complex notation needed to state conditions for the validity of asymptotic expansion of $I_t^{1/2}z_t$ (see Eq. (3.1) from [6], with $R_t \xrightarrow{P_\theta} 0$, as $t \to \infty$.

Note that this convergence is equivalent to the strong consistency of recursive estimate $(\theta_t)_{t\geq 0}$ given by (4) or $(4')$, that is $\theta_t \to \theta$, as $t \to \infty$, P_θ-a.s.

Theorem 1. *Let the following conditions be satisfied: for all* $\theta \in R$

(i) $I^{-1}(\theta+u) < c(\theta)(1+u^2)$, $c(\theta)>0$;

(ii) *for each* ε, $\varepsilon > 0$,

$$\inf_{\varepsilon\le|u|\le\frac{1}{\varepsilon}} \left| uI^{-1}(\theta+u)\int_R \Phi(\theta+u,x)\left(1-\frac{Y(\theta+u,x)}{Y(\theta,x)}\right)\nu(\theta,dx)\right| > 0.$$

Then $z_t \to 0$, *as* $t\to\infty$, P_θ*-a.s.*

Proof. Condition (A) of Theorem 3.1 from [5] follows from the strong monotonicity of $Y(\theta,x)$ w.r.t. θ, for all $x\in R$.

Condition (B) of Theorem 3.1 of [5] is also satisfied, since

$$h_t(u) = \gamma_t^2(\theta+u)I(\theta+u) = \frac{1}{\lambda^2t^2I^2(\theta+u)}\,I(\theta+u) = I^{-1}(\theta+u)\,\frac{1}{\lambda^2t^2}\,.$$

Therefore

$$h_t(u) \le B_t(1+u^2),$$

with $B_t = \frac{c(\theta)}{\lambda^2}\,\frac{1}{t^2}$ and $\int_0^\infty B_s\,ds = \infty$.

Condition (I) of Theorem 3.1 from [5] is satisfied, since

$$\int_0^\infty \inf_{\varepsilon\le|u|\le\frac{1}{\varepsilon}} |uH_t(u)|\,dK_t = \int_0^\infty \inf\left| u\gamma_t(\theta+u)\int_R \Phi(\theta+u,x)\left(1-\frac{Y(\theta+u,x)}{Y(\theta)}\right)\nu(\theta,dx)\right|\lambda\,dt$$
$$= \inf_{\varepsilon\le|u|\le\frac{1}{\varepsilon}} \left| uI^{-1}(\theta+u)\int_R \Phi(\theta+u,x)\left(1-\frac{Y(\theta+u,x)}{Y(\theta)}\right)\nu(\theta,dx)\right| \int_0^\infty \frac{dt}{t} = \infty.$$

□

Below we assume that $z_t\to 0$, as $t\to\infty$, P_θ-a.s.

Denote

$$\beta_t = -\lim_{u\to 0}\frac{H_t(u)}{u}\,, \quad \beta_t(u) = \begin{cases} -\frac{H_t(u)}{u}, & u\neq 0,\\ \beta_t, & u=0.\end{cases}$$

Theorem 2. *Suppose that for each* δ, $0<\delta<1$, *the following conditions are satisfied:*

(i) $$\int_0^\infty \left[\frac{\delta}{t} - 2\beta_t(u)\right]^+ dt < \infty;$$

(ii) $$\int_0^\infty I_t^\delta(\theta)h_t(z_{t-},z_{t-})\,dt < \infty \ \ P_\theta\text{-a.s., where } h_t(u,v) = \frac{d\langle M(u),M(v)\rangle_t}{dK_t}.$$

Then $I_t^\delta(\theta)z_t^2 \to 0$, *as* $t\to\infty$, P_θ*-a.s.*

Proof. It is enough to note that in Theorem 2.1 of [6] we must take $\gamma_t = t$ and $r_t^\delta = \frac{\delta}{t}$. □

References

[1] J. Jacod and A. N. Shiryaev, *Limit theorems for stochastic processes*. Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], 288. Springer-Verlag, Berlin, 1987.

[2] N. Lazrieva, T. Sharia and T. Toronjadze, Semimartingale stochastic approximation procedure and recursive estimation. Martingale theory and its application. *J. Math. Sci. (N. Y.)* **153** (2008), no. 3, 211–261.

[3] N. Lazrieva and T. Toronjadze, Asymptotic theory of M-estimates in general statistical models. Parts I and II. *Centrum Voor Wiskunde en Information*, Reports BS-R9019-20 (1990).

[4] N. Lazrieva and T. Toronjadze, Recursive estimation procedures for one-dimensional parameter of statistical models associated with semimartingales. *Trans. A. Razmadze Math. Inst.* **171** (2017), no. 1, 57–75.

[5] N. Lazrieva, T. Sharia and T. Toronjadze, The Robbins-Monro type stochastic differential equations. I. Convergence of solutions. *Stochastics Stochastics Rep.* **61** (1997), no. 1-2, 67–87.

[6] N. Lazrieva, T. Sharia and T. Toronjadze, T. The Robbins-Monro type stochastic differential equations. II. Asymptotic behaviour of solutions. *Stochastics Stochastics Rep.* **75** (2003), no. 3, 153–180.

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