



# Essentials of Quantitative Finance

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## **PREFACE**

The purpose of current book is to introduce to the students the essence of quantitative finance by the question – answer form with references on the existing textbooks. The book can be used to review and refresh student’s knowledge in the field of quantitative finance.

Main subjects of the book include: World of Finance, General Finance I, General Finance II, Portfolio Theory, Fixed Income Mathematics, Derivatives, Bank Risk Theory, Real Options, Mastering Financial calculations.

Essentials of Quantitative Finance is analogous to Essentials of Business Administration textbook but is concentrated on the issues covered by quantitative finance and is dedicated to students with major in this field



# WORLD OF FINANCE

*Course book: Finance, 5th ed.; Groppelli, Ehsan Nikbakht; Barron's Inc. 2006*

*Course book: Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009*

*Course book: Corporate Finance, 6th ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003*

*Course book: Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008*

## **1. What is Finance? What do financial managers try to maximize, and what is their second objective?**

Finance is the application of a series of economic principles to maximize the wealth or overall value of a business. More specifically, maximizing the wealth of a firm means making the highest possible profits at the least risk. No one really knows when maximum wealth is achieved, though it is assumed to be the ultimate goal of every firm. One way of finding out the wealth of a firm is through the price of its common stock. When the price of a firm's shares increases, it is said that the wealth of the firm's shareholders has increased.

The primary objective of financial managers is to maximize the wealth of the firm or the price of the firm's stock. A secondary objective is to maximize earnings per share. Profit maximization is a short-term objective and is less important than maximizing the wealth of the firm. A firm can achieve high profits in the short run simply by cutting corners. In other words, managers can delay charging of some expenses, they can defer buying expensive albeit cost effective equipment, and they can lay off some of their most productive high-salaried workers. These short-sighted decisions can lower costs and raise profits temporarily. Furthermore, high profits may be obtained by investing in highly uncertain and risky projects. In the long run, these chancy projects can weaken the competitive position of a firm and lower the value of its stock. Therefore, attempts to maximize profits may prove inconsistent with the goal to maximize the wealth of the firm, which calls for attaining the highest expected return possible at any risk level.

Finance is part science, part art. Financial analysis provides a means of making flexible and correct investment decisions at the appropriate and most advantageous time. When financial managers succeed, they help improve the value of the firm's shares.

A good manager knows how to use the factors in arriving at final decisions, factors as: adapting to changes, manager as an agent. Financial managers are charged with the primary responsibility of maximizing the price of the firm's shares while holding risk at the lowest level possible. In order to achieve these goals, a manager must determine which investments will provide the highest profits at least risk. Once this decision is reached, the next step involves the selection of optimal ways to finance these investments. Planning to achieve the best results should be flexible, allowing for alternative strategies to replace existing plans should financial and economic developments diverge from an expected pattern.

## **2. What is the difference between real assets and financial assets?**

The material wealth of a society is ultimately determined by the productive capacity of its economy, that is, the goods and services its members can create. This capacity is a function of the real assets of the economy: the land, buildings, equipment, and knowledge that can be used to produce goods and services. In contrast to such real assets, there are financial assets such as stocks and bonds. Such securities are no more than sheets of paper or, more likely, computer entries and do not contribute directly to the productive capacity of the economy. Instead, these assets are the means by which individuals in well-developed economies hold their claims on real assets.

Financial assets are claims to the income generated by real assets (or claims on income from the government). While real assets generate net income to the economy, financial assets simply define the allocation of income or wealth among investors. The distinction between real and financial assets is apparent when we compare the balance sheet of households, with the composition of national wealth. Household wealth includes financial assets such as bank accounts, corporate stock, or bonds. However, these securities, which are financial assets of households, are liabilities of the issuers of the securities. Therefore, when we aggregate overall balance sheets, these claims cancel out, leaving only real assets as the net wealth of the economy. National wealth consists of structures, equipment, inventories of goods, and land.

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter1; pp.1-2*

## **3. List the categories of financial assets**

It is common to distinguish among three broad types of financial assets: debt, equity, and derivatives. Fixed-income or debt securities promise either a fixed stream of income or a stream of income that is determined according to a specified formula. For example, a corporate bond typically would promise that the bondholder will receive a fixed amount of interest each year. Other so-called floating-rate bonds promise payments that depend on current interest rates. For example, a bond may pay an interest rate that is fixed at two percentage points above the rate paid on U.S. Treasury bills. Unless the borrower is declared bankrupt, the payments on these securities are either fixed or determined by formula. For this reason, the investment performance of debt securities typically is least closely tied to the financial condition of the issuer.

Unlike debt securities, common stock, or equity, in a firm represents an ownership share in the corporation. Equity holders are not promised any particular payment. They receive any dividends the firm may pay and have prorated ownership in the real assets of the firm. If the firm is successful, the value of equity will increase; if not, it will decrease. The performance of equity investments, therefore, is tied directly to the success of the firm and its real assets. For this reason, equity investments tend to be riskier than investments in debt securities.

Finally, derivative securities such as options and futures contracts provide payoffs that are determined by the prices of other assets such as bond or stock prices.

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 1; pp.3-4*

#### **4. Describe money market instruments**

The money market is a subsector of the debt market. It consists of very short-term debt securities that are highly marketable. Many of these securities trade in large denominations and so are out of the reach of individual investors. Below are described some of them.

Treasury bills (T-bills, or just bills, for short) are the most marketable of all money market instruments. T-bills represent the simplest form of borrowing. The government raises money by selling bills to the public. Investors buy the bills at a discount from the stated maturity value. At the bill's maturity, the holder receives from the government a payment equal to the face value of the bill. The difference between the purchase price and the ultimate maturity value represents the investor's earnings. T-bills are highly liquid; that is, they are easily converted to cash and sold at low transaction cost and with little price risk.

A certificate of deposit (CD) is a time deposit with a bank. Time deposits may not be withdrawn on demand. The bank pays interest and principal to the depositor only at the end of the fixed term of the CD. CDs issued in denominations larger than \$100,000 are usually negotiable, however; that is, they can be sold to another investor if the owner needs to cash in the certificate before its maturity date.

The typical corporation is a net borrower of both long-term funds (for capital investments) and short-term funds (for working capital). Large, well-known companies often issue their own short-term unsecured debt notes directly to the public, rather than borrowing from banks. These notes are called commercial paper (CP).

Dealers in government securities use repurchase agreements, also called repos, or RPs, as a form of short-term, usually overnight, and borrowing. The dealer sells securities to an investor on an overnight basis, with an agreement to buy back those securities the next day at a slightly higher price. The increase in the price is the overnight interest. The dealer thus takes out a one-day loan from the investor. The securities serve as collateral for the loan.

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 2; pp.22-24*

#### **5. Describe long term debt securities instruments**

The bond market is composed of longer-term borrowing or debt instruments than those that trade in the money market. This market includes Treasury notes and bonds, corporate bonds, municipal bonds, etc.

These instruments are sometimes said to comprise the fixed-income capital market, because most of them promise either a fixed stream of income or stream of income that is determined according to a specified formula. In practice, these formulas can result in a flow of income that is far from fixed. Therefore, the term "fixed income" is probably not fully

appropriate. It is simpler and more straightforward to call these securities either debt instruments or bonds.

The U.S. government borrows funds in large part by selling Treasury notes and bonds. T-note maturities range up to 10 years, while T-bonds are issued with maturities ranging from 10 to 30 years.

Municipal bonds (“munis”) are issued by state and local governments. They are similar to Treasury and corporate bonds, except their interest income is exempt from federal income taxation. The interest income also is exempt from state and local taxation in the issuing state.

Corporate bonds are the means by which private firms borrow money directly from the public. These bonds are structured much like Treasury issues in that they typically pay semi-annual coupons over their lives and return the face value to the bondholder at maturity. Where they differ most importantly from Treasury bonds is in risk. Default risk is a real consideration in the purchase of corporate bonds. Corporate bonds sometimes come with options attached. Callable bonds give the firm the option to repurchase the bond from the holder at a stipulated call price. Convertible bonds give the bondholder the option to convert each bond into a stipulated number of shares of stock.

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 2; pp. 29-35*

## **6. What is equity stock? What types of stock exist and what is the difference between them?**

Common stocks, also known as equity securities, or equities, represent ownership shares in a corporation. Each share of common stock entitles its owners to one vote on any matters of corporate governance put to a vote at the corporation’s annual meeting and to a share in the financial benefits of ownership (e.g., the right to any dividends that the corporation may choose to distribute). A corporation is controlled by a board of directors elected by the shareholders. The members of the board are elected at the annual meeting. Shareholders who do not attend the annual meeting can vote by proxy, empowering another party to vote in their name.

The two most important characteristics of common stock as an investment are its residual claim and its limited liability features.

**Residual claim** means stockholders are the last in line of all those who have a claim on the assets and income of the corporation. In a liquidation of the firm’s assets, the shareholders have claim to what is left after paying all other claimants, such as the tax authorities, employees, suppliers, bondholders, and other creditors. In a going concern, shareholders have claim to the part of operating income left after interest and income taxes have been paid. Management either can pay this residual as cash dividends to shareholders or reinvest it in the business to increase the value of the shares.

**Limited liability** means that the most shareholders can lose in event of the failure of the corporation is their original investment. Shareholders are not like owners of unincorporated businesses, whose creditors can lay claim to the personal assets of the owner—such as



houses, cars, and furniture. In the event of the firm's bankruptcy, corporate stockholders at worst have worthless stock. They are not personally liable for the firm's obligations: Their liability is limited.

Preferred stock has features similar to both equity and debt. Like a bond, it promises to pay to its holder a fixed stream of income each year. In this sense, preferred stock is similar to an infinite-maturity bond, that is, perpetuity. It also resembles a bond in that it does not give the holder voting power regarding the firm's management. Preferred stock is an equity investment, however. The firm retains discretion to make the dividend payments to the preferred stockholders: It has no contractual obligation to pay those dividends. Instead, preferred dividends are usually cumulative; that is, unpaid dividends cumulate and must be paid in full before any dividends may be paid to holders of common stock. In contrast, the firm does have a contractual obligation to make timely interest payments on the debt. Failure to make these payments sets off corporate bankruptcy proceedings.

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 2; pp. 35-38*

## **7. Define primary and secondary markets**

When firms need to raise capital they may choose to sell or float securities. These new issues of stocks, bonds, or other securities typically are marketed to the public by investment bankers in what is called the primary market. Trading of already-issued securities among investors occurs in the secondary market. Trading in secondary markets does not affect the outstanding amount of securities; ownership is simply transferred from one investor to another.

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 3; pp. 54-55*

## **8. List Financial System Clients**

There appear to be three major players in the financial markets:

- Firms are net borrowers. They raise capital now to pay for investments in plant and equipment. The income generated by those real assets provides the returns to investors who purchase the securities issued by the firm.
- Households typically are net savers. They purchase the securities issued by firms that need to raise funds.
- Governments can be borrowers or lenders, depending on the relationship between tax revenue and government expenditures. Since World War II, the U.S. government typically has run budget deficits, meaning that its tax receipts have been less than its expenditures. The government, therefore, has had to borrow funds to cover its budget deficit. Issuance of Treasury bills, notes, and bonds is the major way that the

government borrows funds from the public. In contrast, in the latter part of the 1990s, the government enjoyed a budget surplus and was able to retire some outstanding debt.

Corporations and governments do not sell all or even most of their securities directly to individuals. For example, about half of all stock is held by large financial institutions such as pension funds, mutual funds, insurance companies, and banks. These financial institutions stand between the security issuer (the firm) and the ultimate owner of the security (the individual investor). For this reason, they are called financial intermediaries. Similarly, corporations do not directly market their securities to the public. Instead, they hire agents, called investment bankers, to represent them to the investing public.

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 1; pp. 11-12*

## **9. What advantage does a stock market provide to corporate investors? What stock markets do you know?**

The main advantage provided by stock markets is trading in corporate securities, the ability to raise capital. The equity shares of most of the large firms in the United States trade in organized auction markets. The largest such market is the New York Stock Exchange (NYSE), which accounts for more than 85 percent of all the shares traded in auction markets. Other auction exchanges include the American Stock Exchange (AMEX) and regional exchanges such as the Pacific Stock Exchange. In addition to the stock exchanges, there is a large OTC market for stocks. In 1971, the National Association of Securities Dealers (NASD) made available to dealers and brokers an electronic quotation system called NASDAQ (NASD Automated Quotation system, pronounced “naz-dak” and now spelled “Nasdaq”). There are roughly two times as many companies on NASDAQ as there are on NYSE, but they tend to be much smaller in size and trade less actively. There are exceptions, of course. Both Microsoft and Intel trade OTC, for example. Nonetheless, the total value of NASDAQ stocks is much less than the total value of NYSE stocks.

There are many large and important financial markets outside the United States, of course, and U.S. corporations are increasingly looking to these markets to raise cash. The Tokyo Stock Exchange and the London Stock Exchange (TSE and LSE, respectively) are two well-known examples. The fact that OTC markets have no physical location means that national borders do not present a great barrier, and there is now a huge international OTC debt market. Because of globalization, financial markets have reached the point where trading in many investments never stops; it just travels around the world.

*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 1, pp. 19-20*

## 10. What role do global financial markets and derivatives play in modern world?

Global financial markets are the economic conduits through which companies and individuals finance a variety of initiatives, such as funding startups as well as corporate mergers and acquisitions. Also known as a securities exchange or stock market, an international financial market allows diverse pools of investors to buy and sell shares of equity.

Corporate finance enables an organization to evaluate its funding needs and take a peek at corporate capital plans. A firm's capital structure provides insight into various sources of funds that it relies on to operate and thrive. Having access to cash is cardinal, especially for firms with moribund sales numbers. Specifically, global financial markets play a central role in allowing businesses to tap a vast pool of liquidity, a strategic tool that top leadership needs to ensure long-term economic success. Organizations raise funds through financial markets by issuing shares of equity, or stocks, and corporate bonds. Buyers of equity, also called stockholders, receive periodic dividends and make profits when share prices rise. Bondholders receive periodic interest payments during the bond term and recover the principal loan amount when the bond matures.

Global financial markets help individuals and companies engage in investment activities, enabling exchange participants to fulfill their short-term and long-term financial objectives. Individual investors buy and sell securities, such as stocks and bonds, for many reasons, including retirement and short-term speculation. Companies also engage in security transactions to generate additional revenue. This complementary revenue can be handy to reduce a company's red ink, especially if the firm's primary operations show a sluggish performance.

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 2*

## 11. How are orders executed on NYSE?

The New York Stock Exchange is by far the largest stock exchange in the United States. Shares of about 2,800 firms trade there, with a combined market capitalization in early 2008 of around \$15 trillion. Daily trading on the NYSE averaged 2.1 billion shares in 2007, with a dollar value of approximately \$87 billion. An investor who wishes to trade shares on the NYSE places an order with a brokerage firm, which either sends the order to the floor of the exchange via computer network or contacts its broker on the floor of the exchange to "work" the order. Smaller orders are almost always sent electronically for automatic execution, while larger orders that may require negotiation or judgment are more likely sent to a floor broker. A floor broker sent a trade order takes the order to the specialist's post. At the post is a monitor called the Display Book that presents current offers from interested traders to buy or sell given numbers of shares at various prices. The specialist can cross the trade with that of another broker if that is feasible or match the trade using its own inventory

of shares. Brokers might also seek out traders willing to take the other side of a trade at a price better than those currently appearing in the Display Book. If they can do so, they will bring the agreed-upon trade to the specialist for final execution.

Brokers must purchase the right to trade on the floor of the NYSE. Originally, the NYSE was organized as a not-for-profit company owned by its members or “seat holders.” For example, in 2005 there were 1,366 seat-holding members of the NYSE. Each seat entitled its owner to place a broker on the floor of the exchange, where he or she could execute trades. Member firms could charge investors for executing trades on their behalf, which made a seat a valuable asset. The commissions that members might earn by trading on behalf of clients determined the market value of the seats, which were bought and sold like any other asset.

Table below gives some of the initial listing requirements for the NYSE. These requirements ensure that a firm is of significant trading interest before the NYSE will allocate facilities for it to be traded on the floor of the exchange. If a listed company suffers a decline and fails to meet the listing criteria, may be delisted.

Minimum annual pretax income in previous 2 years	\$ 2,000,000
Revenue	\$ 75,000,000
Market value of publicly held stock	\$100,000,000
Shares publicly held	1,100,000
Number of holders of 100 shares or more	2,200

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 3; pp. 63-64*

## 12. What was originally NASDAQ meant to be, how is it nowadays and how many level users does it have?

While any security can be traded in the over-the-counter network of security brokers and dealers, not all securities were included in the original National Association of Security Dealers Automated Quotations System. That system, now called the NASDAQ Stock Market, lists about 3,200 firms and offers three listing options. The NASDAQ Global Select Market lists over 1,000 of the largest, most actively traded firms, the NASDAQ Global Market is for the next tier of firms, and the NASDAQ Capital Market is the third tier of listed firms.

Some of the requirements for initial listing are presented in table below:

	NASDAQ Global Market	NASDAQ Capital Market
Shareholders' equity	\$15 million	\$5 million
Shares in public hands	1.1 million	1 million
Market value of publicly traded shares	\$8 million	\$5 million
Minimum price of stock	\$5	\$4
Pretax income	\$1 million	\$750,000
Shareholders	400	300

NASDAQ has three levels of subscribers. The highest, level 3 subscribers are for firms dealing, or “making markets,” in securities. These market makers maintain inventories of a security and constantly stand ready to buy or sell these shares from or to the public at the quoted bid and ask prices. They earn profits from the spread between the bids and ask prices.

Level 3 subscribers may enter the bid and ask prices at which they are willing to buy or sell stocks into the computer network and may update these quotes as desired.

Level 2 subscribers receive all bids and ask quotes, but they cannot enter their own quotes. These subscribers tend to be brokerage firms that execute trades for clients but do not actively deal in the stocks on their own account. Brokers when buying or selling shares trade with the market maker (a level 3 subscriber) displaying the best price quote.

Level 1 subscribers receive only the inside quotes (i.e., the highest bid and lowest ask prices on each stock). Level 1 subscribers tend to be investors who are not actively buying and selling securities but want information on current prices.

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 3; pp. 62-63*

### **13. List and describe the stock indexes known to you.**

Indexes are taken as a measure of the performance of the stock market. The Dow Jones Industrial Average (DJIA) of 30 large, “blue-chip” corporations has been computed since 1896. Its long history probably accounts for its pre-eminence in the public mind. Originally, the DJIA was calculated as the simple average of the stocks included in the index. So, if there were 30 stocks in the index, one would add up the value of the 30 stocks and divide by 30. The percentage change in the DJIA would then be the percentage change in the average price of the 30 shares. The amount of money invested in each company represented in the portfolio is proportional to that company’s share price, so the Dow is called a price-weighted average. Because the Dow Jones averages are based on small numbers of firms, care must be taken to ensure that they are representative of the broad market. As a result, the composition of the average is changed every so often to reflect changes in the economy. In the same way that the divisor is updated for stock splits, if one firm is dropped from the average and another firm with a different price is added, the divisor has to be updated to leave the average unchanged by the substitution.

The Standard & Poor’s Composite 500 (S&P 500) stock index represents an improvement over the Dow Jones Averages in two ways. First, it is a more broadly based index of 500 firms. Second, it is a market value-weighted index. The S&P 500 is computed by calculating the total market value of the 500 firms in the index and the total market value of those firms on the previous day of trading. The percentage increase in the total market value from one day to the next represents the increase in the index. The rate of return of the index equals the rate of return that would be earned by an investor holding a portfolio of all 500 firms in the index in proportion to their market value, except that the index does not reflect cash dividends paid by those firms.

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 2; pp. 38-45*

#### **14. Why are financial intermediaries important?**

In an economy without FIs, the level of fund flows between household savers and the corporate sectors is likely to be quite low. There are several reasons for this. Once they have lent money to a firm by buying its financial claims, households need to monitor, or check, the actions of that firm. They must be sure that the firm's management neither absconds with nor wastes the funds on any projects with low or negative net present values. Such monitoring actions are extremely costly for any given household because they require considerable time and expense to collect sufficiently high-quality information relative to the size of the average household saver's investments. Given this, it is likely that each household would prefer to leave the monitoring to others; in the end, little or no monitoring would be done. The resulting lack of monitoring would reduce the attractiveness and increase the risk of investing in corporate debt and equity.

The relatively long-term nature of corporate equity and debt, and the lack of a secondary market in which households can sell these securities, creates a second disincentive for household investors to hold the direct financial claims issued by corporations. Specifically, given the choice between holding cash and holding long-term securities, households may well choose to hold cash for liquidity reasons, especially if they plan to use savings to finance consumption expenditures in the near future.

Finally, even if financial markets existed (without FIs to operate them) to provide liquidity services by allowing households to trade corporate debt and equity securities among themselves, investors also face a price risk on sale of securities, and the secondary market trading of securities involves various transaction costs. That is, the price at which household investors can sell securities on secondary markets such as the New York Stock Exchange may well differ from the price they initially paid for the securities.

Because of (1) monitoring costs, (2) liquidity costs, and (3) price risk, the average household saver may view direct investment in corporate securities as an unattractive proposition and prefer either not to save or to save in the form of cash.

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett;  
McGraw-Hill, 2008; Chapter 1, pp 3-8*

#### **15. What is financial planning?**

Financial planning, a crucial part of financial management, includes the making of daily decisions to help the firm meet its cash requirements. It requires close attention to intermediate changes in business activity. Analysis of the business cycle helps the financial manager keep financing costs low and avoid excesses in inventories and capacity. Proper timing of business activity leads to better production and inventory decisions to meet changes in economic activity.

Furthermore, financial planning involves proper timing of investments in order to avoid overexpansion and inefficient use of resources. Optimal use of available funds means exploring different options and selecting those that provide the greatest overall value. It also means adopting effective ways of determining how much to borrow in order to reduce financing risks.

Long-range plans must be developed to give proper direction to research and development and to make sound capital expenditure decisions. If properly managed, the lower risk and higher expected returns that result will be recognized by investors, who are likely to raise their assessment of the value of the firm. By adopting these approaches, financial managers have a better-than-even chance of maintaining a healthy firm and maximizing shareholders' wealth.

*Finance, 5th ed.; Groppelli, Ehsan Nikbakht; Barron's Inc. 2006, Chapter 1, p. 2*

## **16. Describe two major activities of financial institution**

### *FI's Function as Brokers*

The first function is the brokerage function. When acting as a pure broker, an FI acts as an agent for the saver by providing information and transaction services. For example, full-service securities firms (e.g., Merrill Lynch) carry out investment research and make investment recommendations for their retail (or household) clients as well as conducting the purchase or sale of securities for commission or fees. Discount brokers (e.g., Charles Schwab) carry out the purchase or sale of securities at better prices and with greater efficiency than household savers could achieve by trading on their own. This efficiency results in reduced costs of trading, or economies of scale. Independent insurance brokers identify the best types of insurance policies household savers can buy to fit their savings and retirement plans. In fulfilling a brokerage function, the FI plays an extremely important role by reducing transaction and information costs or imperfections between households and corporations. Thus, the FI encourages a higher rate of savings than would otherwise exist.

### *FI's Function as Asset Transformers*

The second function is the asset-transformation function. In acting as an asset transformer, the FI issues financial claims that are far more attractive to household savers than the claims directly issued by corporations. That is, for many households, the financial claims issued by FIs dominate those issued directly by corporations as a result of lower monitoring costs, lower liquidity costs, and lower price risk. In acting as asset transformers, FIs purchase the financial claims issued by corporations—equities, bonds, and other debt claims called primary securities—and finance these purchases by selling financial claims to household investors and other sectors in the form of deposits, insurance policies, and so on.

The financial claims of FIs may be considered secondary securities because these assets are backed by the primary securities issued by commercial corporations that in turn invest in

real assets. Specifically, FIs are independent market parties that create financial products whose value added to their clients is the transformation of financial risk.

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 1, pp 3-8*

## 17. Name and describe different types of banks

The necessity for the variety among these banks is because each bank is specialized in their own field. Each bank has its own principles and policies. Different rates of interests are also noted among these banks. All these banks are listed as below:

**Commercial Banks** – These banks collect money from people in various sectors and give the same as a loan to business men and make profits in interests these business men pay. Since the loan is large the interest rates are also high.

**Industrial Development Bank** – these banks are committed towards enhancing the growth of industries by providing loans for a very long period of time. This is vital for the long term growth of the industries

**Federal or National Banks** – these banks control the principles and policies of other banks across the country. These banks are managed and run by the government. This bank provides benchmarks which other banks should follow.

**Credit Unions** – they act just like a co-operative bank except that they provide services to only one employee union in the community. Low interest rates and easy installment paybacks are features of this bank

**Investment Banks** – these banks are pertinent to large organization's investment ventures across the industry. They provide advice in the investments and promote corporate transactions.

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 2*

## 18. Describe the specification of Credit Unions

Generally there are distinguished two types of financial institutions: Depository (Banks, Credit unions) and non-depository (Insurance companies). Commercial banks make up the largest group of depository institutions measured by asset size. They perform functions similar to those of savings institutions and credit unions; that is, they accept deposits (liabilities) and make loans (assets). However, they differ in their composition of assets and liabilities, which are much more varied.

Credit unions (CUs) are non-profit depository institutions mutually organized and owned by their members (depositors). Credit unions (CUs) were first established in the United States in the early 1900s as self-help organizations intended to alleviate widespread poverty.



Members were expected to deposit their savings in the CU, and these funds were lent only to other members. Unlike commercial banks and savings institutions, CUs are prohibited from serving the general public. Rather, in organizing a credit union, members are required to have a common bond of occupation (e.g., police CUs) or association (e.g., university-affiliated CUs), or to cover a well-defined neighbourhood, community, or rural district. CUs may, however, have multiple groups with more than one type of membership. The primary objective of credit unions is to satisfy the depository and lending needs of their members. CU member deposits (shares) are used to provide loans to other members in need of funds. Any earnings from these loans are used to pay higher rates on member deposits, charge lower rates on member loans, or attract new members to the CU. Because credit unions do not issue common stock, the members are legally the owners of a CU. Also, because credit unions are non-profit organizations, their net income is not taxed and they are not subject to the local investment requirements. This tax-exempt status allows CUs to offer higher rates on deposits, and charge lower rates on some types of loans, than do banks and savings institutions.

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 2, pp. 29-47, 53-58*

## **19. Describe specifications of Insurance Companies**

The primary function of insurance companies is to protect individuals and corporations (policyholders) from adverse events. By accepting premiums, insurance companies promise policyholders compensation if certain specified events occur. These policies represent financial liabilities to the insurance company. With the premiums collected, insurance companies invest in financial securities such as corporate bonds and stocks.

Life insurance allows individuals and their beneficiaries to protect against losses in income through premature death or retirement. By pooling risks, life insurance transfers income-related uncertainties from the insured individual to a group. One problem that naturally faces life insurance companies (as well as property-casualty insurers) is the so-called adverse selection problem. Adverse selection is a problem in that customers who apply for insurance policies are more likely to be those most in need of insurance (i.e., someone with chronic health problems is more likely to purchase a life insurance policy than someone in perfect health). Thus, in calculating the probability of having to pay out on an insurance contract and, in turn, determining the insurance premium to charge, insurance companies' use of health (and other) statistics representing the overall population may not be appropriate (since the insurance company's pool of customers is more prone to health problems than the overall population). Insurance companies deal with the adverse selection problem by establishing different pools of the population based on health and related characteristics (such as income). By altering the pool used to determine the probability of losses to a particular customer's health characteristics, the insurance company can more accurately determine the probability of having to pay out on a policy and can adjust the insurance premium accordingly.

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 3, pp. 66-71, 75-76*

## 20. Describe concept of Frequency vs Severity problem

In general, loss rates are more predictable on low severity, high-frequency lines than they are on high-severity, low-frequency lines. For example, losses in fire, auto, and homeowners peril lines tend to involve events expected to occur with a high frequency and to be independently distributed across any pool of the insured. Furthermore, the dollar loss on each event in the insured pool tends to be relatively small. Applying the law of large numbers, insurers can estimate the expected loss potential of such lines—the frequency of loss times the size of the loss (severity of loss)—within quite small probability bounds. Other lines, such as earthquake, hurricane, and financial guaranty insurance, tend to insure very low-probability (frequency) events. Here the probabilities are not always stationary, the individual risks in the insured pool are not independent, and the severity of the loss could be enormous. This means that estimating expected loss rates (frequency times severity) is extremely difficult in these coverage areas. For example, even with the new federal terrorism insurance program introduced in 2002, coverage for high-profile buildings in big cities, as well as other properties considered potential targets, remains expensive. Under the 2002 federal program, the government is responsible for 90 percent of insurance industry losses that arise from any future terrorist incidents that exceed a minimum amount. The government's losses are capped at \$100 billion per year. Each insurer has a maximum amount it would pay before federal aid kicks in.

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 3, pp. 77-78*

## 21. Describe major activities of Investment Banks

Investment banking involves the raising of debt and equity securities for corporations or governments. This includes the origination, underwriting, and placement of securities in money and capital markets for corporate or government issuers. Investment banking also includes corporate finance activities such as advising on mergers and acquisitions (M&A), as well as advising on the restructuring of existing corporations. The firms in the industry can be divided along a number of dimensions. First are the largest firms, the so-called national full-line firms, which service both retail customers (especially in acting as **broker-dealers**, thus assisting in the trading of existing securities) and corporate customers (such as **underwriting**, thus assisting in the issue of new securities). Investment banks engage in as many as seven key activity areas:

***Investing***

Investing involves managing not only pools of assets such as closed- and open end mutual funds but also pension funds in competition with life insurance companies. Securities firms can manage such funds either as agents for other investors or as principals for themselves.

***Investment Banking***

Investment banking refers to activities related to underwriting and distributing new issues of debt and equity. New issues can be either primary, the first-time issues of companies (sometimes called IPOs [initial public offerings]), or secondary issues (the new issues of seasoned firms whose debt or equity is already trading).

***Market Making***

Market making involves creating a secondary market in an asset by a securities firm or investment bank. Thus, in addition to being primary dealers in government securities and underwriters of corporate bonds and equities, investment bankers make a secondary market in these instruments. Market making can involve either agency or principal transactions. Agency transactions are two-way transactions on behalf of customers, for example, acting as a stockbroker or dealer for a fee or commission.

***Trading***

Trading is closely related to the market-making activities just described, where a trader takes an active net position in an underlying instrument or asset. There are at least four types of trading activities:

Position trading involves purchasing large blocks of securities on the expectation of a favorable price move. Such positions also facilitate the smooth functioning of the secondary markets in such securities. In most cases, these trades are held in inventory for a period of time, either after or prior to the trade.

- Pure arbitrage entails buying an asset in one market at one price and selling it immediately in another market at a higher price. Pure arbitrage “locks in” profits are available in the market. This profit position usually occurs with no equity investment, the use of only very short-term borrowed funds, and reduced transaction costs for securities firms.
- Risk arbitrage involves buying blocks of securities in anticipation of some information release, such as a merger or takeover announcement or a Federal Reserve interest rate announcement.
- Program trading is defined by the NYSE as the simultaneous buying and selling of a portfolio of at least 15 different stocks valued at more than \$1 million, using computer programs to initiate such trades. Program trading is often associated with seeking a risk arbitrage between a cash market price (e.g., the Standard & Poor’s 500 Stock Market Index) and the futures market price of that instrument.

***Cash Management***

Investment banks offer bank deposit-like cash management accounts (CMAs) to individual investors

***Mergers and Acquisitions***

Investment banks are frequently involved in providing advice or assisting in mergers and acquisitions. For example, they will assist in finding merger partners, underwriting new securities to be issued by the merged firms, assessing the value of target firms, recommending terms of the merger agreement, and even helping target firms prevent a merger.

***Back-Office and Other Service Functions***

These functions include custody and escrow services, clearance and settlement services, and research and other advisory services—for example, giving advice on divestitures and asset sales. In addition, investment banks are making increasing inroads into traditional bank service areas such as small business lending and the trading of loans.

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 4*

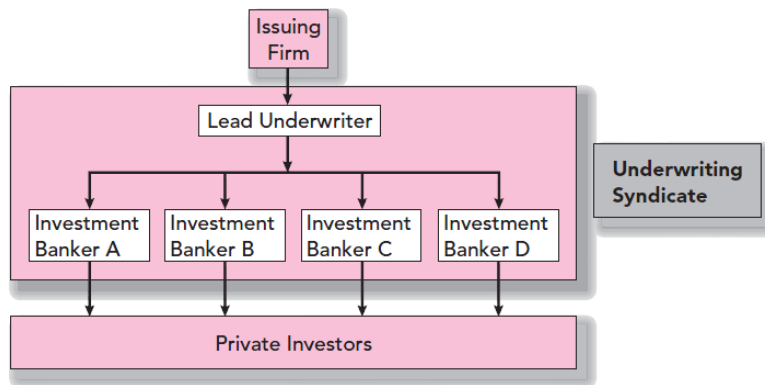
**22. Describe two ways of raising funds via selling stock**

There are two types of primary market issues of common stock: Initial public offerings or Private placement.

Primary offerings also can be sold in a private placement rather than a public offering. In this case, the firm (using an investment banker) sells shares directly to a small group of institutional or wealthy investors. Private placements can be far cheaper than public offerings. On the other hand, because private placements are not made available to the general public, they generally will be less suited for very large offerings. Moreover, private placements do not trade in secondary markets like stock exchanges. This greatly reduces their liquidity and presumably reduces the prices that investors will pay for the issue.

Investment bankers manage the issuance of new securities to the public. Public offerings of both stocks and bonds typically are marketed by investment bankers who in this role are called underwriters. More than one investment banker usually markets the securities. A lead firm forms an underwriting syndicate of other investment bankers to share the responsibility for the stock issue. Investment bankers advise the firm regarding the terms on which it should attempt to sell these securities. A preliminary registration statement must be filed with the Securities and Exchange Commission (SEC), describing the issue and the prospects of the company.

In a typical underwriting arrangement, the investment bankers purchase the securities from the issuing company and then resell them to the public. The issuing firm sells the securities to the underwriting syndicate for the public offering price less a spread that serves as compensation to the underwriters. This procedure is called a firm commitment. In addition to the spread, the investment banker also may receive shares of common stock or other securities of the firm.



*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 2; p. 55*

### 23. Describe major activities of Mutual Fund. What types of Mutual Fund do you know?

Mutual funds and hedge funds are financial intermediaries that pool the financial resources of individuals and companies and invest in diversified portfolios of assets. An open-ended mutual fund (the major type of mutual fund) continuously stands ready to sell new shares to investors and to redeem outstanding shares on demand at their fair market value. Thus, these funds provide opportunities for small investors to invest in financial securities and diversify risk. Mutual funds are also able to generate greater economies of scale by incurring lower transaction costs and commissions than are incurred when individual investors buy securities directly.

The mutual fund industry is usually divided into two sectors: short-term funds and long-term funds. Long-term funds include bond funds (comprised of fixed income securities with a maturity of over one year), equity funds (comprised of common and preferred stock securities), and hybrid funds (comprised of both bond and stock securities). Short-term funds include taxable money market mutual funds (MMMFs) and tax-exempt money market mutual funds.

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 5*

### 24. Describe major activities of Hedge Fund. What types of hedge fund do you know?

Hedge funds are a type of investment pool that solicit funds from (wealthy) individuals and other investors (e.g., commercial banks) and invest these funds on their behalf. Hedge

funds are similar to mutual funds in that they are pooled investment vehicles that accept investors' money and generally invest it on a collective basis.

Hedge funds are also not subject to the numerous regulations that apply to mutual funds for the protection of individuals, such as regulations requiring a certain degree of liquidity, regulations requiring that mutual fund shares be redeemable at any time, regulations protecting against conflicts of interest, regulations to ensure fairness in the pricing of funds shares, disclosure regulations, and regulations limiting the use of leverage. Further, hedge funds do not have to disclose their activities to third parties. Thus, they offer a high degree of privacy for their investors.

Most hedge funds are highly specialized, relying on the specific expertise of the fund manager(s) to produce a profit. Hedge fund managers follow a variety of investment strategies, some of which use leverage and derivatives, while others use more conservative strategies and involve little or no leverage. Generally, hedge funds are set up with specific parameters so that investors can forecast a risk-return profile.

More risky funds seek high returns using leverage, typically investing based on anticipated events.

Moderate Risk Funds have moderate exposure to market risk, typically favoring a longer-term investment strategy.

Low Risk Funds strive for moderate, consistent returns with low risk.

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 5; pp.143-149*

## **25. Describe the concept of Finance Companies. What types of Finance Companies do you know?**

The primary function of finance companies is to make loans to both individuals and corporations. The services provided by finance companies include consumer lending, business lending, and mortgage financing. Some of their loans are similar to commercial bank loans, such as consumer and auto loans, but others are more specialized. Finance companies differ from banks in that they do not accept deposits but instead rely on short- and long-term debt as a source of funds. Additionally, finance companies often lend to customers commercial banks find too risky. This difference can lead to losses and even failure if the high risk does not pay off.

The three major types of finance companies are (1) sales finance institutions, (2) personal credit institutions, and (3) business credit institutions. Sales finance institutions (e.g., Ford Motor Credit and Sears Roebuck Acceptance Corp.) specialize in making loans to the customers of a particular retailer or manufacturer. Because sales finance institutions can frequently process loans faster and more conveniently (generally at the location of purchase) than depository institutions, this sector of the industry competes directly with depository institutions for consumer loans. Personal credit institutions specialize in making installment and other loans to consumers. Personal credit institutions will make loans to customers that

depository institutions find too risky to lend to (due to low income or a bad credit history). These institutions compensate for the additional risk by charging higher interest rates than depository institutions and/or accepting collateral (e.g., used cars) that depository institutions do not find acceptable. Business credit institutions are companies that provide financing to corporations, especially through equipment leasing and factoring, in which the finance company purchases accounts receivable from corporate customers.

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 6*

## **26. List three types of efficient market hypothesis (EMH)**

It is common to distinguish among three versions of the EMH: the weak, semi-strong, and strong forms of the hypothesis. These versions differ by their notions of what is meant by the term “all available information.”

The weak-form hypothesis asserts that stock prices already reflect all information that can be derived by examining market trading data such as the history of past prices, trading volume, or short interest. This version of the hypothesis implies that trend analysis is fruitless. Past stock price data are publicly available and virtually costless to obtain. The weak-form hypothesis holds that if such data ever conveyed reliable signals about future performance, all investors already would have learned to exploit the signals. Ultimately, the signals lose their value as they become widely known because a buy signal, for instance, would result in an immediate price increase.

The semi-strongform hypothesis states that all publicly available information regarding the prospects of a firm already must be reflected in the stock price. Such information includes, in addition to past prices, fundamental data on the firm’s product line, quality of management, balance sheet composition, patents held, earning forecasts, and accounting practices. Again, if investors have access to such information from publicly available sources, one would expect it to be reflected in stock prices.

Finally, the strong-form version of the efficient market hypothesis states that stock prices reflect all information relevant to the firm, even including information available only to company insiders. This version of the hypothesis is quite extreme. Few would argue with the proposition that corporate officers have access to pertinent information long enough before public release to enable them to profit from trading on that information. Indeed, much of the activity of the Securities and Exchange Commission is directed toward preventing insiders from profiting by exploiting their privileged situation.

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 11; pp. 348-349*

## GENERAL FINANCE I

*Course book: Finance, 5th ed.; Groppelli, Ehsan Nikbakht; Barron's Inc. 2006*

*Course book: Corporate Finance, 6th ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003*

*Course book: Fundamentals of Corporate Finance, 6th ed., Ross, Westerfield, Jordan; McGraw-Hill, 2003*

### **27. State the importance of accounting for financial managers**

Financial managers rely on accountants to prepare income and balance sheet statements that provide information on the profitability and the financial status of a firm. The Financial Accounting Standards Board requires firms to report a current Statement of Cash Flows. This statement provides a detailed analysis of the way cash is generated and traces how cash is utilized in the conduct of all phases of a business. As a result, this statement supplies another important financial tool to managers who seek to control and understand the external factors and internal policies that can influence the cash flows of the firm. Financial statements help managers to make business decisions involving the best use of cash, the attainment of efficient operations, the optimal allocation of funds among assets, and the effective financing of investment and operations. The interpretation of financial statements is achieved partly by using financial ratios, pro forma statements, sources and uses of funds and cash budgets.

It should be pointed out that the managers of a firm are supplied with more detailed statistical information than appears in published financial statements. These data are especially important in developing cash flow concepts for evaluating the relative merits of different investment projects. This information permits managers to determine incremental cash flows (an approach that looks at the net returns a given project generates in comparison with alternative investments), thus enabling them to make more accurate assessments of the profit abilities of specific investments. It is the responsibility of managers to direct their accountants to prepare internal statements that include this information so that they can make the best investment decisions possible.

The primary objectives of financial accounting are to provide information that is useful in making investment and credit decisions; in assessing the amount, timing, and uncertainty of future cash flows; and in learning about the enterprise's economic resources, claims to resources, and changes in claims to resources.

The importance of accounting for investors is huge! Accounting is the language of business. That is what management uses to 'communicate' performance to the outside world. However management often wants to communicate only favorable news and this communication is often 'mis' communication.

*Finance, 5th ed.; Groppelli, Ehsan Nikbakht; Barron's Inc. 2006, Chapter 1, pp.7-8*



## 28. What are the three types of business organizations? Define them

The three types of business organizations are proprietorships, partnerships, and corporations. A proprietorship is the oldest form of organization for a business owned by only one individual. Anyone with money can buy basic working tools and start a proprietorship to produce goods or services thought to be marketable.

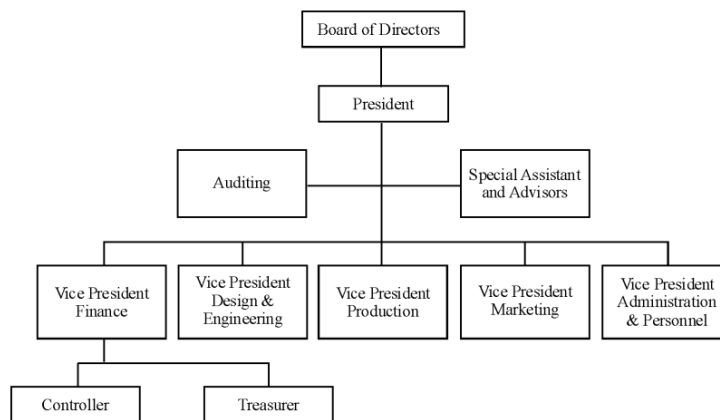
A main advantage of a proprietorship is its easy formation process. The formation of a proprietorship doesn't require the approval of any regulatory agency. Once the working conditions of the business are present the sole proprietorship is in existence. (The only exception is that certain professions require a license in order to practice.) Another advantage is the straightforward taxation method used for proprietorships: The proprietor's income is simply included on the owner's individual tax return each year.

A main disadvantage of this form of organization is that the owner is responsible for the entire liability of the proprietorship. Since the owner has unlimited liability, personal properties that are not used in the business may be lost to creditors.

Another disadvantage is that a proprietorship can't use organized capital markets — such as stock and bond markets — to raise needed capital. A proprietorship therefore has limited opportunities for growth, because its capital can be expanded only so far. Capital, in the form of either debt or equity, is the means to buy assets and expand a company.

A partnership is a form of business organization in which two or more individuals are the owners. A partnership can be viewed as a proprietorship with more than one owner. There are two kinds of partners: general partners and limited partners. General partners have unlimited liability in running a business, but limited partners are liable only up to the amount of their investments or for a specified amount of money. In a general partnership all partners have unlimited liability. In a limited partnership there is at least one limited partner in the business.

The third form of organization is the corporation, which, in terms of dollars, dominates today's business world. A corporation can be formed by a person or a group of persons. The "personality" of the corporation, under the law, is totally separate from its owners. Precisely speaking, a corporation is a "legal entity"; therefore, the corporation, rather than the owners, is responsible for paying all debts.



A Typical Corporate Organization

Because of the legal status of a corporation, the owners have limited liability and can't lose more than their invested money. Unlike a proprietor, the owners of a corporation do not have to withdraw from their personal savings or sell their personal belongings to satisfy creditors if the corporation goes bankrupt. An owner of a corporation is called a stockholder or shareholder.

Advantages of a corporate form include limited liability, unlimited life, separation of ownership and management (ability to own shares in several companies without having to work for all of them), ease of transferring ownership and ease of raising capital.

Disadvantages include separation of ownership and management (agency costs) and double taxation (income tax and dividend tax).

*Finance, 5th ed.; Gropelli, EhsanNikbakht; Barron's Inc. 2006, Chapter 2, pp. 1-3*

## **29. What are the advantages and disadvantages of organizing a business as a corporation?**

Starting a corporation is more complicated than starting a proprietorship or partnership. The incorporators must prepare articles of incorporation and a set of bylaws. The articles of incorporation must include the following:

- Name of the corporation.
- Intended life of the corporation (it may be forever).
- Business purpose.
- Number of shares of stock that the corporation is authorized to issue, with a statement of limitations and rights of different classes of shares.
- Nature of the rights granted to shareholders.
- Number of members of the initial board of directors.

The bylaws are the rules to be used by the corporation to regulate its own existence, and they concern its shareholders, directors, and officers. Bylaws range from the briefest possible statement of rules for the corporation's management to hundreds of pages of text.

In closely held corporations with few shareholders there may be a large overlap among the shareholders, the directors, and the top management. However, in larger corporations the shareholders, directors, and the top management are likely to be distinct groups.

The potential separation of ownership from management gives the corporation several advantages over proprietorships and partnerships:

- Because ownership in a corporation is represented by shares of stock, ownership can be readily transferred to new owners. Because the corporation exists independently of those who own its shares, there is no limit to the transferability of shares as there is in partnerships.

- The corporation has unlimited life. Because the corporation is separate from its owners, the death or withdrawal of an owner does not affect its legal existence. The corporation can continue on after the original owners have withdrawn.
- The shareholders' liability is limited to the amount invested in the ownership shares. For example, if a shareholder purchased \$1,000 in shares of a corporation, the potential loss would be \$1,000. In a partnership, a general partner with a \$1,000 contribution could lose the \$1,000 plus any other indebtedness of the partnership.

Limited liability, ease of ownership transfer, and perpetual succession are the major advantages of the corporation form of business organization. These give the corporation an enhanced ability to raise cash. There is, however, one great disadvantage to incorporation. The federal government taxes corporate income. This tax is in addition to the personal income tax that shareholders pay on dividend income they receive. This is double taxation for shareholders when compared to taxation on proprietorships and partnerships.

*Corporate Finance, 6<sup>th</sup> ed, Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 1, p. 13*

### **30. List several examples of agency problem and the costs associated with them. What forms of managerial compensation do you know?**

At one time or another, most people have had occasion to hire agents to take care of a specific matter. In doing so, responsibility is delegated to another person. For example, when suing for damages, individuals may represent themselves or may hire a lawyer to plead their case in court. As an agent, the lawyer is given the assignment to get the highest possible award. And so it is with stockholders when they delegate the task of running a firm to a financial manager, who, though not strictly correct, legally may be thought of as an agent of the company. Obviously, the goal is to achieve the highest value of a share of stock for the firm's owners. But there are no standard rules that indicate which course of action should be followed by managers to achieve this. The ultimate guideline is how investors perceive the actions of managers.

In general, managers should seek to use sound investment policies that minimize risk. However, some managers interpret their mission differently. As agents, they envision their role as one of avoiding big mistakes. As a result, these agents may overlook good opportunities with acceptable levels of risk. This conflict between agent and stockholders is unlikely to produce the best results.

There are no easy answers to ensure compatibility between agents and their stockholders. It is up to the stockholders-acting through the board of directors to hire the right managers and to make sure they are properly compensated. This means meeting the market price to attract the right talent. Offering stock to these managers also helps ensure that they will seek to maximize the value of the firm's shares.

In any event, capable managers have the right judgment and instincts to know what policies to implement and when to implement them. They know when to raise funds and how

to control assets. These correct decisions are translated into favorable signals to investors, usually resulting in a higher valuation of the firm's common stock.

*Finance, 5th ed.; Groppelli, Ehsan Nikbakht, Barron's Inc. 2006, Chapter 1, pp. 2-3*

### **31. What is time value of money? List and explain two main reasons why money changes its value through time**

The old saying "A bird in the hand is worth two in the bush" makes a great deal of sense when applied to finance. In monetary terms, it means that cash today is worth more than cash in the future. In other words, the value of money changes over time. Investors have a natural preference for cash now rather than later, so they can increase its value. This, of course, is a major goal of a financial manager. Aside from this basic reason why cash now is worth more than cash later, you should also be aware of factors that decrease the value of money over time. Two important reasons why the value of money decreases progressively over time are as follows:

1. Risk
2. Preference for Liquidity

Risk, or uncertainty about the future, also causes a decline in the value of money. Because the future is uncertain, risk increases with time. Most people wish to avoid risk, so they value cash today more than the promise of cash in the future. Most people are willing to give up cash for promised cash only if properly compensated for the risk they are asked to take.

No one can predict with certainty either the future of the domestic economy or economic and financial trends in other parts of the world. It is impossible to predict accurately whether money invested today will be available tomorrow. There is no assurance that a financially sound firm will remain so in the years ahead. Investors cannot be guaranteed dividends or price appreciation in stocks they purchase, nor can they be completely certain that the interest and principal on fixed-income securities will be paid as agreed by the issuer. Financial analysts or sophisticated investors, no matter how competent they are, cannot be assured that the returns they project from a given investment will turn out as originally visualized.

Since uncertainty increases the further one looks into the future, risk also increases — and the value of money promised in the future diminishes accordingly.

Liquidity is important to an investor or a firm. Liquidity refers to how easily assets can be converted into cash without significant loss. Cash, government bonds, and other marketable securities (company assets guaranteed to lenders to ensure repayment of a loan) increase the liquidity of a firm. By the same token, fixed assets such as plant and equipment are not considered very liquid. Investors have a preference for liquidity; that is, they prefer to hold ready cash for unexpected emergencies and financial claims rather than commit funds into future-yielding assets. If they do give up current liquidity by buying assets that promise future returns, they are trading an assured cash asset for a riskier future asset. The trade will take

place only if the promised rewards of the future assets are sufficiently high to warrant taking the risk.

When lenders or investors give up cash for very risky future returns, they require high premiums, or returns, on their invested cash to compensate for less liquidity. Conversely, when they invest in low-risk assets, the premiums they expect in return are relatively low.

*Example: Liquidity versus Future Returns*

If a person deposits cash in a bank that is FDIC (Federal Deposit Insurance Corporation) insured, she will be willing to accept 5% interest, whereas if she buys the long-term bond of an unknown company, a higher rate of interest, say 15%, would be required. In both cases cash, and 100% liquidity is given up, and the return must compensate for the risk.

It is clearly essential for lenders or investors to know how much their cash investments will grow so they can determine whether their investments are worthwhile. Borrowers also want to know how much, and over what period of time, they will have to repay the lenders, and whether the returns from these borrowed funds will be greater than the costs of borrowing. This all boils down to the concept of future value, as determined by the compound rate of interest and the present value of future returns once they are adjusted for risk.

To sum up, aside from the fact that money invested wisely today will yield a return in the future (a fact that creates a natural investor desire for cash today), money loses value over time because of risk, and preference for cash. The concept that the value of a dollar today is more than the value of a dollar tomorrow is central to financial theory.

*Finance, 5<sup>th</sup> ed.; Groppelli, Ehsan Nikbakht; Barron's Inc. 2006, Chapter 3, pp. 1-2*

### **32. How do you compare cash flows at different points in time?**

Only cash flows in the same units can be compared or combined. A dollar today and a dollar in one year are not equivalent. Having money now is more valuable than having money in the future; if you have the money today you can earn interest on it. To compare or combine cash flows that occur at different points in time, you first need to convert the cash flows into the same units or *move* them to the same point in time.

Let's define three rules of time travel:

- *First rule is that it is only possible to compare or combine values at the same point in time.*
- *Second rule stipulates that to move a cash flow forward in time, you must compound it.*
- *Third rule stipulates that to move a cash flow back in time, we must discount it.*

*Finance, 5th ed.; Groppelli, Ehsan Nikbakht; Barron's Inc. 2006, Chapter 3*

### 33. What is Future Value/Present Value of an investment?

Any reasonable investment or commitment of cash must provide for an increase in value over time. Given the amount of cash that you want to commit, you can find out how much that cash value will increase in the future once the expected rate of return is known. This calculation is called finding the future value of an investment.

*Example: Future Value after One Year*

Suppose an investor saves \$100. This cash is deposited in the bank at a 10% annual interest rate. After one year the investor will have the original \$100 plus \$10 in interest:

$$\begin{aligned} \text{Original deposit} + \text{Interest on deposit} &= \text{FV} \\ \$100 + (10\%) (\$100) &= \$110 \end{aligned}$$

At the time of deposit the \$100 was worth 100%, or \$100. Since the bank promised to pay an additional 10%, the future value of the \$100 one year from now is equal to \$110 (\$100 plus 10).

Calculating future value for 1 year is perfectly straightforward, but what happens when someone wants to know how much money will be in an account after 20 years? Luckily, there is an easy formula to calculate future values:

$$FV = PV (1 + R)^N$$

Where

$FV$  = future value

$PV$  = initial deposit (principal)

$R$  = annual rate of interest

$N$  = number of years

*Example: Future Value after Any Number of Years*

The equation just introduced can be used for any number of years. Here are two instances involving a \$100 deposit at a 10% interest rate:

1 Year on Deposit	2 Years on Deposit
$FV = P (1+R)^1$	$FV = P (1+R)^2$
$FV = \$100 (1 + .10)$	$FV = \$100 (1 + .10)^2$
$FV = \$100 (1.10)$	$FV = \$100 (1.10) (1.10)$
$FV = \$110$	$FV = \$121$

If the preceding example had involved 10 years, you would have had to calculate  $(1.10)^{10}$ , which is equal to 2.594. So the future value of \$100 in 10 years would be  $\$100(2.594)$ , or \$259.40. Note that each year the cash value increases, not by 10% of the

original \$100, but by 10% of each subsequently higher amount. In other words, you earn interest not only on your initial deposit, but also on your interest:

Original \$100 x 1.10 = \$110 future value (*FV*) after 1 year

←  
\$100 x 1.10 = \$121 *FV* after 2 years

←  
\$121 x 1.10 = \$133 *FV* after 3 years

We now know that an annual interest rate of 10 percent enables the investor to transform \$1 today into \$1.21 two years from now. In addition, we would like to know:

How much would an investor need to lend today so that she could receive \$1 two years from today?

Algebraically, we can write this as

$$PV \times (1.09)^2 = \$1$$

In the preceding equation, *PV* stands for present value, the amount of money we must lend today in order to receive \$1 in two years' time. Solving for *PV* in this equation, we have

$$PV = \frac{\$1}{(1.09)^2} = \$0.84$$

To be certain that \$0.84 is in fact the present value of \$1 to be received in two years, we must check whether or not, if we loaned out \$0.84 and rolled over the loan for two years, we would get exactly \$1 back. If this were the case, the capital markets would be saying that \$1 received in two years' time is equivalent to having \$0.84 today. Checking the exact numbers, we get

$$\$0.84168 \times 1.09 \times 1.09 = \$1$$

In other words, when we have capital markets with a sure interest rate of 9 percent, we are indifferent between receiving \$0.84 today or \$1 in two years. We have no reason to treat these two choices differently from each other, because if we had \$0.84 today and loaned it out for two years, it would return \$1 to us at the end of that time. In the multiperiod case, the formula for Present Value of Investment can be written as:

$$PV = \frac{C_T}{(1+r)^T}$$

Where  $C_T$  is cash flow at date  $T$  and  $r$  is the appropriate interest rate.

*Corporate Finance, 6th ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 4, pp. 67-75*

### 34. What are discount rates? Discount Factors? Write formula for Discount Factor

To calculate present value, a discount rate must be determined that takes into consideration how much risk is associated with each project or investment. Risk levels follow a simple rule: High risk means a high discount (capitalization) rate, and low risk means a low discount rate.

For example, if an investor decides that the discount rate assigned to a stock should be 5%, another stock having double this risk will have a discount rate of 10. Once the risk level is determined, the next step is to adjust returns or future income for the uncertainty of time. Generally speaking, the following principles apply to evaluating discount rates. Evaluating discount rates is accomplished by the following principles:

1. Between two future incomes, the one that will take longer to reach maturity should have a higher discount rate.
2. The lower the perceived risk, the lower the discount rate should be.
3. If general interest rates in the market rise, the discount rate should increase also.

Risk can decline because of a more favorable business outlook, the prospect of declining inflation and interest rates, or less uncertain economic conditions. As risk declines, the present value of future income will increase, as illustrated in the table below.

Inverse Relationship between Present Value and Risk

Future Income (3 years from now) (dollars)	Discount Rate (%)	PV of \$1 in 3 Years	PV of Future Income (dollars)
1,000	15 (high risk)	.658	658
1,000	10 (average risk)	.751	751
1,000	5 (low risk)	.684	864

The present value of any future returns declines the further out into the future you look. Obviously, this procedure employs a mathematical adjustment for the time value of money. As it turns out, the principle involved is not a difficult one to grasp — the present value of future returns is merely the reverse of future value compounding.

An arithmetic illustration will provide a better understanding of this principle. Assume you wish to find out the present value of \$1,000 3 years from now, and you expect the level of risk associated with the project to be 10% annually. Thus, if

$$FV = PV (1 + R)^N$$

Then

$$PV = \frac{FV}{(1 + R)^N}$$



It is evident from the above table that the factors increase as time passes and as the compound interest rate raises. You can observe that, if these factors are plugged into the denominator in the last equation, the present value of \$1,000 3 years hence is

$$\frac{\$1,000}{(1 + .10)^3} = \$751$$

How was this value found? Simply by multiplying 1.10 three times ( $1.10 \times 1.10 \times 1.10 = 1.33$ ), and using this factor to discount:

$$\frac{\$1,000}{1.33} = \$751$$

While 10% is the discount rate, it is clear that the discount factor can be defined as the present value of \$1 coming in  $N$  years discounted by the appropriate rate, which in our case is:

$$\frac{\$1}{(1 + .10)^3} = 0.751$$

And we can calculate the present value by simply multiplying future cash flow by the discount factor, that is:

$$\$1,000 \times 0.751 = \$751$$

To sum up, if the discount rate for one period is  $R$  and  $N$  is the number of periods, the discount factor is:

$$\frac{1}{(1 + R)^N}$$

*Finance, 5th ed.; Gropelli, Ehsan Nikbakht; Barron's Inc. 2006, Chapter 3, pp. 6-8*

### **35. How does the change in the number of compounding per year affect PV and FV calculations?**

From the formula for calculating future value of an investment, it is easily seen that increasing compounding per year will raise FV of investment.

$$FV = P \left( 1 + \frac{R}{n} \right)^{N \times n}$$

Where

$FV$  = future value

$P$  = initial deposit (principal)

$R$  = annual rate of interest

$N$  = number of years

$n$  = number of compounding periods

Thus interim-year compounding serves as additional source of income.

Vice versa happens when calculating PV of investment. Increase in compounding frequency decreases PV of future cash flows as can also be seen from the formula for calculating present value of an investment:

$$PV = \frac{C_N}{\left(1 + \frac{R}{n}\right)^{N \times n}}$$

*Finance, 5th ed.; Groppelli, Ehsan Nikbakht; Barron's Inc. 2006, Chapter 3, pp. 13-14*

### **36. What is the general definition of the value of an asset? What are three necessary conditions for continuous increase in value of asset?**

The value of an asset, such as a share of common stock or a bond, is influenced by three major factors: cash flow of the asset, growth rate of the cash flow, and risk or uncertainty of the cash flow.

An increase in the amount of cash flow tends to raise the price of an asset. Conversely, the price declines if cash flow becomes more uncertain. These relationships are fundamental to the valuation of an asset. Accordingly, the responsibility of a financial officer is to increase cash flows as much as possible while controlling risk.

Since profits raise the price of an asset and risk reduces the price, all of the following three conditions are required for a continuous increase in the value of any asset:

- The asset must continuously produce cash flow.
- Cash flow must have a positive rate of growth (cash flow must increase over time).
- Risk must be controlled.

Of the three factors, estimation of risk is the most difficult task. As a result, the subject of risk in the valuation of an asset should receive more attention when the outlook for the economy becomes more uncertain. The discount rate reflects risk, or uncertainty about the future income. To find the current price of an asset, then, future cash flows must be discounted back to present value at an appropriate rate to reflect each aspect of risk. Therefore, the price of an asset is the same as the present value of its future cash flows.

*Value of an Asset = The Present Value of its future expected cash flows*

*Finance, 5th ed.; Groppelli, Ehsan Nikbakht; Barron's Inc. 2006, Chapter 5, p. 1*

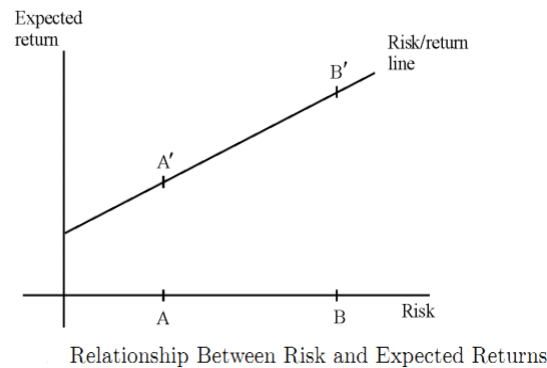
### **37. What is the risk/return trade-off principle?**

In finance, higher risks are generally associated with higher possible gains — but there is also a greater chance of loss. The same principle applies to Aunt Jane or John Doe. Both

would like to strike it rich, but they know that to do so they must be willing to face the dangers of heavy losses. Aunt Jane and John Doe are aware that it is safer to purchase a U.S. treasury bill than to buy pork belly contracts. The chances of incurring losses from owning a treasury bill are slim indeed, while pork belly contracts could mean high gains or large losses.

Managers are faced with a similar dilemma. Some projects may be more profitable than others, but the risk associated with them could be too high and might jeopardize the solvency of the firm. This situation is analogous to gamblers who insist on betting on long-shot horses. Although the payout is great if the long-shot wins, the chances are that these gamblers will be consistent losers. Bettors who play to “show” or put their money on the favorite horse have a better chance of winning, but their gains will be lower.

The same applies to the policies adopted by managers. There is a constant conflict between engaging in highly profitable ventures and maintaining a sound financial status. These managerial decisions involve a compromise between taking excessive risks to maximize profits and accepting investments that will probably result in lower risk and lower profitability-but will lead to a sound financial posture for the firm.



So, risk/return trade-off is the tradeoff between the high returns bearing high risk and the low risk accepting low returns. The goal of maximizing the value of the firm is accomplished by investing in those projects that have the best risk/return trade-off. Relationship between risk and expected return can be graphically illustrated on figure above.

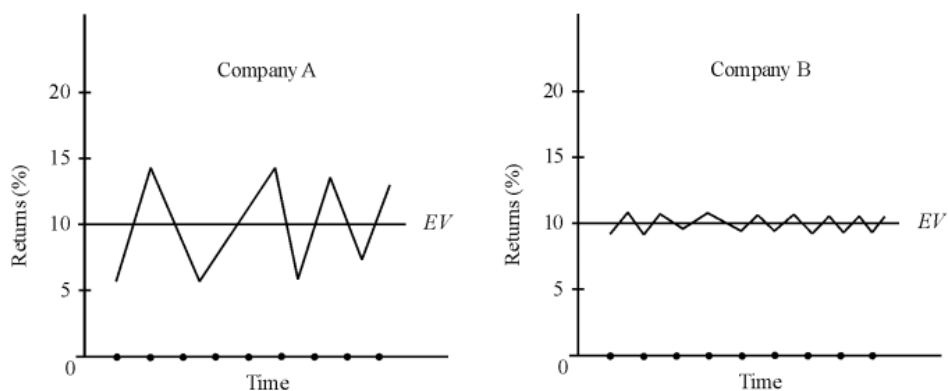
*Finance, 5th ed.; Groppelli, Ehsan Nikbakht; Barron's Inc. 2006, Chapter 1, pp. 3-4*

### 38. Define risk. How can risk be measured?

Risk can be defined as the statistical possibility of incurring financial losses, above and beyond expected, as changes occur in the environment (market rates, firms in distress, fraud, etc.) used in transaction valuation, processing, accounting, etc. Risk and return are the foundations upon which rational and intelligent investment decisions are made. Broadly speaking, risk is a measure of the volatility, or uncertainty of returns, and returns are the expected receipts or cash flows anticipated from any investment.

The following example may help explain the meaning of risk. Everyone knows that deposits at a savings bank are safer than money bet on a horse race. Bank deposits yield a steady but low rate of interest year by year and are. There is a high degree of confidence that these returns and the original deposit will be paid back. The returns from bank deposits don't fluctuate very much, and for this reason they are considered to be safe and to have a low degree of risk. On the other hand, when people gamble they don't know the outcome. They may win big, but they can also lose everything. Returns from horse betting are highly uncertain, very volatile, and subject to a high degree of risk. When two investments yield the same returns, the final choice will be based on the evaluation of the riskiness of each project. The project having the lower risk will be selected.

Risk is the degree of uncertainty associated with an investment. The more volatile the returns from an investment – the greater its risk. When two projects have the same expected returns, choose the one with the least risk.



Volatility of Returns for Company A and Company B

Note: Company A and Company B have same percentage scale of returns and  $EV =$  expected value.

Low risk is associated with low returns and high risk with high returns. In finance, risk is measured by the degree of volatility associated with expected returns. Volatility is the amount of fluctuation that occurs in a series of figures as they deviate from a representative average. For example, the average of the series 1, 2, 3 is 2, and the average of the series 1, 3, 5 is 3. The second series is considered more volatile than the first series of figures. Thus, the higher is volatility the higher is level of risk.

Risk is defined as the deviation of expected outcomes from a mean or expected value. It can also be regarded as the chance of incurring a loss or gain by investing in an asset or project. The chances of making a profit or incurring a loss can be high or low depending on the degree of risk (variability of expected returns) associated with a given investment.

The volatility of the returns of any asset measures the level of risk. The wider dispersion of Company A's returns in above figure indicates that there is a greater chance that an actual return will fall either below or above the straight line, that is expected value.

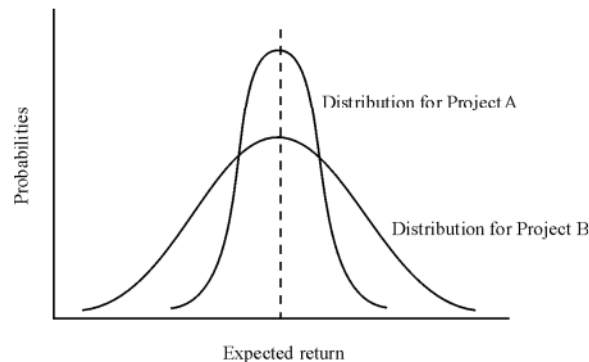
One common way to measure the risk of an asset is to calculate its deviation from a mean or an expected return. By assuming that all values are distributed normally — that the returns

are distributed equally between the higher and lower sides of expected returns — it is possible to measure the volatility of returns for each project and, in turn, to measure their comparative risk. This can be done by subtracting the actual returns  $R_i$  from the expected return  $ER$ . The values derived from these calculations are then squared to eliminate the problem of minus signs. In a world of uncertainty, probabilities are assigned to each deviation to obtain a single representative value, which is called **variance**. The square root of variance is none other than the **standard deviation**.

$$\text{Standard deviation } (\sigma) = \sqrt{\sum_{i=1}^N (R_i - ER)^2 P_i}$$

Where

- $N$  = number of observations
- $i$  = time periods
- $ER$  = expected return
- $P_i$  = probability of  $i$  – th return
- $R_i$  = actual return at  $i$  – th time period



Normal Probability Distributions of Two Projects, A and B

What does all this mean? First, you must assume that the probability distribution is normal. This implies that half the values in the distribution are likely to fall below the expected value and half to fall above the expected value. The closer a distribution is to the expected value; the more likely it is that the actual outcomes will be closer to the mean or expected value. Chances will be higher that the outcomes will be close to the expected value in a narrow distribution than in a wide distribution.

As the above figure shows, probability distributions for both A and B are normal, but B has a wider dispersion away from the expected value. Consequently, the distribution for B is considered riskier than the distribution for A. Note: Both probability distributions have the same expected value, but A has a narrower distribution, indicating less volatility relative to the expected value and hence less risk.

### **39. What is the difference between current expenses and capital expenditures?**

Current expenses are short-term expenses that are completely written off in the year when the expenses occur. In contrast, capital expenditures refer to spending on long-term assets that are capitalized and amortized over their useful life. Examples of current expenses include wages, salaries, raw material costs, and administrative expenses. In accounting, current expenses are treated like other short-term expenses. They are fully expensed during the fiscal period in which they are incurred. Unlike capital expenditures, which are first recorded on the balance sheet as assets before hitting the income statement as amortization expenses, current expenses are recorded directly on the income statement as expenses in the current fiscal period. Basically, if the capital outlay is invested in an asset that will last longer than one year, it is considered a capital expenditure and treated accordingly. On the other hand, if the capital outlay is invested in an asset that will last less than one year, it is considered a current expense.

Capital Expenditures are long-term expenditures that are amortized over a period of time. A capital expenditure is incurred when a business spends money either to buy fixed assets or to add to the value of an existing fixed asset with a useful life extending beyond the taxable year. In accounting, a capital expenditure is added to an asset account ("capitalized"), thus increasing the asset's basis (the cost or value of an asset adjusted for tax purposes).

*Finance, 5th ed.; Gropelli, Ehsan Nikbakht; Barron's Inc. 2006, Chapter 6, pp. 1-2*

### **40. What methods of financial statements analysis do you know? How are financial statements standardized?**

Most common ways of financial statements analysis are:

1. Vertical analysis – analysis of *common-size financial statements*
2. Horizontal analysis – analysis of history, so called *time-trend analysis*
3. *Peer group* analysis

To start making comparisons, one obvious thing we might try to do is to somehow standardize the financial statements. One very common and useful way of doing this is to work with percentages instead of total dollars. Example of standardized financial statements is common-size financial statements.

One way, though not the only way, to construct a common-size balance sheet is to express each item as a percentage of total assets. In this form, financial statements are relatively easy to read and compare. A useful way of constructing a common-size income statement is to express each item as a percentage of total sales. This income statement tells us what happens to each dollar in sales.

Time-trend analysis is a standard way of using history. Suppose we found that the current ratio for a particular firm is 2.4 based on the most recent financial statement information. Looking back over the last 10 years, we might find that this ratio had declined fairly steadily over that period. Based on this, we might wonder if the liquidity position of the firm has deteriorated. It could be, of course, that the firm has made changes that allow it to more efficiently use its current assets, that the nature of the firm's business has changed, or that business practices have changed. If we investigate, we might find any of these possible explanations behind the decline. This is an example management by exception – a deteriorating time trend may not be bad, but it does merit investigation.

Peer group analysis establishes a benchmark to identify firms similar in the sense that they compete in the same markets, have similar assets, and operate in similar ways. In other words, we need to identify a peer group. There are obvious problems with doing this since no two companies are identical. Ultimately, a choice of what companies to use as a basis for comparison is subjective. Some examples of benchmark are industry average, top ten best companies in the industry, or top ten worst companies in the industry.

*Fundamentals of Corporate Finance, 6th ed., Ross, Westerfield, Jordan; McGraw-Hill, 2003; Chapter 3, pp. 59-79*

#### 41. What is EFN? How can it be calculated?

Identifying the funds which must be raised in order to support the forecasted sales level is one of the key outputs of the forecasting process. This amount is known as the External Financing Needed (EFN). EFN can be defined as the difference between the forecasted increase in assets and the forecasted increase in liabilities and equity. It is the amount of external financing needed to achieve the forecasted growth in sales. EFN can be calculated in two ways: (1) based on projection of growth in sales pro-forma income statement and balance sheet can be constructed using percentage of sales approach and the difference amount between the forecasted increase in assets and the forecasted increase in liabilities and equity calculated, and (2) using the formula for EFN as follows:

$$\frac{\text{Assets}}{\text{Sales}} \times \Delta \text{Sales} + \frac{\text{Spont. Liab.}}{\text{Sales}} \times \Delta \text{Sales} - (\text{PM} \times \text{Projected Sales}) \times (1 - d)$$

Where

$\Delta \text{Sales}$  – change in sales

*Spontaneous Liabilities* – liabilities that are affected by growth in sales

*PM* – profit margin

*Projected Sales* – new projected sales to be achieved

*d* – Dividends payout ratio

*Fundamentals of Corporate Finance, 6th ed., Ross, Westerfield, Jordan; McGraw-Hill, 2003; Chapter 4, pp. 99-109*

#### 42. What is the difference between internal and sustainable growth rates?

The internal growth rate is the maximum growth rate that can be achieved with no external financing of any kind. It is called this the internal growth rate because this is the rate the firm can maintain with internal financing only. It is the case when the required increase in assets is exactly equal to the addition to retained earnings, and EFN is therefore zero. We have seen that this happens when the growth rate is slightly less than 10 percent. With a little algebra, we can define this growth rate more precisely as:

$$\text{Internal Growth Rate} = \frac{ROA \times b}{1 - ROA \times b}$$

Where

*ROA* – Return on Assets

*b* – Plowback ratio, or retention ratio (the percentage of net income contributed to retained earnings)

The sustainable growth rate is the maximum growth rate a firm can achieve with no external equity financing while it maintains a constant debt-equity ratio. This rate is commonly called the sustainable growth rate because it is the maximum rate of growth a firm can maintain without increasing its financial leverage. We can calculate the sustainable growth rate as:

$$\text{Sustainable Growth Rate} = \frac{ROE \times b}{1 - ROE \times b}$$

*Fundamentals of Corporate Finance, 6th ed., Ross, Westerfield, Jordan; McGraw-Hill, 2003; Chapter 4, pp. 112-116*

#### 43. What are the main financial ratios used for analysis? Divide them into categories

Another way of avoiding the problems involved in comparing companies of different sizes is to calculate and compare financial ratios. Such ratios are ways of comparing and investigating the relationships between different pieces of financial information. Using ratios eliminates the size problem because the size effectively divides out. We're then left with percentages, multiples, or time periods. Financial ratios are traditionally grouped into the following categories:

1. Short-term solvency, or liquidity, ratios – current ratio, the quick (or acid-test) ratio, cash ratio



2. Long-term solvency, or financial leverage, ratios – total debt ratio, debt-equity ratio, equity multiplier, long-term debt ratio, times earned ratio, cash coverage ratio
3. Asset management, or turnover, ratios – inventory turnover, day's sales in inventory, receivables turnover, day's sales in receivables, total asset turnover
4. Profitability ratios - profit margin, return on assets (ROA), return on equity (ROE),
5. Market value ratios – earnings per share (EPS), price-earnings ratio (PE), market-to-book ratio

As the name suggests, short-term solvency ratios as a group are intended to provide information about a firm's liquidity, and these ratios are sometimes called liquidity measures. The primary concern is the firm's ability to pay its bills over the short run without undue stress. Consequently, these ratios focus on current assets and current liabilities.

Long-term solvency ratios are intended to address the firm's long-run ability to meet its obligations, or, more generally, its financial leverage. These are sometimes called financial leverage ratios or just leverage ratios.

Asset management measures are intended to describe how efficiently or intensively a firm uses its assets to generate sales. These measures are sometimes called asset utilization ratios.

Profitability ratios are probably the best known and most widely used of all financial ratios. In one form or another, they are intended to measure how efficiently the firm uses its assets and how efficiently the firm manages its operations. The focus in this group is on the bottom line, net income.

Final group of measures is based, in part, on information not necessarily contained in financial statements – the market price per share of the stock. Obviously, these measures can only be calculated directly for publicly traded companies.

*Fundamentals of Corporate Finance, 6th ed., Ross, Westerfield, Jordan; McGraw-Hill, 2003; Chapter 3, pp. 62-73*

#### **44. Why P/E ratio is important? What does it measure?**

The P/E ratio is defined as:

$$P/E \text{ ratio} = \frac{\text{Market price per share}}{\text{Earnings per share}}$$

For example, if stock A is trading at \$24 and the earnings per share for the most recent 12 month period is \$3, and then stock A has a P/E ratio of 24/3 or 8. Put another way, the purchaser of the stock is paying \$8 for every dollar of earnings. Companies with losses (negative earnings) or no profit have an undefined P/E ratio sometimes; however, a negative P/E ratio may be shown.

By comparing price and earnings per share for a company, one can analyze the market's stock valuation of a company and its shares relative to the income the company is actually generating. Stocks with higher (and/or more certain) forecast earnings growth will usually

have a higher P/E, and those expected to have lower (and/or riskier) earnings growth will usually have a lower P/E. Investors can use the P/E ratio to compare the value of stocks: if one stock has a P/E twice that of another stock, all things being equal (especially the earnings growth rate), it is a less attractive investment. Companies are rarely equal, however, and comparisons between industries, companies, and time periods may be misleading. P/E ratio in general is useful for comparing valuation of peer companies in similar sector or group.

*Finance, 5th ed.; Groppelli, Ehsan Nikbakht; Barron's Inc. 2006, chapter 18, p.18*

#### 45. Write Modified Du Pont Identity. Explain how is it used?

The famous decomposition of return on equity (ROE), called Du Pont Identity, after Du Pont Corporation, who popularized its use, is as follows:

$$\begin{aligned} ROE &= \frac{Net\ Income}{Total\ Equity} = \frac{Net\ Income}{Total\ Equity} \times \frac{Assets}{Assets} = \frac{Net\ Income}{Assets} \times \frac{Assets}{Total\ Equity} \\ &= \frac{Sales}{Sales} \times \frac{Net\ Income}{Assets} \times \frac{Assets}{Total\ Equity} \\ &= \frac{Net\ Income}{Sales} \times \frac{Sales}{Assets} \times \frac{Assets}{Total\ Equity} \end{aligned}$$

Remind that *Net Income/Sales* is profit margin, *Sales/Assets* is total asset turnover and *Assets/Total Equity* is equity multiplier. Thus, Du Pont Identity becomes:

$$ROE = Profit\ Margin \times Total\ Asset\ Turnover \times Equity\ Multiplier$$

The Du Pont identity tells us that ROE is affected by three things:

1. Operating efficiency (as measured by profit margin)
2. Asset use efficiency (as measured by total asset turnover)
3. Financial leverage (as measured by the equity multiplier)

The decomposition of ROE is a convenient way of systematically approaching financial statement analysis. If ROE is unsatisfactory by some measure, then the Du Pont identity tells you where to start looking for the reasons.

*Fundamentals of Corporate Finance, 6th ed., Ross, Westerfield, Jordan; McGraw-Hill, 2003; Chapter 3, pp. 73-75*

**46. What is perpetuity? Give example and write a formula for present value of perpetuity**

Perpetuity is a constant stream of cash flows without end. If you are thinking that perpetuities have no relevance to reality, it will surprise you that there is a well-known case of an unending cash flow stream: the British bonds called consols. An investor purchasing a consol is entitled to receive yearly interest from the British government forever.

How can the price of a consol be determined? Consider a consol that pays a coupon of  $C$  dollars each year and will do so forever. Simply applying the PV formula gives us:

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \dots$$

Here the dots at the end of the formula stand for the infinite string of terms that continues the formula. Series like the preceding one are called geometric series. It is well known that even though they have an infinite number of terms, the whole series has a finite sum because each term is only a fraction of the preceding term. Before turning to our calculus books, though, it is worth going back to our original principles to see if a bit of financial intuition can help us find the PV.

The present value of the consol is the present value of all of its future coupons. In other words, it is an amount of money that, if an investor had it today, would enable him to achieve the same pattern of expenditures that the consol and its coupons would. Suppose that an investor wanted to spend exactly  $C$  dollars each year. If he had the consol, he could do this. How much money must he have today to spend the same amount? Clearly he would need exactly enough so that the interest on the money would be  $C$  dollars per year. If he had any more, he could spend more than  $C$  dollars each year. If he had any less, he would eventually run out of money spending  $C$  dollars per year.

The amount that will give the investor  $C$  dollars each year, and therefore the present value of the consol, is simply

$$PV \text{ of perpetuity} = \frac{C}{r}$$

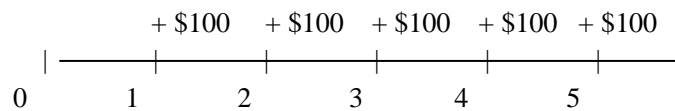
To confirm that this is the right answer, notice that if we lend the amount  $C/r$ , the interest it earns each year will be:

$$\text{Interest} = \frac{C}{r} \times r = C$$

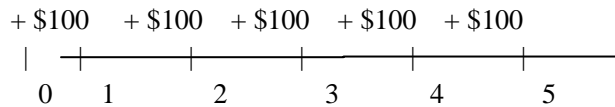
#### 47. What is Annuity? What is Annuity Due? What is the difference between them?

An annuity is a series of equal payments (or receipts) made at any regular interval of time. An annuity can be a payment or an investment each year, each half-year (semiannually), each quarter, or each month. Not surprisingly, annuities are among the most common kinds of financial instruments. The pensions that people receive when they retire are often in the form of an annuity. Other examples are the monthly mortgage payments on a house, quarterly investments in a trust account for a child's future education, and periodic loan payments.

Below is illustrated the timeline of \$100 annuity for a period of 5 years. Cash flows at the end of period



The only difference between annuity and annuity due is the timing of the first cash flow. Ordinary annuity starts at the end of each period, annuity due starts at the beginning of each period. So, the first payment of annuity due is at the time 0, and the first payment of ordinary annuity is at the time 1. Below is illustrated the timeline of \$100 annuity due for a period of 5 years. Cash flows at the beginning of period



*Corporate Finance, 6th ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 4, p. 86*

#### 48. What is the future value and present values of ordinary annuity? Annuity Due?

Let us first outline the convention with ordinary annuity that the first payment occurs at the end of each period that is one period from today, at date 1. While for annuity due the first payment occurs at the beginning of every period, thus, the first payment is today, at date 0.

So, from the timeline of ordinary annuity, the present value of an  $N$ -period annuity with payment  $C$  and interest rate  $r$  is:

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \dots + \frac{C}{(1+r)^N} = \sum_{n=1}^N \frac{C}{(1+r)^n}$$

To simplify the calculations of the present value of an annuity, let's first remind that the present value of perpetuity is  $C/r$ . and annuity is just a perpetuity that ends after some fixed

number of payments  $N$ . Now, the simplest way to derive the formula for present value of an annuity is accomplished in 3 steps:

1. Take the present value of perpetuity starting at time 0
2. Take the present value of perpetuity starting at some time  $N$
3. Subtract the results

So, we have:

$$PV \text{ of ordinary annuity} = \frac{C}{r} - \frac{\frac{C}{r}}{(1+r)^N} = \frac{C}{r} \left( 1 - \frac{1}{(1+r)^N} \right)$$

You can easily see this on timeline as follows:

	Now								
Date (or end of year)	0	1	2	...	$N$		$(N+1)$	$(N+2)$	
Consol 1		$C$	$C$	...	$C$	$C$	$C...$		
Consol 2							$C$	$C...$	
Annuity	$C$	$C$	...	$C$					

In case of annuity due for  $N$  payments, we should calculate the present value of an ordinary annuity for  $N-1$  periods and add one annuity payment of  $C$  coming at the beginning of the period that is today, at time 0. Note that the present value of  $C$  coming today is  $C$  itself.

$$PV \text{ of annuity due} = C + \frac{C}{r} \left( 1 - \frac{1}{(1+r)^{N-1}} \right)$$

Now that we know how to calculate the present value of annuities, we can also provide a formula for the future value of annuities just by taking its future value. So, for an ordinary annuity we have future value of the present value of an ordinary annuity:

$$FV \text{ of ordinary annuity} = \frac{C}{r} \left( 1 - \frac{1}{(1+r)^N} \right) \times (1+r)^N = \frac{C}{r} ((1+r)^N - 1)$$

And, in case of annuity due we calculate the future value of the present value of annuity due. That is:

$$FV \text{ of annuity due} = \left[ C + \frac{C}{r} \left( 1 - \frac{1}{(1+r)^{N-1}} \right) \right] \times (1+r)^N$$

*Corporate Finance, 6th ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 4, pp. 87-91*

#### 49. How incremental cash flows are calculated?

You may not have thought about it, but there is a big difference between corporate finance courses and financial accounting courses. Techniques in corporate finance generally

use cash flows, whereas financial accounting generally stresses income or earnings numbers. When considering a single project, we discount the cash flows that the firm receives from the project not accounting incomes. When valuing the firm as a whole, we discount dividends – not earnings – because dividends are the cash flows that an investor receives.

There are many differences between earnings and cash flows. In fact, much of a standard financial accounting course delineates these differences. Consider a firm buying a building for \$100,000 today. The entire \$100,000 is immediate cash outflow. However, assuming straight-line depreciation over 20 years, only \$5,000 ( $\$100,000/20$ ) is considered an accounting expense in the current year. Current earnings are thereby reduced only by \$5,000. The remaining \$95,000 is expensed over the following 19 years.

Because the seller of the property demands immediate payment, the cost at date  $t=0$  of the project to the firm is \$100,000. Thus, the full \$100,000 figure should be viewed as an immediate outflow for capital budgeting purposes. This is not merely our opinion but the unanimous verdict of both academics and practitioners.

In addition, it is not enough to use cash flows. In calculating the NPV of a project, only cash flows that are incremental to the project should be used. These cash flows are the changes in the firm's cash flows that occur as a direct consequence of accepting the project. That is, we are interested in the difference between the cash flows of the firm with the project and the cash flows of the firm without the project.

There are three difficulties in determining incremental cash flows:

- Sunk costs – costs that have already occurred. Because sunk costs are in the past, they cannot be changed by the decision to accept or reject the project. Just as we “let bygones be bygones,” we should ignore such costs. Sunk costs are not incremental cash outflows.
- Opportunity costs – your firm may have an asset that it is considering selling, leasing, or employing elsewhere in the business. If the asset is used in a new project, potential revenues from alternative uses are lost. These lost revenues can meaningfully be viewed as costs. They are called opportunity costs because, by taking the project, the firm forgoes other opportunities for using the assets. Incremental cash flows matter this cost.
- Side effects-in determining incremental cash flows come from the side effects of the proposed project on other parts of the firm. The most important side effect is erosion. Erosion is the cash flow transferred to a new project from customers and sales of other products of the firm. Incremental cash flows matter side effects.

## **50. What is capital budgeting? What methods are used in Capital Budgeting?**

Capital budgeting refers to the methods for evaluating, comparing, and selecting projects to achieve maximum return or maximum wealth for stockholders. Maximum return is measured by profit, and maximum wealth is reflected in stock price.

Capital budgeting is answering the question: in what long-term assets the firm Capital budgeting is answering the question invest? This question concerns the left-hand side of the balance sheet. Of course, the type and proportions of assets the firm needs tend to be set by the nature of the business. We use the terms capital budgeting to describe the process of making and managing expenditures on long-term assets.

Some of the methods used in capital budgeting decision making are: the payback period, the discounted payback period, the net present value (NPV), the internal rate of return (IRR), and the profitability index (PI).

*Corporate Finance, 6th ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 6*

## **51. Why financial analysts are interested in the cash flows and not in the accounting incomes?**

Financial analysts are interested in the cash flows because for capital budgeting decision making you need to know the initial investment and future expected cash flows. As a general rule to value any asset we need to find the present value of its future cash flows. Accounting incomes include non-cash expense such as depreciation, credit sales, non-paid expenses, etc. and couldn't be used in assets valuation. The price of an asset is stated in cash, dollars today you should pay to obtain it, and for the price of an asset to be fair it must be equal to the present value of its future cash flows, not non-cash measures like accounting incomes.

In order to decide whether the initial cost will payoff it is necessary to estimate future cash flows. Management should be concerned only with the incremental cash flow. The incremental cash flow is the additional cash flow that the firm will receive over the existing cash flow after the project is accepted. Suppose that the existing cash flow of a firm is \$100. If cash flow increases to \$150 after starting a new project, the incremental cash flow is \$50. Therefore, only \$50 is considered as the relevant cash flow, or the benefit of the project. There is a simple method to determine the incremental cash flow of a new project for each year.

*Incremental cash flow is the only relevant cash flow for the capital budgeting decisions; it is all you need to compare projects.*

For capital budgeting analysis, investment is in the form of cash outflow. So, naturally the management needs to compare the costs (outflows) and benefits (inflows) arising out of the project. This can be effectively measured only by means of cash flow method. You need cash to buy an asset. It is an outflow. But accounting profit method ignores expenditure of

buying asset at the time of purchase. It records the expenditure of an asset over the entire economic life of the project in the form of depreciation, which is a non-cash item. Hence, even in this case, time value is ignored. The accounting profit does not reflect the requirement of cash at outflow and inflow stages of time. Moreover, this does not actually reflect the actual outflows and inflows. So, only the cash flow method is the right choice for evaluating a capital budgeting decision.

*Finance, 5th ed.; Groppelli, Ehsan Nikbakht; Barron's Inc. 2006, Chapter 6, pp. 5-8*

## 52. Define sunk costs

A sunk cost is a cost that has already occurred. Because sunk costs are in the past, they cannot be changed by the decision to accept or reject the project. Just as we “let bygones be bygones,” we should ignore such costs. Sunk costs are not incremental cash outflows.

*Example:*

The General Milk Company is currently evaluating the NPV of establishing a line of chocolate milk. As part of the evaluation the company had paid a consulting firm \$100,000 to perform a test-marketing analysis. This expenditure was made last year. Is this cost relevant for the capital budgeting decision now confronting the management of General Milk Company?

The answer is no. The \$100,000 is not recoverable, so the \$100,000 expenditure is a sunk cost, or spilled milk. Of course, the decision to spend \$100,000 for a marketing analysis was a capital budgeting decision itself and was perfectly relevant before it was sunk. Our point is that once the company incurred the expense, the cost became irrelevant for any future decision.

*Corporate Finance, 6th ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 7, p.170*

## 53. Define NPV and PI

If the present value of a project's future cash flow is greater than the initial cost, the project is worth undertaking. On the other hand, if the present value is less than the initial cost, a project should be rejected because the investor would lose money if the project were accepted. By definition, the net present value of an accepted project is zero or positive, and the net present value of a rejected project is negative. The net present value (NPV) of a project can be calculated as follows:

$$NPV = PVCF - I$$

Where

*PVCF* – present value of future cash flows from the project

*I* – initial investment



To derive an algebraic formula for net present value of a cash flow, recall that the *PV* of receiving a cash flow,  $C$  one year from now is:

$$PV = \frac{C_1}{(1+r)}$$

And the *PV* of receiving a cash flow  $C$  two years from now is:

$$PV = \frac{C_1}{(1+r)^2}$$

We can write the *NPV* of a  $T$ -period project as:

$$NPV = -C_0 + \frac{C_1}{(1+r)} + \frac{C_1}{(1+r)^2} + \dots + \frac{C_T}{(1+r)^T} = -C_0 + \sum_{i=1}^T \frac{C_i}{(1+r)^i}$$

The initial flow,  $-C_0$ , is assumed to be negative because it represents an investment. If the initial investment is not negative, we can use more general formula for *NPV* that considers all cash flow from time  $0$  to  $T$ :

$$NPV = \sum_{i=0}^T \frac{C_i}{(1+r)^i}$$

The net present value method has three main advantages. First, it uses cash flows rather than net earnings. Cash flows (net earnings + depreciation) include depreciation as a source of funds. This works because depreciation is not cash expenditure in the year the asset is depreciated. In contrast with accounting, the field of finance considers cash flows rather than net earnings.

Second, the *NPV* method recognizes the time value of money. The longer the time is the higher the discount. Simply speaking, if the cash flows of a project with an average risk are discounted at 10%, another project with a higher degree of risk should be discounted at more than 10%. Therefore, the time value of money for a project is reflected in the discount rate, which should be selected carefully by the financial analyst. Generally, the discount rate tends to rise if the money supply is tight and the interest rate is expected to go up.

Third, by accepting only projects with positive *NPVs*, the company will also increase its value. An increase in the value of the company is, in fact, an increase in the stock price or in the wealth of stockholders. The *NPV* method of capital budgeting, therefore, should ultimately lead to more wealth for the owners of the company. Since the objective of modern finance is to continuously increase the wealth of stockholders, the *NPV* method should be viewed as the most modern technique of capital budgeting.

There are also some limitations, however, to the *NPV* approach. The method assumes that management is able to make detailed predictions of cash flows for future years. In reality, however, the more distant the date, the more difficult it is to estimate future cash flows. Future cash flows are influenced by future sales, costs of labor, materials and overhead, interest rates, consumer tastes, government policies, demographic changes, and so on. Overestimation or underestimation of future cash flows may lead to the acceptance of a project that should be rejected, or the rejection of a project that should be accepted.

Additionally, the *NPV* approach usually assumes that the discount rate is the same over the life of the project. The discount rate of a project, like the interest rate, actually changes from one year to another. Opportunities to reinvest future cash flows, future interest rates, and the costs of raising new capital can all affect the discount rate. It may be suggested that the problem can be resolved by predicting future interest rates, and then discounting the cash flow of each future year at the predicted discount rate. While this is an intelligent suggestion, you may agree that the prediction of the interest rate for the next 5 or 10 years is as uncertain as the outcome of flipping a coin 5 or 10 times! Despite its limitations, however, the *NPV* method is still the best method of capital budgeting.

The profitability index, or *PI*, method compares the present value of future cash inflows with the initial investment on a relative basis. Therefore, the *PI* is the ratio of the present value of cash flows (*PVCF*) to the initial investment of a project:

$$PI = \frac{PVCF}{Initial\ Investment}$$

In this method, a project with a *PI* greater than 1 is accepted, but a project is rejected when its *PI* is less than 1. Note that the *PI* method is closely related to the *NPV* approach. In fact, if the net present value of a project is positive, the *PI* will be greater than 1. On the other hand, if the net present value is negative, the project will have a *PI* of less than 1. The same conclusion is reached, therefore, whether the net present value or the *PI* is used. In other words, if the present value of cash flows exceeds the initial investment, there is a positive net present value and a *PI* greater than 1, indicating that the project is acceptable.

An important comment about the *PI* and the *NPV* methods is that, although the two techniques generally lead to the same major decision — whether to accept or reject a project — they often rank alternative projects in different orders.

*Finance, 5th ed.; Groppelli, Ehsan Nikbakht; Barron's Inc. 2006, Chapter 7, pp. 4-6*

## 54. Define IRR

The internal rate of return, or *IRR*, is a popular measure used in capital budgeting. The *IRR* is a measure of the rate of profitability. By definition, *IRR* is a discount rate that makes the present value of cash flows equal to the initial investment. In simple terms, the *IRR* is a discount rate that makes the *NPV* equal to zero.

The basic rationale behind the *IRR* is that it tries to find a single number that summarizes the merits of a project. That number does not depend on the interest rate that prevails in the capital market. That is why it is called the internal rate of return; the number is internal or intrinsic to the project and does not depend on anything except the cash flows of the project.

The rate below which projects are rejected is called the cutoff rate, the target rate, the hurdle rate, or the required rate of return. Firms determine their cutoff rates by the cost of financing and the riskiness of the project. Next, they predict future cash flows and calculate the *IRR*. If the calculated *IRR* exceeds the cutoff rate, the project is added to the list of recommended investments.

By definition, the IRR of a project is calculated by assigning various discount rates in the net present value formula and choosing the one that makes NPV equal to zero. Thus, we can solve IRR by trial and error or simply by the use of computer from:

$$0 = \sum_{i=0}^T \frac{C_i}{(1 + IRR)^i}$$

A number of surveys have shown that, in practice, the IRR method is more popular than the NPV approach. The reason may be that the IRR is straightforward, like the ARR, but it uses cash flows and recognizes the time value of money, like the NPV. In other words, while the IRR is easy and understandable, it does not have the drawbacks of the ARR and the payback period, both of which ignore the time value of money.

The main problem with the IRR method is that it often gives unrealistic rates of return. Suppose the cutoff rate is 11% and the IRR is calculated as 40%. Does this mean that management should immediately accept the project because its IRR is 40%? The answer is no! An IRR of 40% assumes that a firm has the opportunity to reinvest future cash flows at 40%. If past experience and the economy indicate that 40% is an unrealistic rate for future reinvestments, an IRR of 40% is suspect. Simply speaking, an IRR of 40% is too good to be true! So unless the calculated IRR is a reasonable rate for reinvestment of future cash flows, it should not be used as a yardstick to accept or reject a project.

*Finance, 5th ed.; Groppelli, Ehsan Nikbakht; Barron's Inc. 2006, Chapter 7, pp. 7-9*

## **55. State the criterion for accepting or rejecting independent projects under each rule: Payback Period, IRR, PI, NPV**

*Payback Period:*

If the payback period of a project is shorter than the target payback period set by the management of the company, *accept* the project, otherwise, *reject*.

*Internal Rate of Return (IRR):*

To evaluate projects by IRR method we need to know the required rate of return (RRR) of a project. Criterion for accepting or rejecting projects under IRR differs for different types of projects:

1. For investing projects, projects with negative initial investment and positive cash flow:
  - If  $IRR > RRR$ , => *accept* the project
  - If  $IRR < RRR$ , => *reject* the project
2. For financing projects, projects with positive initial investment and negative cash flow:

If  $IRR > RRR$ , => *reject* the project

If  $IRR < RRR$ , => *accept* the project

- For projects with multiple sign cash flow, some negative, some positive, no general rule can be defined as such projects have multiple IRRs.

*Profitability Index (PI):*

If  $PI > 1$ , => *accept* the project

If  $PI < 1$ , => *reject* the project

*Net Present Value (NPV):*

If  $NPV > 0$ , => *accept* the project

If  $NPV < 0$ , => *reject* the project

*Finance, 5th ed.; Groppelli, Ehsan Nikbakht; Barron's Inc. 2006, Chapter 7*

## **56. What is sensitivity analysis? What is the purpose of simulation in capital budgeting?**

Sensitivity analysis is a popular way to find out how, say, the NPV of a project changes if sales, labor or material costs, the discount rate, or other factors vary from one case to another. In simple terms, sensitivity analysis is a “what if” study. For example, you might be interested in knowing what happens to the NPV of a project if cash flow increases by 10%, 20%, or 30% each year. Will the NPV still be positive if there is no cash flow in the second year? Which project’s NPV will fall more sharply if the discount rate goes up from 8% to 11%? These are the kinds of questions financial analysts raise when they want to measure the risk of a project through sensitivity analysis. Remember that risk is measured by variation. The more variation or change there is in the NPV of a project, the more risky that investment would be.

Sensitivity analysis measures NPV, IRR, and other indicators of profit or risk change as sales, costs, the discount rate, or other variables change. The purpose is to find out how sensitive those indicators are to a change in one variable. Of two projects, the one more sensitive to a change is the project considered to have more risk.

The word simulation comes from the Latin word *similis*, which means “like.” Accordingly, the idea behind simulation is basically to make hypothetical situations like real ones. Since the actual cash flows or discount rate that will exist in the future are not known, various cash flows and discount rates are assumed, and the results are studied. These cases based on assumptions are called simulated events.

Simulated events in capital budgeting are used to study the NPVs or the IRRs of a project for different cash flows at different reinvestment rates. After different NPVs are computed, the average NPV and standard deviation of the project are studied to see if the project is worth undertaking. If you have more than one project, you can simulate the NPV or the IRR of each project a number of times and compute the average NPVs or IRRs and standard deviations.

Then rank the projects, starting with the one that has the highest NPV or IRR and the lowest standard deviation. Ranking projects is much easier if you first divide the average standard deviation by the average NPV of each simulated project. The result, as you may remember is the coefficient of variation. Finally, rank the projects according to their coefficients of variation, giving the highest rank to the project with the lowest coefficient.

There is a variety of simulation software for various personal computers. These programs use variables at random and calculate many more scenarios than anyone would do by hand. The results can also be used to draw curves to show the distribution of the NPVs or the IRRs. The shapes of distribution curves help financial analysts get a good idea about the riskiness of a project.

As a conclusion, it can be said that the purpose of simulation in capital budgeting is to determine which project has a more stable NPV, IRR, or PI when the values of cash flow, discount rate, and other factors change.

*Finance, 5th ed.; Groppelli, EhsanNikbakht; Barron's Inc. 2006, Chapter 8, pp. 5-7*

## **57. Define the concept of inflation in capital budgeting**

Inflation is a general price increase in the economy. When inflation increases, the real value of expected cash flows decreases. If the analyst does not adjust for risk of inflation, the NPV or the IRR may be artificially high. In other words, you might accept a project with an unadjusted IRR or NPV, while the real IRR or NPV, adjusted for inflation, could be unacceptable.

Therefore, capital budgeting techniques that ignore inflation are often misleading. Since inflation has probably become a permanent problem in the economy, you should plan to deal with it anytime you make a major decision. How can you deal with inflation in capital budgeting? The answer is that you should adjust both the cash flows and the discount rate for the annual rate of inflation.

Suppose that the one-year interest rate that the bank pays is 10 percent. This means that an individual who deposits \$1,000 at date 0 will get \$1,100 ( $\$1,000 \times 1.10$ ) in one year. While 10 percent may seem like a handsome return, one can only put it in perspective after examining the rate of inflation.

Suppose that the rate of inflation is 6 percent over the year and it affects all goods equally. For example, a restaurant that charges \$1.00 for a hamburger at date 0 charges \$1.06 for the same hamburger at the end of the year. You can use your \$1,000 to buy 1,000 hamburgers at date 0. Alternatively, if you put all of your money in the bank, you can buy 1,038 ( $\$1,100/\$1.06$ ) hamburgers at date 1. Thus, you are only able to increase your hamburger consumption by 3.8 percent by lending to the bank. Since the prices of all goods rise at this 6-percent rate, lending lets you increase your consumption of any single good or any combination of goods by only 3.8 percent. Thus, 3.8 percent is what you are really earning through your savings account, after adjusting for inflation. Economists refer to the 3.8-percent number as the real interest rate. Economists refer to the 10-percent rate as the nominal interest rate or simply the interest rate.

In general, there is Fischer's formula between real and nominal cash flows as:

$$1 + \textit{nominal interest rate} = (1 + \textit{real interest rate}) \times (1 + \textit{inflation rate})$$

Rearranging terms, we have:

$$\textit{real interest rate} = \frac{1 + \textit{nominal interest rate}}{1 + \textit{inflation rate}} - 1$$

The formula indicates that the real interest rate in our example is 3.8 percent ( $1.10/1.06 - 1$ ).

The above formula determines the real interest rate precisely. The following formula is an approximation:

$$\textit{real interest rate} \approx \textit{nominal interest rate} - \textit{inflation rate}$$

The symbol  $\approx$  indicates that the equation is approximately true. This latter formula calculates the real rate in our example as:

$$4\% \approx 10\% - 6\%$$

This approximation is reasonably accurate for low rates of interest and inflation. In our example, the difference between the approximate calculation and the exact one is only .2 percent (4 percent - 3.8 percent). Unfortunately, the approximation becomes poor when rates are higher.

Besides the above techniques, financial practitioners correctly stress the need to maintain consistency between cash flows and discount rates. That is,

*Nominal* cash flows must be discounted at the *nominal* rate.

*Real* cash flows must be discounted at the *real* rate.

*Corporate Finance, 6th ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003;  
Chapter 7, pp. 177-183*

## **58. What types of dividends do you know? Which type is more preferable? Why?**

Any direct payment by the corporation to the shareholders may be considered part of dividend policy. The most common type of dividend is in the form of cash. Public companies usually pay regular cash dividends four times a year. Sometimes firms will pay a regular cash dividend and an extra cash dividend. Paying a cash dividend reduces the corporate cash and retained earnings shown in the balance sheet.

Another type of dividend is paid out in shares of stock. This dividend is referred to as a stock dividend. It is not a true dividend, because no cash leaves the firm. Rather, a stock dividend increases the number of shares outstanding, thereby reducing the value of each share. A stock dividend is commonly expressed as a ratio; for example, with a 2 – percent stock dividend a shareholder receives one new share for every 50 currently owned.

When a firm declares a stock split, it increases the number of shares outstanding. Because each share is now entitled to a smaller percentage of the firm's cash flow, the stock

price should fall. For example, if the managers of a firm whose stock is selling at \$90 declare a 3:1 stock split, the price of a share of stock should fall to about \$30. A stock split strongly resembles a stock dividend except it is usually much larger.

Cash dividends are probably more preferable because no stock dilution follows and the stock price doesn't decline.

*Corporate Finance, 6th ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003;  
Chapter 16, pp.495-497*

### 59. Why does the value of a share of stock depend on dividends? Define dividend growth model

To value common stocks like any asset we should find the present value of its future cash flows. A stock provides two kinds of cash flows. First, most stocks pay dividends on a regular basis. Second, the stockholder receives the sale price when they sell the stock. But as the sale price is determined by the dividends after the sale date, then, the value of a firm's common stock to the investor is equal to the present value of all of the expected future dividends.

Over time, the annual dividend per share may remain fixed, may grow at a constant rate, or may rise at a relatively high rate for a few years and then grow at a constant rate. Because of all these possibilities, calculation of the price of common stock calls for careful projection of future dividends. Since a company is considered to operate forever, the price of common stock is not influenced by the number of years an investor wants to maintain ownership.

The price of common stock is largely determined by three factors: the annual dividends, growth of dividends, and discount rate. The rate at which future dividends are to be discounted is called the required rate of return. If a company has a high level of risk, a high required rate of return is expected by investors. To encourage investors to invest their money in a risky venture, a higher payoff must be offered. The following are the procedures to determine the value of common stock in three possible cases:

Case 1 (Zero Growth) The value of a stock with a constant dividend is given by:

$$S_0 = \frac{Div_1}{(1+r)} + \frac{Div_2}{(1+r)^2} + \dots = \frac{Div}{r}$$

Here it is assumed that  $Div_1 = Div_2 = \dots = Div$ . This is just an application of the perpetuity formula.

Case 2 (Constant Growth/Gordon Model) Dividends grow at rate  $g$ , as follows:

End of year	1	2	3	4	...
Dividend	$Div$	$Div(1+g)$	$Div(1+g)^2$	$Div(1+g)^3$	

Note that  $Div$  is the dividend at the end of the first period. The value of a common stock with dividends growing at a constant rate is:

$$S_0 = \frac{Div}{(1+r)} + \frac{Div(1+g)}{(1+r)^2} + \frac{Div(1+g)^2}{(1+r)^3} + \frac{Div(1+g)^3}{(1+r)^4} + \dots = \frac{Div}{r-g}$$

Where  $g$  is the growth rate and  $Div$  is the dividend on the stock at the end of the first period. This is just an application of the growing perpetuity formula.

Case 3 (Differential Growth) In this case, an algebraic formula would be too unwieldy. Instead, we present simple rule: in a differential dividend growth model you should discount all dividends separately and calculate the sum, which will result in the current value of common stock. General formula can be written as follows:

$$Stock\ Value = \sum_{i=1}^{\infty} \frac{Div_i}{(1+r)^i}$$

*Corporate Finance, 6th ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003;  
Chapter 27, pp.108-112*



## GENERAL FINANCE II

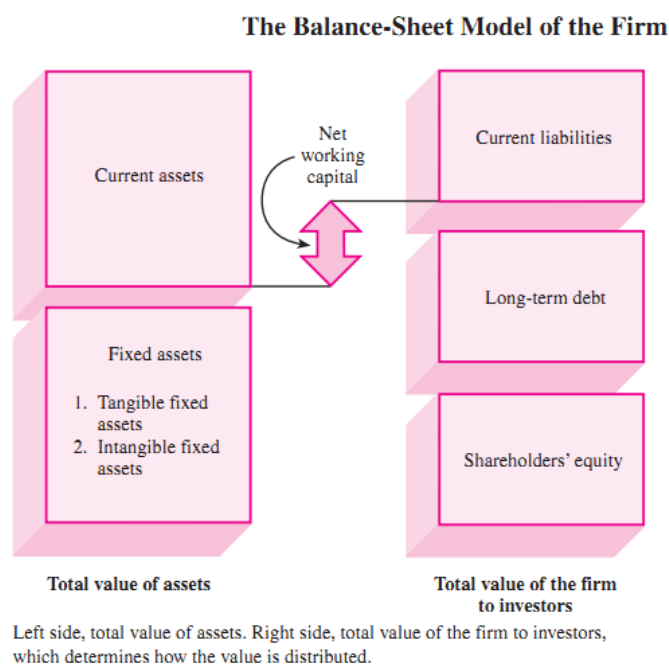
*Course book: Corporate Finance, 6th ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003*

*Course book: Fundamentals of Corporate Finance, 6th ed., Ross, Westerfield, Jordan; McGraw-Hill, 2003*

*Course book: Finance, 5th ed.;Groppelli, Ehsan Nikbakht; Barron's Inc. 2006*

### 60. Graph the balance sheet model of the firm. What questions does corporate finance study? Define each of them

Balance sheet model of the firm can be graphed as follows:



From the balance-sheet model of the firm it is easy to see why corporate finance can be thought of as the study of the following three questions:

1. *Capital Budgeting.* In what long-lived assets should the firm invest? This question concerns the left-hand side of the balance sheet. Of course, the type and proportions of assets the firm needs tend to be set by the nature of the business. We use the terms capital budgeting and capital expenditures to describe the process of making and managing expenditures on long-lived assets.
2. *Capital Structure.* How can the firm raise cash for required capital expenditures? This question concerns the right-hand side of the balance sheet. The answer to this

involves the firm's capital structure, which represents the proportions of the firm's financing from current and long-term debt and equity.

3. *Net Working Capital*. How should short-term operating cash flows be managed? This question concerns the upper portion of the balance sheet. There is often a mismatch between the timing of cash inflows and cash outflows during operating activities. Furthermore, the amount and timing of operating cash flows are not known with certainty. The financial managers must attempt to manage the gaps in cash flow. From a balance-sheet perspective, short-term management of cash flow is associated with a firm's net working capital. Net working capital is defined as current assets minus current liabilities. From a financial perspective, the short-term cash flow problem comes from the mismatching of cash inflows and outflows. It is the subject of short-term finance.

*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 1, pp. 2-9*

### **61. How do firms interact with financial markets? Show on the graph how the cash is generated**

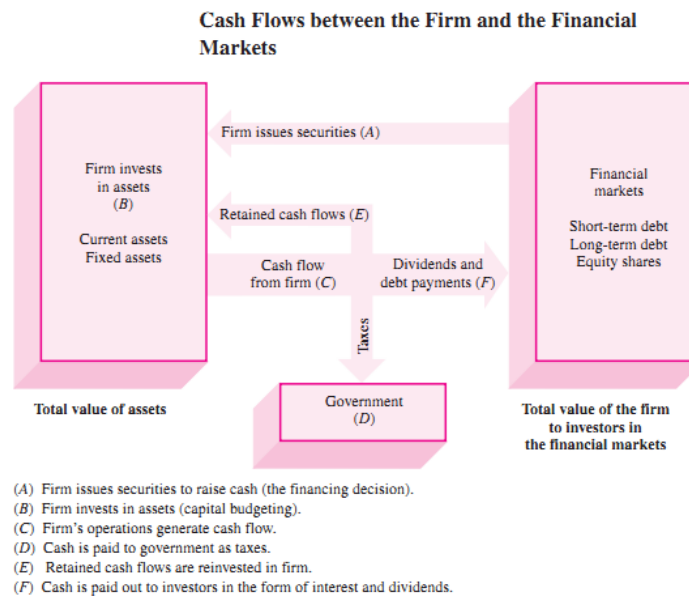
The most important job of a financial manager is to create value from the firm's capital budgeting, financing, and liquidity activities. How do financial managers create value?

1. The firm should try to buy assets that generate more cash than they cost.
2. The firm should sell bonds and stocks and other financial instruments that raise more cash than they cost.

Thus the firm must create more cash flow than it uses. The cash flows paid to bondholders and stockholders of the firm should be higher than the cash flows put into the firm by the bondholders and stockholders. To see how this is done, we can trace the cash flows from the firm to the financial markets and back again.

The interplay of the firm's finance with the financial markets is illustrated in the graph below. The arrows in the graph trace cash flow from the firm to the financial markets and back again. Suppose we begin with the firm's financing activities. To raise money the firm sells debt and equity shares to investors in the financial markets. This results in cash flows from the financial markets to the firm (A). This cash is invested in the investment activities of the firm (B) by the firm's management. The cash generated by the firm (C) is paid to shareholders and bondholders (F). The shareholders receive cash in the form of dividends; the bondholders who lent funds to the firm receive interest and, when the initial loan is repaid, principal. Not all of the firm's cash is paid out. Some is retained (E), and some is paid to the government as taxes (D).

Over time, if the cash paid to shareholders and bondholders (F) is greater than the cash raised in the financial markets (A), value will be created.



*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 1, pp. 5-9*

## 62. What are the four financial statements that all public companies must produce?

Financial statements, which are accounting reports, serve as the principal method of communicating financial information about a business entity or an individual to outside parties such as banks and investors.

The basic financial statements of businesses include the (1) balance sheet (or statement of financial position), (2) income statement, (3) the retained earnings statement, (4) cash flow statement.

The balance sheet is an accountant's snapshot of the firm's accounting value on a particular date, as though the firm stood momentarily still. The balance sheet has two sides: on the left are the assets and on the right are the liabilities and stockholders' equity. The balance sheet states what the firm owns and how it is financed. The accounting definition that underlies the balance sheet and describes the balance is

We have put three-line equality in the balance equation to indicate that it must always hold, by definition. In fact, the stockholders' equity is defined to be the difference between the assets and the liabilities of the firm. In principle, equity is what the stockholders would have remaining after the firm discharged its obligations.

The income statement measures performance over a specific period of time, say, a year. The accounting definition of income is:

$$\text{Revenue} - \text{Expenses} = \text{Income}$$

The income statement usually includes several sections. The operations section reports the firm's revenues and expenses from principal operations. One of particular importance is earnings before interest and taxes (EBIT), which summarizes earnings before taxes and financing costs. Among other things, the non-operating section of the income statement includes all financing costs, such as interest expense. Usually a second section reports as a separate item the amount of taxes levied on income. The last item on the income statement is the bottom line, or net income. Net income is frequently expressed per share of common stock, that is, earnings per share.

The statements of retained earnings explain the changes in a company's retained earnings over the reporting period. They break down changes in the owners' interest in the organization, and in the application of retained profit or surplus from one accounting period to the next. Line items typically include profits or losses from operations, dividends paid, issue or redemption of stock, and any other items charged or credited to retained earnings.

The statements are expected by Generally Accepted Accounting Principles and explain the owners' equity and retained earnings shown on the balance sheet, where:

$$\text{Owners' Equity} = \text{Assets} - \text{Liabilities}$$

Perhaps the most important item that can be extracted from financial statements is the actual cash flow of the firm. There is an official accounting statement called the statement of cash flows. This statement helps to explain the change in accounting cash and equivalents,

The first point we should mention is that cash flow is not the same as net working capital. For example, increasing inventory requires using cash. Because both inventories and cash are current assets, this does not affect net working capital. In this case, an increase in a particular net working capital account, such as inventory, is associated with decreasing cash flow.

Just as we established that the value of a firm's assets is always equal to the value of the liabilities and the value of the equity, the cash flows received from the firm's assets (that is, its operating activities),  $CF(A)$ , must equal the cash flows to the firm's creditors,  $CF(B)$ , and equity investors,  $CF(S)$ :

$$CF(A) = CF(B) + CF(S)$$

*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 2, pp. 29-35*

### **63. What is a firm worth with outstanding stock issued and without it?**

Like the general rule for valuing any asset, one way of thinking about the question of how much a firm is worth is to calculate the present value of its future expected cash flows. The value of the firm is found by multiplying its cash flows by the appropriate present value factor. The value of the firm, then, is simply the sum of the present values of the individual cash flows.

Though the above method is correct, it is difficult to implement, costly, time-consuming and even might be unnecessary if the firm under consideration has outstanding stocks issued that are publicly traded. When a stock is traded on a public exchange, in case of efficient

markets the stock price reflects all information about the company's future expected cash flows and its value today is measured by the stock's current market price. So, to value a firm with publicly traded stocks outstanding we can simply multiply the current market price of the stock by the number of stocks currently outstanding.

*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 4, pp.94-95*

#### 64. How the changes in capital structure affect the overall value of a firm? Show on the pie diagram

Sometimes it is useful to think of the firm as a pie. Initially, the size of the pie will depend on how well the firm has made its investment decisions. After a firm has made its investment decisions, it determines the value of its assets (e.g., its buildings, land, and inventories).

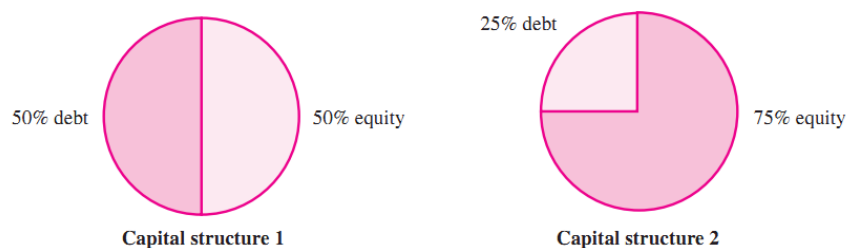
The firm can then determine its capital structure. The firm might initially have raised the cash to invest in its assets by issuing more debt than equity; now it can consider changing that mix by issuing more equity and using the proceeds to buy back some of its debt. Financing decisions like this can be made independently of the original investment decisions. The decisions to issue debt and equity affect how the pie is sliced.

The pie we are thinking of is depicted in figure below. The size of the pie is the value of the firm in the financial markets. We can write the value of the firm,  $V$ , as:

$$V = B + S$$

Where  $B$  is the value of the debt and  $S$  is the value of the equity. The pie diagrams consider two ways of slicing the pie: 50 percent debt and 50 percent equity, and 25 percent debt and 75 percent equity. The way the pie is sliced could affect its value. If so, the goal of the financial manager will be to choose the ratio of debt to equity that makes the value of the pie – that is, the value of the firm,  $V$  – as large as it can be.

**Two Pie Models of the Firm**



*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 1, pp.4-5*

## **65. Define the optimal capital structure for a firm**

Changing the capital structure of the firm changes the way the firm pays out its cash flows. Firms that borrow pay lower taxes than firms that do not. Because of corporate taxes, the value of a firm that borrows may be higher than the value of one that does not. However, with costly bankruptcy, a firm that borrows may have lower value. The combined effects of taxes and bankruptcy costs can produce an optimal capital structure.

*Changes in capital structure benefit the stockholders if and only if the value of the firm increases.*

Thus, Managers should choose the capital structure that they believe will have the highest firm value, because this capital structure will be most beneficial to the firm's stockholders.

*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003;  
Chapter 29, pp. 390-393*

## **66. What is Opportunity Cost of Capital**

The required rate of return is sometimes referred to as the cost of capital, or the opportunity cost of capital. Individuals have to decide where to invest the income they have saved. The goal, obviously, is to gain the highest return possible. To determine which assets are profitable and which are not, investors need a point of reference. This point of reference is known as the required rate of return. A manager of a firm, with the responsibility for making investment decisions, uses a similar point of reference. This point of reference, the firm's required rate of return, is called the cost of capital. The firm must earn a minimum rate of return to cover the cost of generating funds to finance investments; otherwise, no one will be willing to buy its bonds, preferred stock, and common stock. The goal of a financial officer is to achieve the highest efficiency and profitability from asset and, at the same time, keep the cost of the funds that the firm generates from various financing sources as low as possible. In other words, the cost of capital is the rate of return (cost) that a firm must pay investors to induce them to risk their funds and purchase the bonds, preferred stock, and common stock issued by the firm.

Clearly, the cost of capital is one of the major factors used in the determination of the value of the firm. In finance, the cost of capital is the same as the discount rate. High risk means a high cost of capital, while low risk means a low cost of capital. Moreover, a high cost of capital (high discount rate) usually means a low valuation for securities, and a low discount rate means a high value for the securities of a firm. Since the sale of these securities provides firms with funds for investments, the cost of financing increases when the value of securities is low and it decreases when their value is high. The benchmark for determining whether the returns of a firm's securities are high or low is the cost of capital.

*Finance, 5th ed.; Groppelli, Ehsan Nikbakht; Barron's Inc. 2006, Chapter 9*

### 67. What is weighted average cost of capital (WACC)?

Suppose a firm uses both debt and equity to finance its investments. If the firm pays  $r_B$  for its debt financing and  $r_S$  for its equity, what is the overall or average cost of its capital? The cost of equity is  $r_S$ , as discussed in earlier sections. The cost of debt is the firm's borrowing rate,  $r_B$ . If a firm uses both debt and equity, the cost of capital is a weighted average of each. This works out to be

$$\frac{S}{S+B}r_S + \frac{B}{S+B}r_B$$

If the firm had issued no debt and was therefore an all-equity firm, its average cost of capital would equal its cost of equity,  $r_S$ . At the other extreme, if the firm had issued so much debt that its equity was valueless, it would be an all-debt firm, and its average cost of capital would be its cost of debt,  $r_B$ .

Of course, interest is tax deductible at the corporate level

$$\text{Cost of Debt (After Corporate Tax)} = r_B(1 - T_C)$$

Assembling these results, we get the average cost of capital (after tax) for the firm:

$$\text{Average Cost of Capital} = \frac{S}{S+B}r_S + \frac{B}{S+B}r_B$$

Because the average cost of capital is a weighting of its cost of equity and its cost of debt, it is usually referred to as the **weighted average cost of capital**,  $r_{WACC}$

*Corporate Finance, 4<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 12, pp. 321*

### 68. Describe three methods of valuation using leverage

**The adjusted-present-value (APV) approach:** first values the project on an all-equity basis. That is, the project's after-tax cash flows under all-equity financing (called unlevered cash flows, or UCF) are placed in the numerator of the capital-budgeting equation. The discount rate, assuming all-equity financing, appears in the denominator. Then add the net present value of the debt. Point out that the net present value of the debt is likely to be the sum of four parameters: tax effects, flotation costs, bankruptcy costs, and interest subsidies.

$$APV = \sum_{t=1}^{\infty} \frac{UCF_t}{(1+r_0)^t} + \text{Additional effects of debt} - \text{initial investment}$$

$UCF_t =$  The project's cash flows at date  $t$  to the equity holders of an unlevered firm

$r_0 =$  Cost of capital for project in an unlevered firm

**The flow-to-equity (FTE) approach** discounts the after-tax cash flow from a project going to the equity holders of a levered firm (LCF). LCF, which stands for levered cash flow,

is the residual to equityholders after interest has been deducted. The discount rate is  $r_s$ , the cost of capital to the equityholders of a levered firm. For a firm with leverage,  $r_s$  must be greater than  $r_0$ , the cost of capital for an unlevered firm.

$$FTE = \sum_{t=1}^{\infty} \frac{LCF_t}{(1+r_s)^t} - (\text{initial investment} - \text{Amount borrowed})$$

$UCF_t =$  The project's cash flows at date  $t$  to the equityholders of a levered firm

$r_s =$  Cost of equity capital with levered firm

**The weighted-average-cost-of-capital (WACC) method:** This technique calculates the project's after-tax cash flows assuming all-equity financing (UCF). The UCF is placed in the numerator of the capital-budgeting equation. The denominator,  $r_{WACC}$ , is a weighted average of the cost of equity capital and the cost of debt capital. The tax advantage of debt is reflected in the denominator because the cost of debt capital is determined net of corporate tax. The numerator does not reflect debt at all.

$$WACC = \sum_{t=1}^{\infty} \frac{UCF_t}{(1+r_{WACC})^t} - \text{initial investment}$$

$r_{WACC} =$  Weighted average cost of capital

Notes:

- The middle term in the APV formula implies that the value of a project with leverage is greater than the value of the project without leverage. Since  $r_{WACC} < r_0$ , the WACC formula implies that the value of a project with leverage is greater than the value of the project without leverage.
- In the FTE method, cash flow after interest (LCF) is used. Initial investment is reduced by amount borrowed as well.

Guidelines:

- Use WACC or FTE if the firm's target debt-to-value ratio applies to the project over its life.
- Use APV if the project's level of debt is known over the life of the project.

*Corporate Finance, 6th ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003;  
Chapter 17, pp. 474-476*

## 69. What is homemade leverage?

Modigliani and Miller show a blindingly simple result: If levered firms are priced too high, rational investors will simply borrow on their personal accounts to buy shares in unlevered firms. This substitution is often called *homemade leverage*. As long as individuals



borrow (and lend) on the same terms as the firms, they can duplicate the effects of corporate leverage on their own.

*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 15, pp. 397*

### 70. Define MM Propositions I & II with and without Taxes. Show the results graphically

Modigliani and Miller (or simply MM) showed that leverage would not affect the total value of the firm and this result holds more generally under a set of conditions referred to as perfect capital markets:

- No taxes. Investors and firms can trade the same set of securities at competitive market prices equal to the present value of their future cash flows
- No transaction costs. There are no taxes, transaction costs, or issuance costs associated with security trading.
- Individuals and corporations borrow at same rate. A firm's financing decisions do not change the cash flows generated by its investments, nor do they reveal new information about them.

Under these conditions, MM demonstrated the following result regarding the role of capital structure in determining firm value:

*MM Proposition I: In a perfect capital market, the total value of a firm is equal to the market value of the total cash flows generated by its assets and is not affected by its choice of capital structure.*

*MM Proposition II: The cost of capital of levered equity increases with the firm's market value debt – equity ratio.*

MM Proposition I (no taxes):  $V_L = V_U$  (Value of levered firm equals value of unlevered firm)

MM Proposition II (no taxes):  $r_S = r_0 + \frac{B}{S}(r_0 - r_B)$ , where  $r_0$  is the cost of unlevered equity

Intuition:

Proposition I: Through **homemade leverage**, individuals can either duplicate or undo the effects of corporate leverage. If investors would like more leverage than the firm have chosen they can borrow and add leverage to their own portfolio.

Proposition II: The cost of equity rises with leverage, because the risk to equity rises with leverage.

After incorporating  $t_C$ , corporate tax rate on earnings after interest Modigliani-Miller propositions showed that the total value of the levered firm exceeds the value of the firm without leverage due to the present value of the tax savings from debt:

MM Proposition I (with taxes):  $V_L = V_U + T_C B$  (for a firm with perpetual debt)

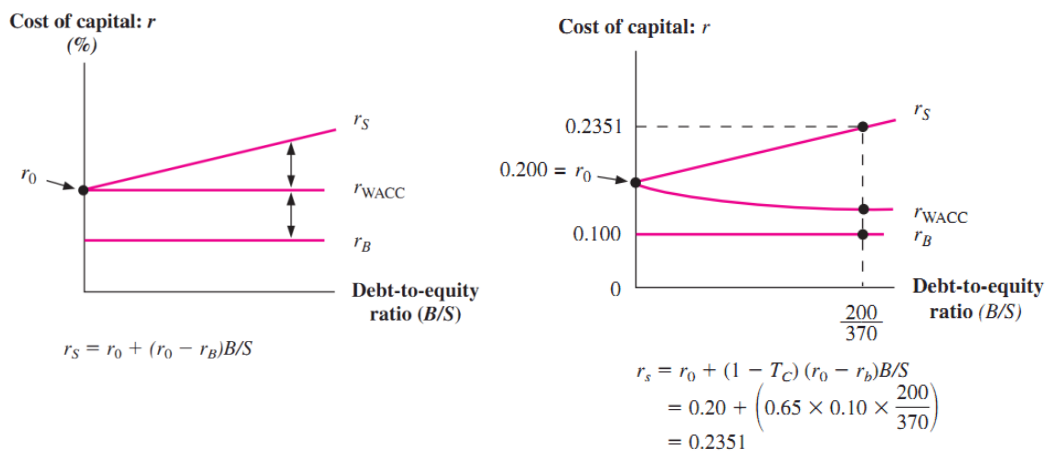
MM Proposition I (with taxes):  $r_S = r_0 + \frac{B}{S}(1 - t_C)(r_0 + r_B)$

Intuition:

Proposition I: Since corporations can deduct interest payments but not dividend payments, corporate leverage lowers tax payments.

Proposition II: The cost of equity rises with leverage, because the risk to equity rises with leverage.

Graphically the above propositions can be illustrated as follows:



*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 15, pp.393-407*

*Corporate finance, 3rd ed., Jonathan Berk, Peter DeMarzo; Pearson 2014; Chapter 14-15, pp. 483-511*

### 71. Explain the limits to the use of debt. What costs are associated with the financial distress?

A frequently asked question is, “Does the MM theory with taxes predict the capital structure of typical firms?” The typical answer is “yes”, but unfortunately, “no.” The theory states that  $V_L = V_U + T_C B$ . According to this equation, one can always increase firm value by increasing leverage, implying that firms should issue maximum debt. This is inconsistent with the real world, where firms generally employ only moderate amounts of debt.

However, the MM theory tells us where to look when searching for the determinants of capital structure. For example, the theory ignores bankruptcy and its attendant costs. Because these costs are likely to get out of hand for a highly levered firm, the moderate leverage of most firms can now easily be explained. Our discussion leads quite naturally to the idea that a firm's capital structure can be thought of as a trade-off between the tax benefits of debt and the *costs of financial distress and bankruptcy*. This trade-off of benefits and costs leads to an optimum amount of debt.

In addition, the MM theory ignores personal taxes. In the real world, the personal tax rate on interest is higher than the effective personal tax rate on equity distributions. Thus, the personal tax penalties to bondholders tend to offset the tax benefits to debt at the corporate level. Even when bankruptcy costs are ignored, this idea can be shown to imply that there is an optimal amount of debt for the economy as a whole.

If these obligations are not met, the firm may risk some sort of financial distress. The ultimate distress is bankruptcy, where ownership of the firm's assets is legally transferred from the stockholders to the bondholders. These debt obligations are fundamentally different from stock obligations. While stockholders like and expect dividends, they are not legally entitled to dividends in the way bondholders are legally entitled to interest and principal payments.

The possibility of bankruptcy has a negative effect on the value of the firm. However, it is not the risk of bankruptcy itself that lowers value. Rather it is the costs associated with bankruptcy that lower value.

#### *Direct Costs of Financial Distress*

Those are legal and administrative costs of liquidation or reorganization. In the process, lawyers are involved throughout all the stages before and during bankruptcy. With fees often in the hundreds of dollars an hour, these costs can add up quickly. A wag once remarked that bankruptcies are to lawyers what blood is to sharks. In addition, administrative and accounting fees can substantially add to the total bill. And if a trial takes place, we must not forget expert witnesses. Each side may hire a number of these witnesses to testify about the fairness of a proposed settlement. Their fees can easily rival those of lawyers or accountants.

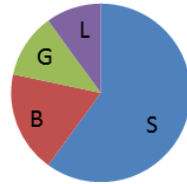
#### *Indirect Costs of Financial Distress*

Indirect costs of financial distress, in other words, can be stated as impaired ability to conduct business. Bankruptcy hampers conduct with customers and suppliers. Sales are frequently lost because of both fear of impaired service and loss of trust. For example, many loyal Chrysler customers switched to other manufacturers when Chrysler skirted insolvency in the 1970s. These buyers questioned whether parts and servicing would be available were Chrysler to fail. Sometimes the taint of impending bankruptcy is enough to drive customers away. Though these costs clearly exist, it is quite difficult to measure them.

Taxes and bankruptcy costs can be viewed as just another claim on the cash flows of the firm. Let  $G$  and  $L$  stand for payments to the government and bankruptcy lawyers, respectively. Then value of a firm,  $V$ , is:

$$V = S + B + G + L$$

The more are  $G$  and  $L$ , the less is left to  $S$  and  $B$ . Graphically this looks as follows on a pie diagram:



When a firm has debt, conflicts of interest arise between stockholders and bondholders. Because of this, stockholders are tempted to pursue selfish strategies. These conflicts of interest, which are magnified when financial distress is incurred, impose agency costs on the firm. Three kinds of selfish strategies that stockholders use to hurt the bondholders and help themselves are:

1. Selfish Investment Strategy 1 – Incentive to Take Large Risks: Firms near bankruptcy often take great chances, because they believe that they are playing with someone else's money.
2. Selfish Investment Strategy 2 – Incentive toward Underinvestment: Stockholders of a firm with a significant probability of bankruptcy often find that new investment helps the bondholders at the stockholders' expense.
3. Selfish Investment Strategy 3 – Milking the Property: Another strategy is to pay out extra dividends or other distributions in times of financial distress, leaving less in the firm for the bondholders. This is known as milking the property, a phrase taken from real estate.

It's worth mentioning that strategies 2 and 3 are very similar. In strategy 2, the firm chooses not to raise new equity. Strategy 3 goes one step further, because equity is actually withdrawn through the dividend.

*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 16, pp.423-430*

## **72. What difference is between authorized and issued common stock? Define the treasury stock?**

Shares of common stock are the fundamental ownership units of the corporation. The articles of incorporation of a new corporation must state the number of shares of common stock the corporation is authorized to issue.

The board of directors of the corporation, after a vote of the shareholders, can amend the articles of incorporation to increase the number of shares authorized; there is no limit to the number of shares that can be authorized. For example in 1999 Anheuser-Busch had authorized 1.6 billion shares and had issued 716.1 million shares. There is no requirement that all of the authorized shares actually be issued. Although there are no legal limits to authorizing shares of stock, some practical considerations may exist:

1. Some states impose taxes based on the number of authorized shares.
2. Authorizing a large number of shares may create concern on the part of investors, because authorized shares can be issued later with the approval of the board of directors but without a vote of the shareholders.

Companies also might maintain *treasury stock*. The shares bought back are called treasury stock. Consequently, treasury stock is the name given to previously issued stock that has been repurchased by the firm.

*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 14, p.372*

### **73. What happens when the firm issues either too much debt or too much stock?**

Financial leverage is related to the extent to which a firm relies on debt financing rather than equity. Measures of financial leverage are tools in determining the probability that the firm will default on its debt contracts. The more debt a firm has, the more likely it is that the firm will become unable to fulfill its contractual obligations. In other words, too much debt can lead to a higher probability of insolvency and financial distress.

On the positive side, debt is an important form of financing, and provides a significant tax advantage because interest payments are tax deductible. If a firm uses debt, creditors and equity investors may have conflicts of interest. Creditors may want the firm to invest in less risky ventures than those the equity investors prefer.

Too much stock might also be a bad thing for the company. First of all, issuing new shares is not always welcomed by existing shareholders mainly due to decreased percentage share and voting rights. Secondly, too many new shareholders with additional voting rights might have significant power in selecting new members of board of directors, thus, possibly causing restructuring or some other pleasant or unpleasant changes in the management of the company.

*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 2, p. 35*

### **74. Explain the market, book and replacement value of a firm**

Market value is a concept distinct from market price, which is “the price at which one can transact”, while market value is “the true underlying value” according to theoretical

standards. The concept is most commonly invoked in inefficient markets or disequilibrium situations where prevailing market prices are not reflective of true underlying market value. For market price to equal market value, the market must be informational efficient and rational expectations must prevail. Market value is also distinct from fair value in that fair value depends on the parties involved, while market value does not. Fair value is frequently used when undertaking due diligence in corporate transactions, where particular synergies between the two parties may mean that the price that is fair between them is higher than the price that might be obtainable in the wider market.

$$\text{Market Value} = \text{price of the stock} \times \text{number of shares outstanding}$$

Book Value or carrying value is the sum of par value, capital surplus, and accumulated retained earnings are the common equity of the firm, usually referred to as the book value of the firm.

*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003;  
Chapter 14, pp. 373-374*

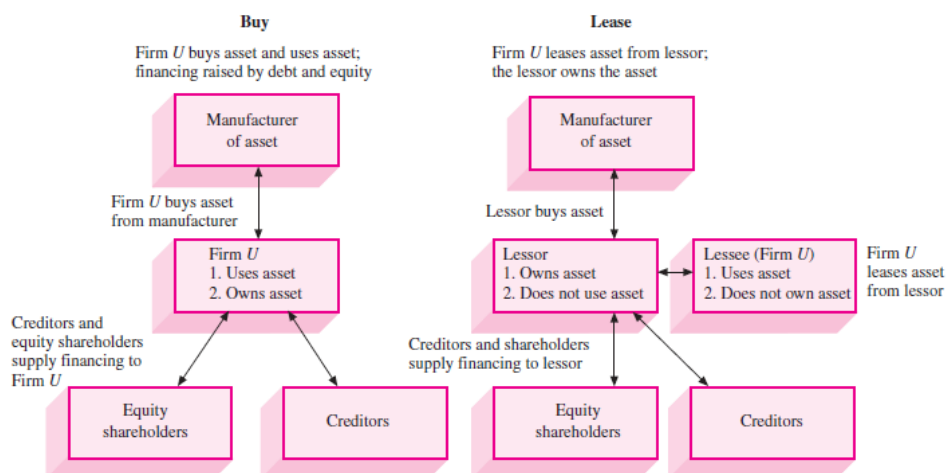
## 75. Describe buying versus leasing asset decision

A lease is a contractual agreement between a lessee and lessor. The agreement establishes that the lessee has the right to use an asset and in return must make periodic payments to the lessor, the owner of the asset. The lessor is either the asset's manufacturer or an independent leasing company. If the lessor is an independent leasing company, it must buy the asset from a manufacturer. Then the lessor delivers the asset to the lessee, and the lease goes into effect.

As far as the lessee is concerned, it is the use of the asset that is most important, not who owns the asset. The use of an asset can be obtained by a lease contract. Because the user can also buy the asset, leasing and buying involve alternative financing arrangements for the use of an asset. This is illustrated in Figure below.

The specific example in the figure happens often in the computer industry. Firm U, the lessee, might be a hospital, a law firm, or any other firm that uses computers. The lessor is an independent leasing company who purchased the equipment from a manufacturer such as IBM or Apple. Leases of this type are called *direct leases*. In the figure, the lessor issued both debt and equity to finance the purchase.

Of course, a manufacturer like IBM could lease its own computers, though we do not show this situation in the example. Leases of this type are called *sales-type leasing*. In this case, IBM would compete with the independent computer-leasing company.



*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 21, pp. 586-587*

## 76. Define operating leases

Years ago, a lease where the lessee received an operator along with the equipment was called an *operating lease*. Though the operating lease defies an exact definition today, this form for leasing has several important characteristics.

- Operating leases are usually not fully amortized. This means that the payments required under the terms of the lease are not enough to recover the full cost of the asset for the lessor. This occurs because the term or life of the operating lease is usually less than the economic life of the asset. Thus, the lessor must expect to recover the costs of the asset by renewing the lease or by selling the asset for its residual value.
- Operating leases usually require the lessor to maintain and insure the leased assets.
- Perhaps the most interesting feature of an operating lease is the cancellation option. This option gives the lessee the right to cancel the lease contract before the expiration date. If the option to cancel is exercised, the lessee must return the equipment to the lessor. The value of a cancellation clause depends on whether future technological and/or economic conditions are likely to make the value of the asset to the lessee less than the value of the future lease payments under the lease.

*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 21, pp. 587*

## 77. Define financial leases

*Financial leases* are the exact opposite of operating leases, as is seen from their important characteristics:

- Financial leases do not provide for maintenance or service by the lessor.
- Financial leases are fully amortized.
- The lessee usually has a right to renew the lease on expiration.
- Generally, financial leases cannot be canceled. In other words, the lessee must make all payments or face the risk of bankruptcy.

Because of the above characteristics, particularly (2), this lease provides an alternative method of financing to purchase. Hence, its name is a sensible one. Two special types of financial leases are the sale *and lease-back arrangement* and *the leveraged lease*.

A *sale and lease-back* occurs when a company sells an asset it owns to another firm and immediately leases it back. In a sale and lease-back, two things happen:

- The lessee receives cash from the sale of the asset.
- The lessee makes periodic lease payments, thereby retaining use of the asset.

A *leveraged lease* is a three-sided arrangement among the lessee, the lessor, and the lenders:

- As in other leases, the lessee uses the assets and makes periodic lease payments.
- As in other leases, the lessor purchases the assets, delivers them to the lessee, and collects the lease payments. However, the lessor puts up no more than 40 to 50 percent of the purchase price.
- The lenders supply the remaining financing and receive interest payments from the lessor.

The lenders in a leveraged lease typically use a nonrecourse loan. This means that the lessor is not obligated to the lender in case of a default. However, the lender is protected in two ways:

1. The lender has a first lien on the asset.
2. In the event of loan default, the lease payments are made directly to the lender.

The lessor puts up only part of the funds but gets the lease payments and all the tax benefits of ownership. These lease payments are used to pay the debt service of the nonrecourse loan. The lessee benefits because, in a competitive market, the lease payment is lowered when the lessor saves taxes.

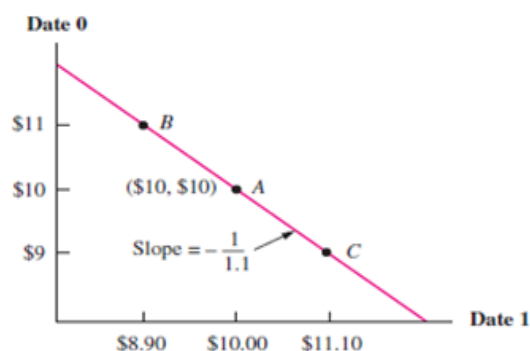
*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 21, pp. 588*



## 78. Explain the concept of homemade dividends

Suppose individual investor X prefers dividends per share of \$10 at both dates 0 and 1. Would she be disappointed when informed that the firm's management is adopting the alternative dividend policy (dividends of \$11 and \$8.90 on the two dates, respectively)? Not necessarily, because she could easily reinvest the \$1 of unneeded funds received on date 0, yielding an incremental return of \$1.10 at date 1. Thus, she would receive her desired net cash flow of  $\$11 - \$1 = \$10$  at date 0 and  $\$8.90 + \$1.10 = \$10$  at date 1.

Conversely, imagine investor Z preferring \$11 of cash flow at date 0 and \$8.90 of cash flow at date 1, who finds that management, will pay dividends of \$10 at both dates 0 and 1. Here he can sell off shares of stock at date 0 to receive the desired amount of cash flow. That is, if he sells off shares (or fractions of shares) at date 0 totaling \$1, his cash flow at date 0 becomes  $\$10 + \$1 = \$11$ . Because a sale of \$1 stock at date 0 will reduce his dividends by \$1.10 at date 1, his net cash flow at date 1 would be  $\$10 - \$1.10 = \$8.90$ .



The graph illustrates both (1) how managers can vary dividend policy and (2) how individuals can undo the firm's dividend policy.

*Manager varying dividend policy.* A firm paying out all cash flows immediately is at point A on the graph. The firm could achieve point B by issuing stock to pay extra dividends or achieve point C by buying back old stock with some of its cash.

*Individuals undoing the firm's dividend policy.* Suppose the firm adopts the dividend policy represented by point B: dividends of \$11 at date 0 and \$8.90 at date 1. An investor can reinvest \$1 of the dividends at 10 percent, which will place her at point A.

Suppose, alternatively, the firm adopts the dividend policy represented by point A. An individual can sell off \$1 of stock at date 0, placing him at point B. No matter what dividend policy the firm establishes, a shareholder can undo it. The example illustrates how investors can make *homemade dividends*.

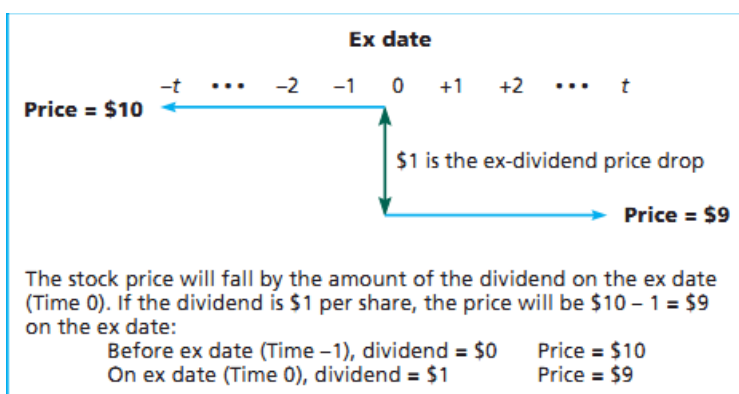
### 79. Explain the importance of dividend policy and what effect it has on a stock price

The dividend decision is important because it determines the payout received by shareholders and the funds retained by the firm for investment. Dividend policy is usually reflected by the current dividend-to-earnings ratio. This is referred to as the payout ratio. Unfortunately, the optimal payout ratio cannot be determined quantitatively. Rather, one can only indicate qualitatively what factors lead to low – or high – dividend policies.

The mechanics of a cash dividend payment are divided into four dates:

1. Declaration date – the board of directors passes a resolution to pay a dividend
2. Ex – dividend date – two business days before the date of record
3. Date of record – preparation of a list of all individuals believed to be stockholders
4. Date of payment – dividend checks are mailed

Interesting to note, that the stock price will fall by the amount of the dividend on the ex – dividend date as illustrated below:



Dividend policy is strategic decision that hugely determines the price of the firm's stock. According to Gordon model, investors price a security by forecasting and discounting future dividends. So, once dividend policy is declared, it is expected to be committed and drive the stock price. Thus, changes in dividend policy will be followed by changes in current stock price.

*Fundamentals of Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jordan; McGraw-Hill, 2003; Chapter 4, pp. 606-616*

### 80. What are the reasons for holding cash? Draw the operating and the cash cycles?

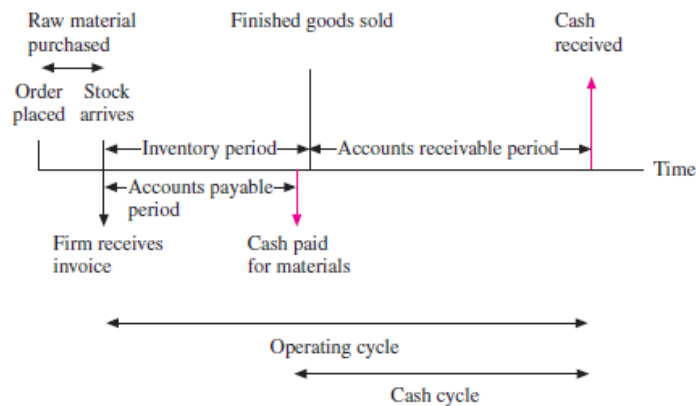
There are two primary reasons for holding cash. First, cash is needed to satisfy the *transactions motive*. Transactions-related needs come from normal disbursement and

collection activities of the firm. The disbursement of cash includes the payment of wages and salaries, trade debts, taxes, and dividends. Cash is collected from sales from operations, sales of assets, and new financing. The cash inflows (*collections*) and outflows (*disbursements*) are not perfectly synchronized, and some level of cash holdings is necessary to serve as a buffer. If the firm maintains too small a cash balance, it may run out of cash. If so, it must sell marketable securities or borrow. Selling marketable securities and borrowing involve trading costs.

Another reason to hold cash is for *compensating balances*. Cash balances are kept at commercial banks to compensate for banking services rendered to the firm. A minimum required compensating balance at banks providing credit services to the firm may impose a lower limit on the level of cash a firm holds.

*The operating cycle* is the time period from the arrival of stock until the receipt of cash. (Sometimes the operating cycle is defined to include the time from placement of the order until arrival of the stock.) The *cash cycle* begins when cash is paid for materials and ends when cash is collected from receivables

### Cash Flow Time Line and the Short-Term Operating Activities of a Typical Manufacturing Firm



*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 28, pp. 772*

### 81. Is it possible for a firm to have too much cash? Why would shareholder care if a firm accumulates large amounts of cash?

Once a firm has more cash than it needs for operations and planned expenditures, the excess cash has an opportunity cost. It could be invested (by shareholders) in potentially more profitable ways.

Large amount of cash is the concern for shareholders because excess cash on hand can lead to poorly thought-out investments. The thought is that keeping cash levels relatively low forces management to pay careful attention to cash flow and capital spending.

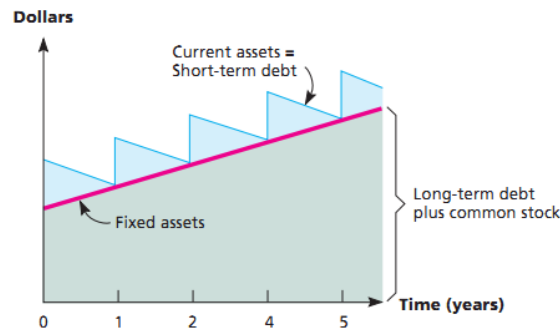
If a firm has too much cash it can simply pay a dividend, or, more likely based on the financial environment, buy back stock. It can also reduce debt. If the firm has insufficient cash, then it must borrow, sell stock, or improve profitability.

*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 27*

## 82. In an ideal company, NWC is always zero. Why might NWC be positive in a real company?

In an ideal economy the firm could perfectly predict its short-term uses and sources of cash, and net working capital could be kept at zero. In the real world, cash and net working capital provides a buffer that lets the firm meet its ongoing obligations. The financial manager seeks the optimal level of each of the current assets.

The situation is shown on a graph below. Long-term assets are assumed to grow over time, whereas current assets increase at the end of the harvest and then decline during the year. Short-term assets end up at zero just before the next harvest. Current (short-term) assets are financed by short-term debt, and long-term assets are financed with long-term debt and equity. Net working capital – current assets minus current liabilities – is always zero.



In an ideal world, net working capital is always zero because short-term assets are financed by short-term debt.

In the real world, it is not likely that current assets will ever drop to zero. For example, a long-term rising level of sales will result in some permanent investment in current assets. Moreover, the firm's investments in long-term assets may show a great deal of variation. A growing firm can be thought of as having a total asset requirement consisting of the current assets and long-term assets needed to run the business efficiently. The total asset requirement may exhibit change over time for many reasons, including (1) a general growth trend, (2) seasonal variation around the trend, and (3) unpredictable day-to-day and month-to-month fluctuations.

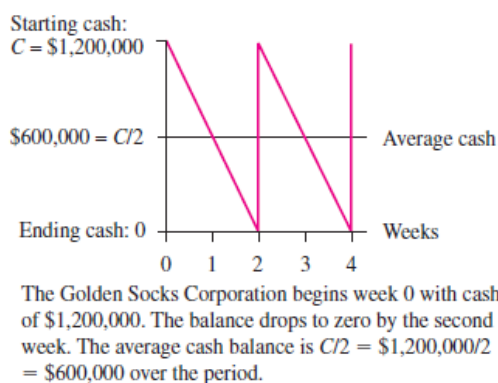
*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 27, pp. 651-654*

### 83. Describe The Baumol Model of cash management

William Baumol was the first to provide a formal model of cash management incorporating opportunity costs and trading costs. His model can be used to establish the target cash balance.

Suppose the Golden Socks Corporation began week 0 with a cash balance of  $C = \$1.2$  million and outflows exceed inflows by  $\$600,000$  per week. Its cash balance will drop to zero at the end of week 2, and its average cash balance will be  $C/2 = \$1.2\text{million}/2 = \$600,000$  over the two-week period. At the end of week 2, Golden Socks must replace its cash either by selling marketable securities or by borrowing. Figure below shows this situation.

**Cash Balances for the Golden Socks Corporation**



If  $C$  were set higher, say, at  $\$2.4$  million, cash would last four weeks before the firm would need to sell marketable securities, but the firm's average cash balance would increase to  $\$1.2$  million (from  $\$600,000$ ). If  $C$  were set at  $\$600,000$ , cash would run out in one week and the firm would need to replenish cash more frequently, but its average cash balance would fall from  $\$600,000$  to  $\$300,000$ .

Because transactions costs must be incurred whenever cash is replenished (for example, the brokerage costs of selling marketable securities), establishing large initial cash balances will lower the trading costs connected with cash management. However, the larger the average cash balance, the greater the opportunity cost (the return that could have been earned on marketable securities).

To solve this problem, Golden Socks needs to know the following three things:

- $F =$  The fixed cost of selling securities to replenish cash
- $T =$  The total amount of new cash needed for transactions purposes over the relevant planning period, say, one year
- and
- $K =$  The opportunity cost of holding cash; this is the interest rate on marketable securities

With this information, Golden Socks can determine the total costs of any particular cash balance policy. It can then determine the optimal cash-balance policy.

The total opportunity costs of cash balances, in dollars, must be equal to the average cash balance multiplied by the interest rate, or

$$\text{Opportunity costs}(\$) = (C/2) \times K$$

The opportunity costs of various alternatives are given here:

Initial Cash Balance	Average Cash Balance	Opportunity Costs (K = 0.10)
<i>C</i>	<i>C/2</i>	$(C/2) \times K$
\$4,800,000	\$2,400,000	\$240,000
2,400,000	1,200,000	120,000
1,200,000	600,000	60,000
600,000	300,000	30,000
300,000	150,000	15,000

Total trading costs can be determined by calculating the number of times that Golden Socks must sell marketable securities during the year. The total amount of cash disbursement during the year is  $\$600,000 \times 52 \text{ weeks} = \$31.2 \text{ million}$ . If the initial cash balance is set at \$1.2 million, Golden Socks will sell \$1.2 million of marketable securities every two weeks. Thus, trading costs are given by

$$\frac{31.2 \text{ million}}{1.2 \text{ million}} \times F = 26F$$

The general formula is:

$$\text{Trading costs}(\$) = (T/C) \times F$$

A schedule of alternative trading costs follows:

Total Disbursements during Relevant Period	Initial Cash Balance	Trading Costs (F = \$1,000)
<i>T</i>	<i>C</i>	$(T/C) \times F$
\$31,200,000	\$4,800,000	\$ 6,500
31,200,000	2,400,000	13,000
31,200,000	1,200,000	26,000
31,200,000	600,000	52,000
31,200,000	300,000	104,000

The total cost of cash balances consists of the opportunity costs plus the trading costs:

Cash Balance	Total Cost	=	Opportunity Costs	+	Trading Costs
\$4,800,000	\$246,500		\$240,000		\$ 6,500
2,400,000	133,000		120,000		13,000
1,200,000	86,000		60,000		26,000
600,000	82,000		30,000		52,000
300,000	119,000		15,000		104,000

$$\text{Total cost} = \text{Opportunity costs} + \text{Trading costs} = (C/2) \times K + (T/C) \times F$$

Limitations of the Baumol model represent an important contribution to cash management. The limitations of the model include the following:

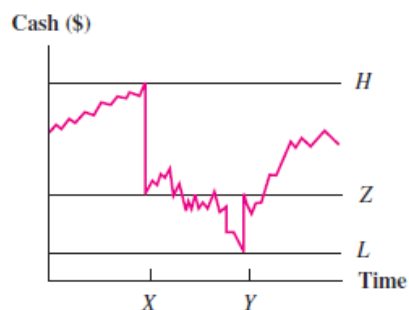
1. *The Model Assumes the Firm Has a Constant Disbursement Rate.* In practice, disbursements can be only partially managed, because due dates differ and costs cannot be predicted with certainty.
2. *The Model Assumes There Are No Cash Receipts during the Projected Period.* In fact, most firms experience both cash inflows and outflows on a daily basis.
3. *No Safety Stock Is Allowed For.* Firms will probably want to hold a safety stock of cash designed to reduce the possibility of a cash shortage or cash-out. However, to the extent that firms can sell marketable securities or borrow in a few hours, the need for a safety stock is minimal.

*Corporate Finance, 6th ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 28, pp. 773-776*

#### 84. Describe Miller-Orr model of cash management

Merton Miller and Daniel Orr developed a cash-balance model to deal with cash inflows and outflows that fluctuate randomly from day to day. In the Miller-Orr model, both cash inflows and cash outflows are included. The model assumes that the distribution of daily

##### The Miller-Orr Model



*H* is the upper control limit. *L* is the lower control limit. The target cash balance is *Z*. As long as cash is between *L* and *H*, no transaction is made.

Net cash flows (cash inflow minus cash outflow) are normally distributed. On each day the net cash flow could be the expected value or some higher or lower value. We will assume that the expected net cash flow is zero.

Figure above shows how the Miller-Orr model works. The model operates in terms of upper (H) and lower (L) control limits, and a target cash balance (Z). The firm allows its cash balance to wander randomly within the lower and upper limits. As long as the cash balance is between H and L, the firm makes no transaction. When the cash balance reaches H, such as at point X, then the firm buys  $H - Z$  units (or dollars) of marketable securities. This action will decrease the cash balance to Z. In the same way, when cash balances fall to L, such as at point Y (the lower limit), the firm should sell  $Z - L$  securities and increase the cash balance to Z. In both situations, cash balances return to Z. Management sets the lower limit, L, depending on how much risk of a cash shortfall the firm is willing to tolerate.

Like the Baumol model, the Miller-Orr model depends on trading costs and opportunity costs. The cost per transaction of buying and selling marketable securities, F, is assumed to be fixed. The percentage opportunity cost per period of holding cash, K, is the daily interest rate on marketable securities. Unlike the Baumol model, the number of transactions per period is a random variable that varies from period to period, depending on the pattern of cash inflows and outflows.

As a consequence, trading costs per period are dependent on the expected number of transactions in marketable securities during the period. Similarly, the opportunity costs of holding cash are a function of the expected cash balance per period.

Given L, which is set by the firm, the Miller-Orr model solves for the target cash balance, Z, and the upper limit, H. Expected total costs of the cash-balance–return policy (Z,H) are equal to the sum of expected transactions costs and expected opportunity costs. The values of Z (the return-cash point) and H (the upper limit) that minimize the expected total cost have been determined by Miller and Orr:

$$Z^* = \sqrt[3]{3F\sigma^2/4K} + L$$

$$H^* = 3Z^* - 2L$$

Where \* denotes optimal values, and  $\sigma^2$  is the variance of net daily cash flows. The average cash balance in the Miller-Orr model is

$$\text{Average cash balance} = \frac{4Z - L}{3}$$

To clarify the Miller-Orr model, suppose  $F = \$1,000$ , the interest rate is 10 percent annually, and the standard deviation of daily net cash flows is \$2,000. The daily opportunity cost, K, is

$$(1 + K)^{365} - 1 = 0.10$$

$$1 + K = \sqrt[365]{1.10} = 1.000261$$

$$K = 0.000261$$

The variance of daily net cash flows is:

$$\sigma^2 = 2000^2 = 4000000$$

Let us assume that  $L = 0$



$$Z^* = \sqrt[3]{\frac{3 \times 1000 \times 4000000}{4 \times 0.000261}} + 0 = \sqrt[3]{11493900000000} = \$22568$$

$$H^* = 3 \times 22568 = \$67704$$

$$\text{Average cash balance} = \frac{4 \times 22568 - 0}{3} = \$30091$$

*Implications of the Miller-Orr Model*, the manager must do four things.

1. Set the lower control limit for the cash balance. This lower limit can be related to a minimum safety margin decided on by management.
2. Estimate the standard deviation of daily cash flows.
3. Determine the interest rate.
4. Estimate the trading costs of buying and selling marketable securities.

These four steps allow the upper limit and return point to be computed. Miller and Orr tested their model using nine months of data for cash balances for a large industrial firm. The model was able to produce average daily cash balances much lower than the averages actually obtained by the firm.

The Miller-Orr model clarifies the issues of cash management. First, the model shows that the best return point,  $Z^*$ , is positively related to trading costs,  $F$ , and negatively related to  $K$ . This finding is consistent with and analogous to the Baumol model. Second, the Miller-Orr model shows that the best return point and the average cash balance are positively related to the variability of cash flows. That is, firms whose cash flows are subject to greater uncertainty should maintain a larger average cash balance.

*Corporate Finance, 6th ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003;  
Chapter 28, pp. 776-778*

## **85. What are the costs of shortage / carrying costs? Describe them**

Managing current assets can be thought of as involving a trade-off between costs that rise with the level of investment and costs that fall with the level of investment. Costs that rise with the level of investment in current assets are called carrying costs. Costs that fall with increases in the level of investment in current assets are called shortage costs.

Carrying costs are generally of two types. First, because the rate of return on current assets is low compared with that of other assets, there is an opportunity cost. Second, there is the cost of maintaining the economic value of the item. For example, the cost of warehousing inventory belongs here.

Shortage costs are incurred when the investment in current assets is low. If a firm runs out of cash, it will be forced to sell marketable securities. If a firm runs out of cash and cannot readily sell marketable securities, it may need to borrow or default on an obligation (this

general situation is called cash-out.) If a firm has no inventory (a stock-out) or if it cannot extend credit to its customers, it will lose customers. There are two kinds of shortage costs:

- *Trading or Order Costs.* Order costs are the costs of placing an order for more cash (brokerage costs) or more inventories (production set-up costs)
- *Costs Related to Safety Reserves.* These are costs of lost sales, lost customer goodwill, and disruption of production schedule.

### Carrying Costs and Shortage Costs

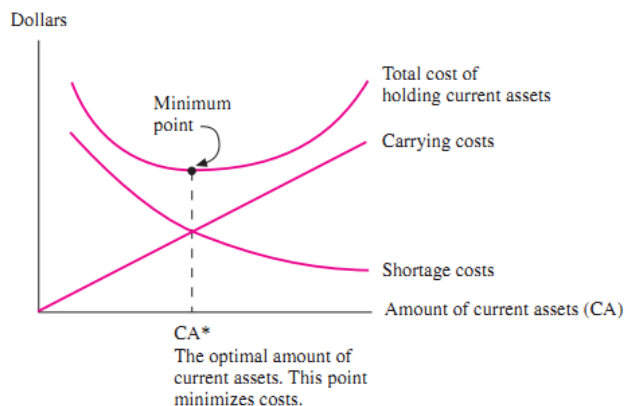


Figure above illustrates the basic nature of carrying costs. The total costs of investing in current assets are determined by adding the carrying costs and the shortage costs. The minimum point on the total cost curve ( $CA^*$ ) reflects the optimal balance of current assets. The curve is generally quite flat at the optimum, and it is difficult, if not impossible, to find the precise optimal balance of shortage and carrying costs. Usually we are content with a choice near the optimum.

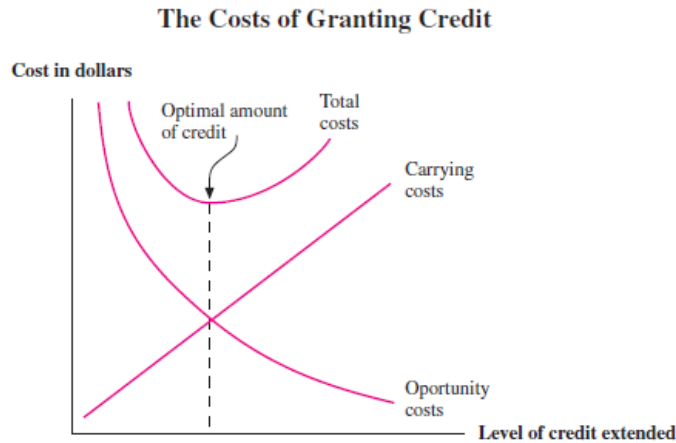
*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003;  
Chapter 27, pp. 753-755*

### 86. What are carrying costs and opportunity costs? Describe optimal credit policy

At the optimal amount of credit, the incremental cash flows from increased sales are exactly equal to the carrying costs from the increase in accounts receivable.

Consider a firm that does not currently grant credit. This firm has no bad debts, no credit department, and relatively few customers. Now consider another firm that grants credit. This firm has lots of customers, a credit department, and a bad-debt expense account. It is useful to think of the decision to grant credit in terms of carrying costs and opportunity costs:

- *Carrying costs* are the costs associated with granting credit and making an investment in receivables. Carrying costs include the delay in receiving cash, the losses from baddebts, and the costs of managing credit.
- *Opportunity costs* are the lost sales from refusing to offer credit. These costs drop as credit is granted.



The sum of the carrying costs and the opportunity costs of a particular credit policy is called the *total-credit-cost curve*. A point is identified as the minimum of the total-credit-cost curve. If the firm extends more credit than the minimum, the additional net cash flow from new customers will not cover the carrying costs of this investment in receivables.

The concept of optimal credit policy in the context of modern principles of finance should be somewhat analogous to the concept of the optimal capital structure discussed earlier in the text. In perfect financial markets there should be no optimal credit policy. Alternative amounts of credit for a firm should not affect the value of the firm. Thus, the decision to grant credit would be a matter of indifference to financial managers.

Just as with optimal capital structure, we could expect taxes, monopoly power, bankruptcy costs, and agency costs to be important in determining an optimal credit policy in a world of imperfect financial markets. For example, customers in high tax brackets would be better off borrowing and taking advantage of cash discounts offered by firms than would customers in low tax brackets. Corporations in low tax brackets would be less able to offer credit, because borrowing would be relatively more expensive than for firms in high tax brackets.

In general, a firm will extend trade credit if it has a comparative advantage in doing so. Trade credit is likely to be advantageous if the selling firm has a cost advantage over other potential lenders, if the selling firm has monopoly power it can exploit, if the selling firm can reduce taxes by extending credit, and if the product quality of the selling firm is difficult to determine. Firm size may be important if there are size economies in managing credit.

The optimal credit policy depends on characteristics of particular firms. Assuming that the firm has more flexibility in its credit policy than in the prices it charges, firms with excess

capacity, low variable operating costs, high tax brackets, and repeat customers should extend credit more liberally than others.

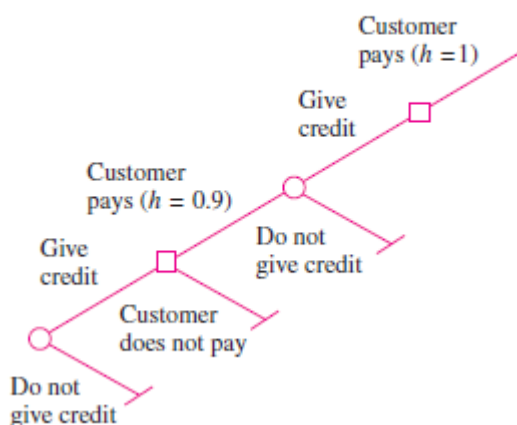
*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003;  
Chapter 29, pp. 805-806*

### 87. Future sales and the credit decision model

Firm has not considered the possibility that offering credit will permanently increase the level of sales in future periods (beyond next month). In addition, payment and nonpayment patterns in the current period will provide credit information that is useful for the next period. These two factors should be analyzed.

There is a 90-percent probability that the customer will pay in period 1. But, if payment is made, there will be another sale in period 2. The probability that the customer will pay in period 2, if the customer has paid in period 1, is 100 percent. Firm can refuse to offer credit in period 2 to customers that have refused to pay in period 1.

#### Future Sales and the Credit Decision



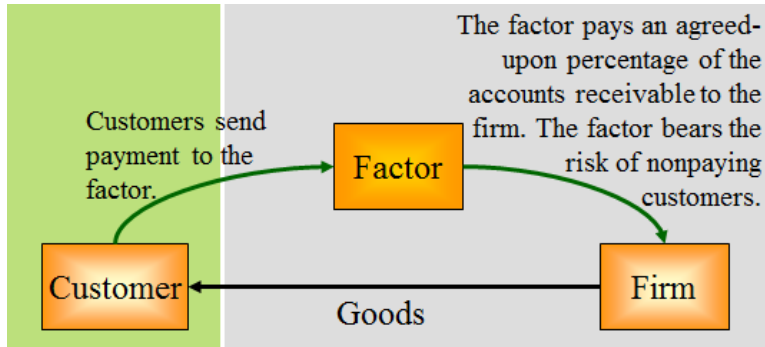
*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003;  
Chapter 29, pp. 804-805*

### 88. What is factoring in credit management?

Factoring refers to the sale of a firm's accounts receivable to a financial institution known as a *factor*. The firm and the factor agree on the basic credit terms for each customer. The customer sends payment directly to the factor, and the factor bears the risk of nonpaying customers. The factor buys the receivables at a discount, which usually ranges from 0.35 to 4

percent of the value of the invoice amount. The average discount throughout the economy is probably about 1 percent.

Graphically, this could be illustrated as follows:



*Corporate Finance, 6<sup>th</sup> ed., Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 29, p. 810*

## PORTFOLIO THEORY

*Course book: Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009*

*Course book: Corporate Finance, 6th ed, Ross, Westerfield, Jaffe; McGraw-Hill, 2003*

*Course book: Finance, 5th ed.; Groppelli, EhsanNikbakht; Barron's Inc. 2006*

### 89. Explain why the bid-ask spread is a transaction cost

The cost of trading a stock reduces the total return that an investor receives. That is, if one buys a stock for \$100 and sells it later for \$105, the gain before trading costs is \$5. If one must pay a dollar of commission when buying and another dollar when selling, the gain after trading costs is only \$3. In addition to the explicit part of trading costs—the broker's commission—there is an implicit part—the dealer's bid–ask spread. The bid–ask spread is the difference between the bid price (the amount of money you get when you sell) and the ask price (the amount of money it costs to buy). Since the ask price is higher than the bid price, it costs you more money to buy the asset than you would receive should you be selling the same asset. This spread is the price (along with a commission) for making the trade.

Sometimes the broker is a dealer in the security being traded and charges no commission but instead collects the fee entirely in the form of the bid–ask spread. Both the bid–ask spread and market-impact costs would reduce this gain still further.

*Corporate Finance, 6<sup>th</sup>ed, Ross, Westerfield, Jaffe; McGraw-Hill, 2003; Chapter 1, p. 13*

### 90. What are Market Orders? Price Contingent Orders?

**Market orders** are buy or sell orders that are to be executed immediately at current market prices. For example, our investor might call her broker and ask for the market price of IBM.

Investors also may place orders specifying prices at which they are willing to buy or sell a security. A limit buy order may instruct the broker to buy some number of shares if and when IBM may be obtained at or below a stipulated price. Conversely, a limit sell instructs the broker to sell if and when the stock price rises above a specified limit. A collection of limit orders waiting to be executed is called a limit order book

**Stop orders** are similar to limit orders in that the trade is not to be executed unless the stock hits a price limit. For stop-loss orders, the stock is to be sold if its price falls below a stipulated level. As the name suggests, the order lets the stock be sold to stop further losses from accumulating. Similarly, stop-buy orders specify that a stock should be bought when its price rises above a limit. These trades often accompany short sales (sales of securities you

don't own but have borrowed from your broker) and are used to limit potential losses from the short position. Short sales are discussed in greater detail later in this chapter.

		Condition	
		Price below the Limit	Price above the Limit
Action	Buy	Limit-Buy Order	Stop-Buy Order
	Sell	Stop-Loss Order	Limit-Sell Order

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 3; pp. 60-61*

### 91. What is meant by short position and long position?

In finance, a long position in a security, such as a stock or a bond, or equivalently to be long in a security, means the holder of the position owns the security and will profit if the price of the security goes up. Going long is the more conventional practice of investing and is contrasted with going short.

In contrast, a short position in an asset means that the holder of the position will profit if the price of the futures contract or derivative goes down.

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 3; pp. 60-62;74*

### 92. Describe the concept of short-selling

A short sale allows investors to profit from a decline in a security's price. An investor borrows a share of stock from a broker and sells it. Later, the short-seller must purchase a share of the same stock in order to replace the share that was borrowed. This is called covering the short position.

The short-seller anticipates the stock price will fall, so that the share can be purchased later at a lower price than it initially sold for; if so, the short-seller will reap a profit. Short-sellers must not only replace the shares but also pay the lender of the security any dividends paid during the short sale.

In practice, the shares loaned out for a short sale are typically provided by the short-seller's brokerage firm, which holds a wide variety of securities of its other investors in street name (i.e., the broker holds the shares registered in its own name on behalf of the client). The owner of the shares need not know that the shares have been lent to the short-seller. If the owner wishes to sell the shares, the brokerage firm will simply borrow shares from another investor. Therefore, the short sale may have an indefinite term. However, if the brokerage

firm cannot locate new shares to replace the ones sold, the short-seller will need to repay the loan immediately by purchasing shares in the market and turning them over to the brokerage house to close out the loan.

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 3; p. 74*  
*Corporate finance, 3rd ed., Jonathan Berk, Peter DeMarzo; Pearson 2014; Chapter 9; p. 274*

### **93. What does “Buying on Margin” mean? Why investors buy securities on margin?**

When purchasing securities, investors have easy access to a source of debt financing called broker’s call loans. The act of taking advantage of broker’s call loans is called buying on margin. Purchasing stocks on margin means the investor borrows part of the purchase price of the stock from a broker. The margin in the account is the portion of the purchase price contributed by the investor; the remainder is borrowed from the broker. The brokers in turn borrow money from banks at the call money rate to finance these purchases; if the stock value were to fall, owners’ equity would become negative; meaning the value of the stock is no longer sufficient collateral to cover the loan from the broker. To guard against this possibility, the broker sets a maintenance margin. If the percentage margin falls below the maintenance level, the broker will issue a margin call, which requires the investor to add new cash or securities to the margin account. If the investor does not act, the broker may sell securities from the account to payoff enough of the loan to restore the percentage margin to an acceptable level. In order to better understand concept of Buying on Margin consider an example:

Let’s suppose an investor is bullish on IBM stock, which is selling for \$100 per share. An investor with \$10,000 to invest expects IBM to go up in price by 30% during the next year. Ignoring any dividends, the expected rate of return would be 30% if the investor invested \$10,000 to buy 100 shares. But now assume the investor borrows another \$10,000 from the broker and invests it in IBM, too. The total investment in IBM would be \$20,000 (for 200 shares). Assuming an interest rate on the margin loan of 9% per year, what will the investor’s rate of return be (again ignoring dividends) if IBM stock goes up 30% by year’s end? The 200 shares will be worth \$26,000. Paying off \$10,900 of principal and interest on the margin loan leaves \$15,100 (i.e., \$26,000 - \$10,900). The rate of return in this case will be

$$\frac{\$15,100 - \$10,000}{\$10,000} = 51\%$$

The investor has parlayed a 30% rise in the stock’s price into a 51% rate of return on the \$10,000 investment. Doing so, however, magnifies the downside risk. Suppose that, instead of going up by 30%, the price of IBM stock goes down by 30% to \$70 per share. In that case, the 200 shares will be worth \$14,000, and the investor is left with \$3,100 after paying off the \$10,900 of principal and interest on the loan. The result is a disastrous return of



$$\frac{\$3,100 - \$10,000}{\$10,000} = -69\%$$

Change in Stock Price	End-of-Year Value of Shares	Repayment of Principal and Interest*	Investor's Rate of Return
30% increase	\$26,000	\$10,900	51%
No change	20,000	10,900	-9
30% decrease	14,000	10,900	-69

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 3; p. 71-73*

#### 94. Describe the difference between Real and Nominal interest rates

An interest rate is a promised rate of return denominated in some unit of account (dollars, yen, euros, or even purchasing power units) over some time period (a month, a year, 20 years, or longer). Thus, when we say the interest rate is 5%, we must specify both the unit of account and the time period.

For example, interest rates that are absolutely safe in dollar terms will be risky when evaluated in terms of purchasing power because of inflation uncertainty. To illustrate, consider a 1-year dollar (nominal) risk-free interest rate. Suppose exactly 1 year ago you deposited \$1,000 in a 1-year time deposit guaranteeing a rate of interest of 10%. You are about to collect \$1,100 in cash. What is the real return on your investment? That depends on what money can buy these days, relative to what you could buy a year ago. The consumer price index (CPI) measures purchasing power by averaging the prices of goods and services in the consumption basket of an average urban family of four. Suppose the rate of inflation (the percent change in the CPI, denoted by  $i$ ) for the last year amounted to  $i = 6\%$ . This tells you that the purchasing power of money is reduced by 6% a year. The value of each dollar depreciates by 6% a year in terms of the goods it can buy. Therefore, part of your interest earnings are offset by the reduction in the purchasing power of the dollars you will receive at the end of the year. With a 10% interest rate, after you net out the 6% reduction in the purchasing power of money, you are left with a net increase in purchasing power of about 4%. Thus we need to distinguish between a nominal interest rate—the growth rate of your money—and a real interest rate—the growthrate of your purchasing power. If we call  $R$  the nominal rate,  $r$  the real rate, and  $i$  the inflation rate, then we conclude

$$r \approx R - i$$

In words, the real rate of interest is the nominal rate reduced by the loss of purchasing power resulting from inflation. If inflation turns out higher than 6%, your realized real return will be lower than 4%; if inflation is lower, your real rate will be higher. In fact, the exact relationship between the real and nominal interest rate is given by

$$1 + r = \frac{1 + R}{1 + i}$$

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 3; p. 114-115*

**95. What is effective annual rate (EAR)? What is the difference between EAR and APR quote?**

Annual Percentage Rate (APR) is interest rate when banks lend money borrowers and earn interest calculated on many consumer loans, mortgages and credit lines. It is nominal interest rate offered by the bank, and is not the actual interest rate earned because it does not include the effects of intra-year compounding. Effective Annual Rate (EAR) is the effective return assuming that nominal interest paid is reinvested at the same rate and by this way it includes the effects of intra-year compounding. EAR is used when comparing investments with different compounding periods per year and it is always higher than APR.

*Example: A bank offers Periodic Rates with 10% APR*

a. APR:

Annual Percentage Rate: APR = 10%

Semi-Annual Rate (compounds twice a year)

$$10\%/2 = 5\%$$

Monthly Rate (compounds twelve times a year)

$$10\%/12 = 0.833\%$$

b. EAR:

$$\text{Effective Annual Rate: } \text{EAR} = \left(1 + \frac{\text{APR}}{n}\right)^n - 1$$

$$\text{Semi-Annual Rate (n = 2), } \text{EAR} = \left(1 + \frac{0.1}{2}\right)^2 - 1 = 10.25\%$$

$$\text{Monthly Rate (n = 12), } \text{EAR} = \left(1 + \frac{0.1}{12}\right)^{12} - 1 = 10.47\%$$

*Investments, 8th ed.; Bodie, Kane, Marcus, McGraw-Hill 2009, Chapter 5*

**96. Write the formulas for calculating EAR when APR is given and vice versa. How the formulas change in the case of continuous compounding?**

Write the formulas for calculating EAR when APR is given and vice versa. How the formulas change in the case of continuous compounding?

$$\text{APR} = \text{Per} - \text{periodrate} \times \text{Periodsperyear}$$

Therefore, to obtain the *EAR* if there are  $n$  compounding periods in the year, we first recover the rate per period as  $APR/n$  and then compound that rate for the number of periods in a year.

$$1 + EAR = (1 + \text{rate per period})^n = \left(1 + \frac{APR}{n}\right)^n$$

Rearranging,

$$APR = [(1 + EAR)^{1/n} - 1] \times n$$

The formula assumes that you can earn the *APR* each period. Therefore, after one year (when  $n$  periods have passed), your cumulative return would be  $(1 + APR/n)^n$ . Note that one needs to know the holding period when given an *APR* in order to convert it to an effective rate.

The *EAR* diverges by greater amounts from the *APR* as  $n$  becomes larger (that is, as we compound cash flows more frequently). In the limit, we can envision continuous compounding when  $n$  becomes extremely large in equation above. With continuous compounding, the relationship between the *APR* and *EAR* becomes

$$1 + EAR = e^{APR}$$

Or equivalently

$$APR = \ln(1 + EAR)$$

*Investments, 8<sup>th</sup> ed.; Bodie, Kane, Marcus, McGraw-Hill 2009, Chapter 5, p. 119*  
*Finance, 5<sup>th</sup> ed.; Groppelli, Ehsan Nikbakht; Barron's Inc. 2006, Chapter 3, pp. 12-13*

## 97. Define holding period return

A key measure of investors' success is the rate at which their funds have grown during the investment period. The total holding-period return (HPR) of a share of stock depends on the increase (or decrease) in the price of the share over the investment period as well as on any dividend income the share has provided. The rate of return is defined as dollars earned over the investment period (price appreciation as well as dividends) per dollar invested:

$$HPR = \frac{\text{ending price} - \text{beginning price} + \text{cash dividend}}{\text{beginning price}}$$

This definition of the HPR assumes that the dividend is paid at the end of the holding period.

To the extent that dividends are received earlier, the definition ignores reinvestment income between the receipt of the dividend and the end of the holding period. Recall also that the percentage return from dividends is called the dividend yield, and so the dividend yield plus the capital gains yield equals the HPR.

$$HPR = \underbrace{\frac{\text{ending price} - \text{beginning price}}{\text{beginning price}}}_{\text{capital gain/loss}} + \underbrace{\frac{\text{cash dividend}}{\text{beginning price}}}_{\text{dividend yield}}$$

*Investments, 8<sup>th</sup> ed.; Bodie, Kane, Marcus, McGraw-Hill2009, Chapter 5, p. 124*

## 98. Define Expected Return and Standard Deviation

There is considerable uncertainty about the price of a share plus dividend income 1 year from now, however, so you cannot be sure about your eventual HPR. We can quantify our beliefs about the state of the economy and the stock market.

The expected rate of return is a probability-weighted average of the rates of return in each scenario. Calling  $p(s)$  the probability of each scenario and  $r(s)$  the HPR in each scenario, where scenarios are labeled or “indexed” by  $s$ , we may write the expected return as

$$E(r) = \sum_s p(s)r(s)$$

The standard deviation of the rate of return is a measure of risk. It is defined as the square root of the variance, which in turn is the expected value of the squared deviations from the expected return. The higher the volatility in outcomes, the higher will be the average value of these squared deviations. Therefore, variance and standard deviation measure the uncertainty of outcomes. Symbolically,

$$\sigma^2 = \sum_s p(s)[r(s) - E(r)]^2$$

and

$$\sigma = \sqrt{\sigma^2}$$

*Investments, 8<sup>th</sup> ed.; Bodie, Kane, Marcus, McGraw-Hill2009, Chapter 5, p. 124-125*

## 99. What information do Arithmetic Average Return and Geometric Average Return carry?

When analyzing investment returns it is important to differentiate between the simple arithmetic return and the geometric return. The geometric return is the more accurate as it is the average compounded return. The arithmetic average is always higher than the geometric average, hence the arithmetic average return is usually the one posted in ads for mutual funds and other investments.

*Example: You made investment of \$1000. After a year, your investment doubled, thus return amounted to 100%. After one more year, your investment decreased to \$1000 as at the starting point. i.e. return amounted to -50%. What is the average rate of return?*

$$\text{Arithmetic Average return: } \frac{100\% + (-50\%)}{2} = 25\%$$

$$\text{Geometric Average return: } \sqrt[2]{(1 + 100\%)(1 + (-50\%))} - 1 = 0$$

Arithmetic Average return tells us the Average Annual Rate of return, i.e. in case of average profitability in future; my investment will yield 25%

Geo metric Average return tells us past growth rate of investment.

*Investments, 8<sup>th</sup> ed.; Bodie, Kane, Marcus, McGraw-Hill2009, Chapter 5, p. 126-128*

### **100. How is Sharpe Ratio calculated? What does it show?**

Sharpe Ratio is calculated as follows:

$$S = \frac{E(r_p) - r_f}{\sigma_p}$$

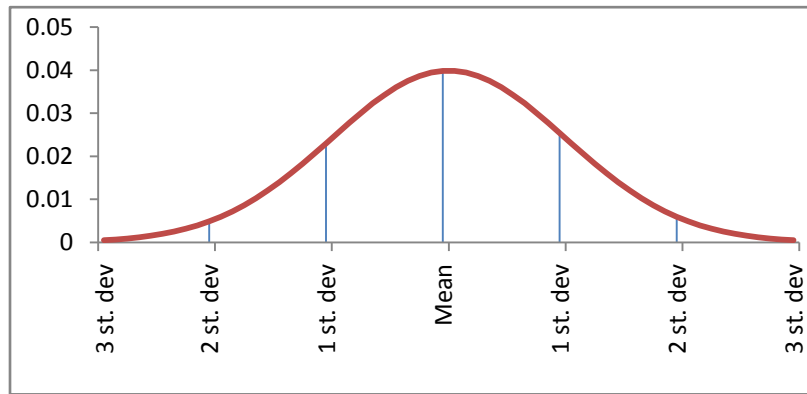
It is slope of CAL and shows increase in the expected return of the complete portfolio per unit of additional standard deviation—in other words, incremental return per incremental risk. Sometimes it is called as Reward-To-Volatility ratio as well.

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 5; p. 170*

### **101. Define Empirical Rule**

The Empirical Rule is most often used in statistics for forecasting final outcomes. After the standard deviation is calculated and before the exact data could be collected, the empirical rule can be used as a rough estimate of the outcome of the impending data. This probability can be used in the meantime as gathering appropriate data may be time consuming, or even impossible to obtain.

About 68.27% of the values lie within 1 standard deviation of the mean. Similarly, about 95.45% of the values lie within 2 standard deviations of the mean. Nearly all (99.73%) the values lie within 3 standard deviations from the mean.

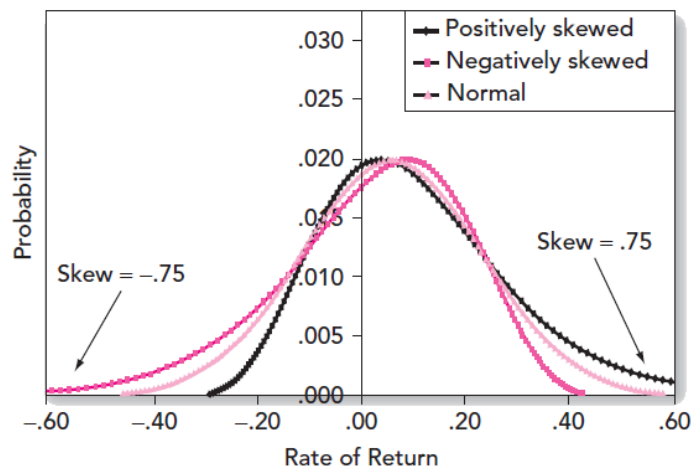


*Investments, 8<sup>th</sup> ed.; Bodie, Kane, Marcus, McGraw-Hill 2009, Chapter 5, p. 130-132*

## 102. State parameters that measure deviation from normality

To assess the adequacy of the assumption of normality we focus on deviations from normality that would invalidate the use of standard deviation as an adequate measure of risk. Our first criterion is symmetry. A measure of asymmetry called skew uses the ratio of the average cubed deviations from the mean, called the third moment, to the cubed standard deviation to measure any asymmetry or “skewness” of a distribution.

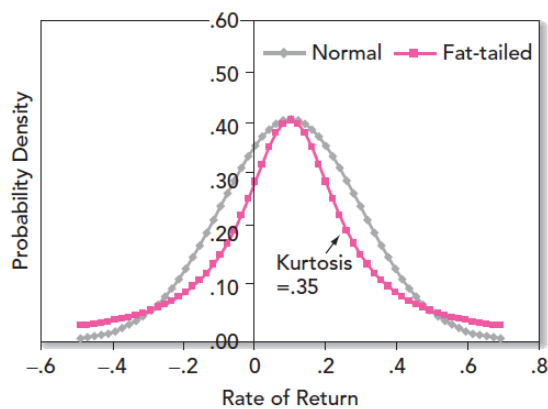
$$Skew = \frac{E[r(s) - E(r)]^3}{\sigma^3}$$



Another potentially important deviation from normality concerns the likelihood of extreme values on either side of the mean at the expense of a smaller fraction of moderate deviations. Graphically speaking, when the tails of a distribution are “fat,” there is

more probability mass in the tails of the distribution than predicted by the normal distribution, at the expense of “slender shoulders”, that is, less probability mass near the center of the distribution. Figure below superimposes a “fattailed” distribution on a normal with the same mean and SD. Although symmetry is still preserved, the SD will underestimate the likelihood of extreme events: large losses as well as large gains. Kurtosis is a measure of the degree of fat tails. In this case, we use the expectation of deviations from the mean raised to the fourth power and standardized by dividing by the fourth power of the SD, that is:

$$Kurtosis = \frac{E[r(s) - E(r)]^4}{\sigma^4} - 3$$



*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 5; p. 132-133*

### 103. What does being Risk Averse, Risk Lover and Risk Neutral mean?

Investors who are **risk averse** reject investment portfolios that are fair games or worse. Risk-averse investors are willing to consider only risk-free or speculative prospects with positive risk premiums. Loosely speaking, a risk-averse investor “penalizes” the expected rate of return of a risky portfolio by a certain percentage (or penalizes the expected profit by a dollar amount) to account for the risk involved. The greater the risk is the larger the penalty.

In contrast to risk-averse investors, risk-neutral investors judge risky prospects solely by their expected rates of return. The level of risk is irrelevant to the **risk neutral** investor, meaning that there is no penalty for risk. For this investor a portfolio’s certainty equivalent rate is simply its expected rate of return.

A **risk lover** is willing to engage in fair games and gambles; this investor adjusts the expected return upward to take into account the “fun” of confronting the prospect’s risk. Risk lovers will always take a fair game because their upward adjustment of utility for risk gives the fair game a certainty equivalent that exceeds the alternative of the risk-free investment.

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 6; p. 157-158*

#### 104. What is utility function? Indifference Curve?

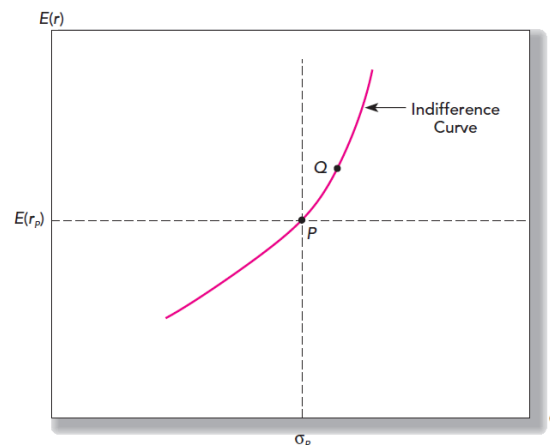
Investor can assign a welfare, or utility, score to competing investment portfolios based on the expected return and risk of those portfolios. Higher utility values are assigned to portfolios with more attractive risk-return profiles. Portfolios receive higher utility scores for higher expected returns and lower scores for higher volatility. Many particular “scoring” systems are legitimate. One of them is:

$$U = E(r) - 0.5A\sigma^2$$

Where  $U$  is the utility value and  
 $A$  is an index of the investor's risk aversion.  
 The factor of  $1/2$  is just a scaling convention.

We can interpret the utility score of risky portfolios as a certainty equivalent rate of return. The certainty equivalent rate is the rate that risk-free investments would need to offer to provide the same utility score as the risky portfolio. In other words, it is the rate that, if earned with certainty, would provide a utility score equivalent to that of the portfolio in question. The certainty equivalent rate of return is a natural way to compare the utility values of competing portfolios.

Suppose an investor identifies all portfolios that are equally attractive as portfolio P. Starting at P, an increase in standard deviation lowers utility; it must be compensated for by an increase in expected return. Thus point Q in Figure 6.2 is equally desirable to this investor as P. Investors will be equally attracted to portfolios with high risk and high expected returns compared with other portfolios with lower risk but lower expected returns. These equally preferred portfolios will lie in the mean– standard deviation plane on a curve called the indifference curve that connects all portfolio points with the same utility value



*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 6; p. 161-162*



### 105. What is Capital Allocation Line? Draw CAL. What determines your position on CAL?

CAL is Plot of risk-return combinations available by varying portfolio allocation between a risk-free asset and a risky portfolio. Its formula is given by

$$E(r_c) = r_f + y[E(r_p) - r_f]$$

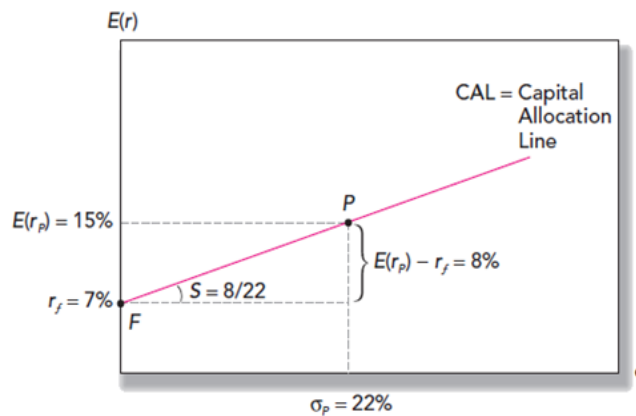
Where,

$E(r_c)$  – Expected return of complete portfolio

$r_f$  – Risk free rate of return

$E(r_p)$  – Expected return on risky asset or portfolio

$y$  – Proportion of your money invested in risky asset or portfolio

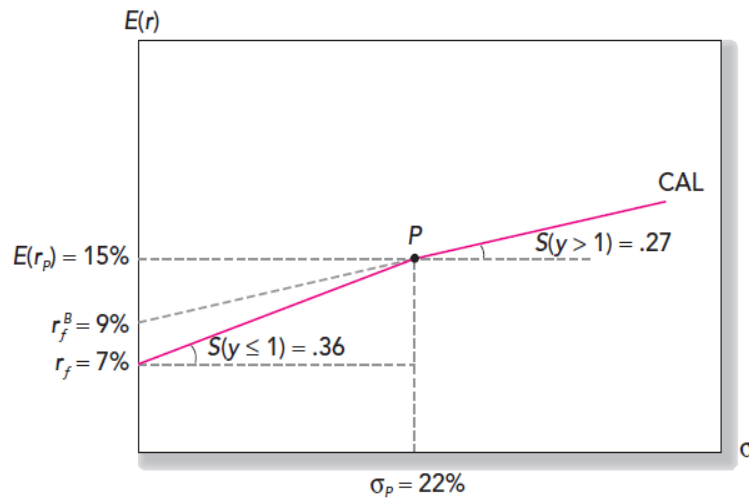


CAL is same for every investor but they decide to hold different complete portfolios, thus appear in different points of the line (i.e. they choose different proportions of risky and riskless asset). Their position on CAL is determined by Risk Aversion factor.

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 6; pp. 168-170*

### 106. What happens if you invest in risky asset more then you have on hand?

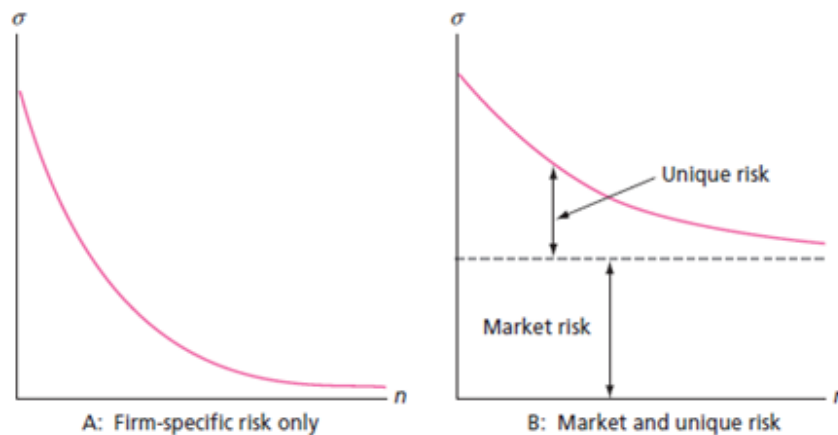
If you are trying to invest in risky asset more then you have on hand, this implies borrowing some of money. Of course, nongovernment investors cannot borrow at the risk-free rate. The risk of a borrower's default causes lender to demand higher interest rates on loans. Therefore, the nongovernment investor's borrowing cost will exceed the lending rate of  $r_f = 7\%$ . Suppose the borrowing rate is  $r_f^B = 9\%$ . Then in the borrowing range, the reward-to-volatility ratio, the slope of the CAL, will be  $6/22 = 0.27$ . The CAL will therefore be "kinked" at point P, as shown in Figure 6.5. To the left of P the investor is lending at 7%, and the slope of the CAL is 0.36. To the right of P, where  $y > 1$ , the investor is borrowing at 9% to finance extra investments in the risky asset, and the slope is 0.27.



*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 6; pp. 171*

### 107. What is diversification? Define market and firm-specific risks

Diversification is the process of investing a portfolio across different asset classes in varying proportions depending on an investor's time horizon, risk tolerance, and goals. While diversification does not assure or guarantee better performance and cannot eliminate the risk of investment losses, this disciplined approach does help alleviate some of the speculation that is often involved with investing. Primary asset classes include domestic equity, foreign equity, and fixed income.



When common sources of risk affect all firms, however, even extensive diversification cannot eliminate risk. In the figure above, portfolio standard deviation falls as the number of securities increases, but it is not reduced to zero. The risk that remains even after diversification is called market risk, risk that is attributable to market wide risk sources. Other

names are systematic risk or non-diversifiable risk. The risk that can be eliminated by diversification is called unique risk, firm-specific risk, non-systematic risk, or diversifiable risk.

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 7; pp. 195-196*

### 108. How do you determine weights for portfolio with minimum variance consisting of two risky assets?

The expected return on the portfolio is a weighted average of expected returns on the component securities with portfolio proportions as weights:

$$E(r_p) = w_1 r_1 + w_2 r_2$$

The variance of the two-asset portfolio is

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \text{cov}(r_1; r_2)$$

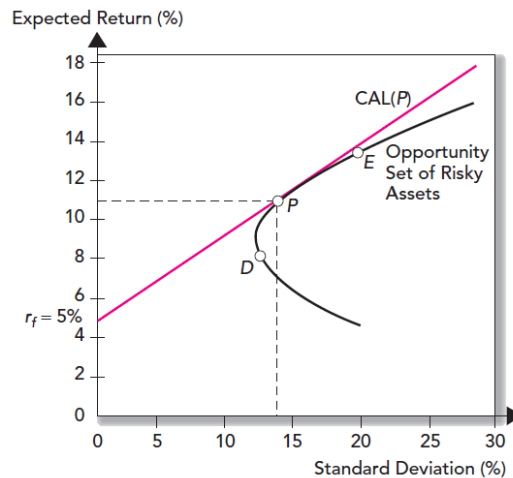
To find minimum of  $\sigma_p^2$  with respect to  $w_1$  remark that  $w_2 = 1 - w_1$ , input this in formula above and derive with respect to  $w_1$ .

$$w_1 = \frac{\sigma_2^2 - \text{cov}(r_1; r_2)}{\sigma_1^2 + \sigma_2^2 - \text{cov}(r_1; r_2)}$$

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 7; pp. 196-203*

### 109. What is Optimal Risky Portfolio? How do you determine it?

CAL with the highest feasible reward-to-volatility ratio is best of all possible ones. Therefore, the tangency portfolio, labeled P is the optimal risky portfolio to mix with T-bills.



The objective is to find the weights  $w_D$  and  $w_E$  that result in the highest slope of the CAL (i.e., the weights that result in the risky portfolio with the highest reward-to-volatility ratio). Therefore, the objective is to maximize the slope of the CAL for any possible portfolio,  $p$ . Thus our objective function is the slope (equivalently, the Sharpe ratio)  $S_p$ :

$$S_p = \frac{E(r_p) - r_f}{\sigma_p}$$

Subject to  $\sum w = 1$

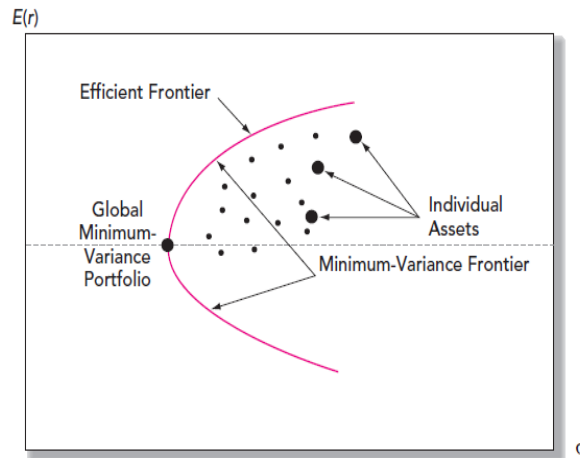
*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 7; pp. 206-207*

### 110. Define Efficient Frontier. What is effective portfolio?

A combination of assets, i.e. a portfolio, is referred to as "efficient" if it has the best possible expected level of return for its level of risk (usually proxy of the standard deviation of the portfolio's return). Here, every possible combination of risky assets, without including any holdings of the risk-free asset, can be plotted in risk-expected return space, and the collection of all such possible portfolios defines a region in this space.

Graph representing a set of portfolios that maximizes expected return at each level of portfolio risk. The inputs are the expected returns and standard deviations of each asset in the universe, along with the correlation coefficients between each pair of assets. These data come from security analysis. Expected return–standard deviation combinations for any individual asset end up inside the efficient frontier, because single-asset portfolios are inefficient—they are not efficiently diversified.

When we choose among portfolios on the efficient frontier, we can immediately discard portfolios below the minimum-variance portfolio. These are dominated by portfolios on the upper half of the frontier with equal risk but higher expected returns. Therefore, the real choice is among portfolios on the efficient frontier above the minimum-variance portfolio.



Effective Portfolio is called portfolio which lies on efficient frontier and is tangent point for capital allocation line.

*Investments, 8<sup>th</sup>ed.; Bodie, Kane, Marcus, McGraw-Hill2009, Chapter 7, pp. 209-213*

### **111. State the assumptions of CAPM**

1. There are many investors, each with an endowment (wealth) that is small compared to the total endowment of all investors. Investors are price-takers, in that they act as though security prices are unaffected by their own trades. This is the usual perfect competition assumption of microeconomics.

2. All investors plan for one identical holding period. This behavior is myopic (shortsighted) in that it ignores everything that might happen after the end of the single period horizon. Myopic behavior is, in general, suboptimal. 3. Investments are limited to a universe of publicly traded financial assets, such as stocks and bonds, and to risk-free borrowing or lending arrangements. This assumption rules out investment in non-traded assets such as education (human capital), private enterprises, and governmentally funded assets such as town halls and international airports. It is assumed also that investors may borrow or lend any amount at a fixed, risk-free rate.

4. Investors pay no taxes on returns and no transaction costs (commissions and service charges) on trades in securities. In reality, of course, we know that investors are in different tax brackets and that this may govern the type of assets in which they invest. For example, tax implications may differ depending on whether the income is from interest, dividends, or capital gains. Furthermore, actual trading is costly, and commissions and fees depend on the size of the trade and the good standing of the individual investor.

5. All investors are rational mean-variance optimizers, meaning that they all use the Markowitz portfolio selection model.

6. All investors analyze securities in the same way and share the same economic view of the world. The result is identical estimates of the probability distribution of future cash flows from investing in the available securities; that is, for any set of security prices, they all derive the same input list to feed into the Markowitz model. Given a set of security prices and the risk-free interest rate, all investors use the same expected returns and covariance matrix of security returns to generate the efficient frontier and the unique optimal risky portfolio. This assumption is often referred to as homogeneous expectations or beliefs.

*Investments, 8<sup>th</sup>ed.; Bodie, Kane, Marcus, McGraw-Hill2009, Chapter 9, p. 280*

### **112. Derive CAPM**

Suppose, for example, that we want to gauge the portfolio risk of GE stock. We measure the contribution to the risk of the overall portfolio from holding GE stock by its covariance

with the market portfolio. To see why this is so, let us look again at the way the variance of the market portfolio is calculated. To calculate the variance of the market portfolio, we use the bordered covariance matrix with the market portfolio weights. We highlight GE in this depiction of the  $n$  stocks in the market portfolio.

Portfolio Weights	$w_1$	$w_2$	...	$w_{GE}$	...	$w_n$
$w_1$	$\text{Cov}(r_1, r_1)$	$\text{Cov}(r_1, r_2)$	...	$\text{Cov}(r_1, r_{GE})$	...	$\text{Cov}(r_1, r_n)$
$w_2$	$\text{Cov}(r_2, r_1)$	$\text{Cov}(r_2, r_2)$	...	$\text{Cov}(r_2, r_{GE})$	...	$\text{Cov}(r_2, r_n)$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
$w_{GE}$	$\text{Cov}(r_{GE}, r_1)$	$\text{Cov}(r_{GE}, r_2)$	...	$\text{Cov}(r_{GE}, r_{GE})$	...	$\text{Cov}(r_{GE}, r_n)$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
$w_n$	$\text{Cov}(r_n, r_1)$	$\text{Cov}(r_n, r_2)$	...	$\text{Cov}(r_n, r_{GE})$	...	$\text{Cov}(r_n, r_n)$

Recall that we calculate the variance of the portfolio by summing over all the elements of the covariance matrix, first multiplying each element by the portfolio weights from the row and the column. The contribution of one stock to portfolio variance therefore can be expressed as the sum of all the covariance terms in the column corresponding to the stock, where each covariance is first multiplied by both the stock's weight from its row and the weight from its column.

For example, the contribution of GE's stock to the variance of the market portfolio is

$$w_{GE} [w_1 \text{Cov}(r_1, r_{GE}) + w_2 \text{Cov}(r_2, r_{GE}) + \dots + w_{GE} \text{Cov}(r_{GE}, r_{GE}) + \dots + w_n \text{Cov}(r_n, r_{GE})]$$

Equation provides a clue about the respective roles of variance and covariance in determining asset risk. When there are many stocks in the economy, there will be many more covariance terms than variance terms. Consequently, the covariance of a particular stock with all other stocks will dominate that stock's contribution to total portfolio risk. Notice that the sum inside the square brackets in equation above is the covariance of GE with the market portfolio. In other words, we can best measure the stock's contribution to the risk of the market portfolio by its covariance with that portfolio:

$$\text{GE's contribution to variance} = w_{GE} \text{Cov}(r_{GE}, r_M)$$

This should not surprise us. For example, if the covariance between GE and the rest of the market is negative, then GE makes a "negative contribution" to portfolio risk: By providing returns that move inversely with the rest of the market, GE stabilizes the return on the overall portfolio. If the covariance is positive, GE makes a positive contribution to overall portfolio risk because its returns reinforce swings in the rest of the portfolio. To demonstrate this more rigorously, note that the rate of return on the market portfolio may be written as

$$r_m = \sum_{k=1}^n w_k r_k$$

Therefore, the covariance of the return on GE with the market portfolio is

$$\text{Cov}(r_{GE}, r_M) = \text{Cov}\left(r_{GE} \sum_{k=1}^n w_k r_k\right) = \sum_{k=1}^n w_k \text{Cov}(r_k, r_{GE}) = w_{GE} \text{Cov}(r_{GE}, r_M)$$

Therefore, the reward-to-risk ratio for investments in GE can be expressed as

$$\frac{\text{GE's contribution to risk premium}}{\text{GE's contribution to variance}} = \frac{w_{GE}[E(r_{GE}) - r_f]}{w_{GE} \text{Cov}(r_{GE}, r_M)} = \frac{E(r_{GE}) - r_f}{\text{Cov}(r_{GE}, r_M)}$$

The market portfolio is the tangency (efficient mean-variance) portfolio. The reward-to-risk ratio for investment in the market portfolio is

$$\frac{\text{Market risk premium}}{\text{Market Variance}} = \frac{E(r_M) - r_f}{\sigma_M^2}$$

The ratio in is often called the market price of risk <sup>6</sup> because it quantifies the extra return that investors demand to bear portfolio risk. Notice that for components of the efficient portfolio, such as shares of GE, we measure risk as the contribution to portfolio variance (which depends on its covariance with the market). In contrast, for the efficient portfolio itself, its variance is the appropriate measure of risk.

A basic principle of equilibrium is that all investments should offer the same reward-to-risk ratio. If the ratio were better for one investment than another, investors would rearrange their portfolios, tilting toward the alternative with the better trade-off and shying away from the other. Such activity would impart pressure on security prices until the ratios were equalized. Therefore we conclude that the reward-to-risk ratios of GE and the market portfolio should be equal:

$$\frac{E(r_{GE}) - r_f}{\text{Cov}(r_{GE}, r_M)} = \frac{E(r_M) - r_f}{\sigma_M^2}$$

To determine the fair risk premium of GE stock, we rearrange equation slightly to obtain

$$E(r_{GE}) - r_f = \frac{\text{Cov}(r_{GE}, r_M)}{\sigma_M^2} E(r_M) - r_f$$

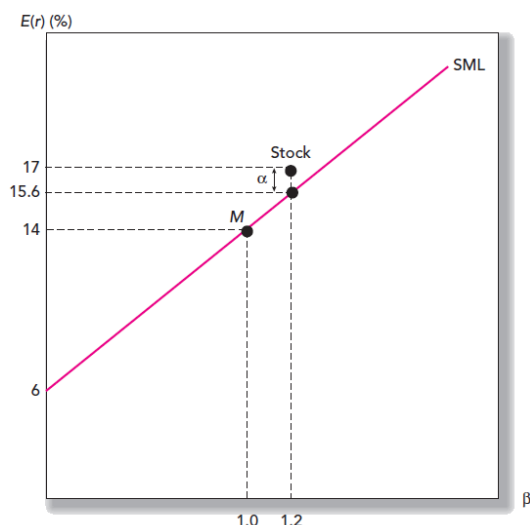
The ratio  $\text{Cov}(r_{GE}, r_M)/\sigma_M^2$  measures the contribution of GE stock to the variance of the market portfolio as a fraction of the total variance of the market portfolio. The ratio is called beta and is denoted by  $\beta$ . Using this measure, we can restate formula as

$$E(r_{GE}) = r_f + \beta_{GE}[E(r_M) - r_f]$$

This expected return–beta relationship is the most familiar expression of the CAPM to practitioners.

### 113. What is the difference between Security Market Line and Capital Market Line? What is stock's Alpha?

The expected return–beta relationship can be portrayed graphically as the **security market line (SML)**



The beta of a stock measures its contribution to the variance of the market portfolio. Hence we expect, for any asset or portfolio, the required risk premium to be a function of beta. The CAPM confirms this intuition, stating further that the security's risk premium is directly proportional to both the beta and the risk premium of the market portfolio; that is, the risk premium equals  $\beta(E(r_m) - r_f)$ . At the point on the horizontal axis where  $\beta = 1$ , we can read off the vertical axis the expected return on the market portfolio.

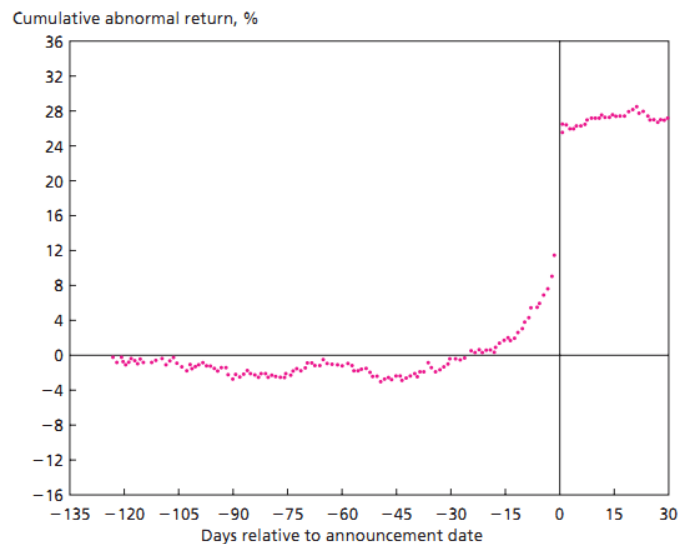
The CML graphs the risk premiums of efficient portfolios (i.e., portfolios composed of the market and the risk-free asset) as a function of portfolio standard deviation. This is appropriate because standard deviation is a valid measure of risk for efficiently diversified portfolios that are candidates for an investor's overall portfolio. The SML, in contrast, graphs individual asset risk premiums as a function of asset risk. The relevant measure of risk for individual assets held as parts of well diversified portfolios is not the asset's standard deviation or variance; it is, instead, the contribution of the asset to the portfolio variance, which we measure by the asset's beta. The SML is valid for both efficient portfolios and individual assets.

Underpriced stocks therefore plot above the SML: Given their betas, their expected returns are greater than dictated by the CAPM. Overpriced stocks plot below the SML. The difference between the fair and actually expected rates of return on a stock is called the stock's alpha, denoted by  $\alpha$ .



#### 114. Draw the graph of reaction of stock price to new information in efficient and inefficient markets

Don't confuse randomness in price changes with irrationality in the level of prices. If prices are determined rationally, then only new information will cause them to change. Therefore, a random walk would be the natural result of prices that always reflect all current knowledge. Indeed, if stock price movements were predictable, that would be damning evidence of stockmarket inefficiency, because the ability to predict prices would indicate that all available information was not already reflected in stock prices. Therefore, the notion that stocks already reflect all available information is referred to as the efficient market hypothesis.



**The figure shows that stock prices jump dramatically on the day the news becomes public.** However, there is no further drift in prices after the announcement date, suggesting that prices reflect the new information, including the likely magnitude of the takeover premium, by the end of the trading day.

*Investments, 8th ed., Bodie, Kane, Marcus; McGraw-Hill, 2009; Chapter 11, pp. 345-349*

## FIXED INCOME MATHEMATICS

*Course Book: FixedIncomeMathematics, 4th ed., Frank J. Fabozz, McGraw-Hill*

### 115. What is Asset-Backed Securities?

An asset-backed security (ABS) is a security whose income payments and hence value is derived from and collateralized (or "backed") by a specified pool of underlying assets. The pool of assets is typically a group of small and illiquid assets which are unable to be sold individually. Pooling the assets into financial instruments allows them to be sold to general investors, a process called securitization, and allows the risk of investing in the underlying assets to be diversified because each security will represent a fraction of the total value of the diverse pool of underlying assets. The pools of underlying assets can include common payments from credit cards, auto loans, and mortgage loans, to esoteric cash flows from aircraft leases, royalty payments and movie revenues. A financial security backed by a loan, lease or receivables against assets other than real estate and mortgage-backed securities. For investors, asset-backed securities are an alternative to investing in corporate debt.

*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 2, pp. 12.*

### 116. What is day count Conversions? How can it be used?

Market conventions for the number of days in a coupon period and the number of days in a year differ by type of bond issuer (i.e., government, government-related entity, local government, and corporate) and by country. The following notation is typically used to denote a day count convention: Number of days in a month/Number of days in a year and "NL is used to denote no leap year and "E to denote European.

In practice, there are eight day count conventions:

- Actual/actual (in period);
- Actual/365;
- Actual (NL)/365;
- Actual/365 (366 in leap year);
- Actual/360;
- 30/360;
- 30/365;
- 30E/360.

In calculation of the actual number of days, only one of the two bracketing dates in question is included. For example, the actual number of days between August 20 and August 24 is 4 days.

Actual (NL)/365 are the same as Actual/365 with the exception that February 29 is not counted in the former method.

In the day count conventions where “30 is used for the number of days (i.e., the last three methods), there are rules for computing the number of days in between two days in assuming a 30-day month.

Day count conventions are used for calculating:

- Accrued interest to be paid;
- Accrued interest for price/yield;
- Next coupon payment;
- The exponent used to compute the present value of the cash flow from the next coupon date back to the settlement date;
- The coupon payment on the maturity date;
- The exponent used to compute the present value of the cash flow at the maturity date back to the last coupon date.

Government bond markets in the United States and European countries, the same day count convention is used for all of the calculations above, but it is not necessary the same for other bond markets. For example, in the Canadian government bond market, Actual/365 is used for computing accrued interest to be paid and the exponent used to compute the present value of the cash flow at the maturity date back to the last coupon date. However, for all other calculations, the Actual/Actual convention is used.

*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 6, pp. 81-84.*

### **117. Determining the price when the settlement date falls between coupon periods**

Once the number of days between the settlement date and the next coupon date is determined, the present value formula must be modified to take into account that the cash flows will not be received 6 months (one full period) from now. The “street convention is to compute the price as follows:

1. Determine the number of days in the coupon period.
2. Compute the ratio:

$$W = \frac{\text{Number of days between settlement and next coupon payment}}{\text{Number of days in the coupon period}}$$

For a corporate bond, municipal bond, or agency security, the number of days in the coupon period will be 180, since a year is assumed to have 360 days. For a coupon-

bearing Treasury security, the number of days is the actual number of days. The number of days in the coupon period is called the basis.

3. For a bond with  $n$  coupon payments remaining to maturity, the price is:

$$P = \frac{c}{(1+y)^w} + \frac{c}{(1+y)^{w+1}} + \frac{c}{(1+y)^{w+2}} + \dots + \frac{c}{(1+y)^{n-1+w}} + \frac{M}{(1+y)^{n-1+w}}$$

where

$p$  = Price (\$);

$c$  = Semiannual coupon payment (\$);

$Y$  = Periodic interest rate (required yield divided by 2) (in decimal form);

$n$  = Number of coupon payments remaining;

$M$  = Maturity value.

The period (exponent) in the formula for determining the present value can be expressed generally as  $t - 1 + w$ . For example, for the first cash flow, the period is  $1 - 1 + w$ , or simply  $w$ . For the second cash flow it is  $2 - 1 + w$ , or simply  $1 + w$ . If the bond has 20 coupon payments remaining, the period is  $20 - 1 + w$ , or simply  $19 + w$ .

*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 6, pp. 84-85.*

## 118. What is Clean price, Dirty price and Accrued interest?

### **Accrued interest:**

The buyer must compensate the seller for the portion of the next coupon interest payment the seller has earned but will not receive from the issuer because the issuer will send the next coupon payment to the buyer. This amount is called *accrued interest*. Interest accrues on a bond from and including the date of the previous coupon up to but *excluding* a date called the *value date*. The value date is usually, but not always, the same as the settlement date. Unlike the settlement date, the value date is not constrained to fall on a business day.

Calculation of accrued interest assumes the coupon payment takes place on the scheduled date, even if in practice it will be delayed because the scheduled date is a non-business day. In the formulas below we use the settlement date rather than the value date.

The accrued interest is calculated as follows:

$$AI = C \left( \frac{\text{Number of days from last coupon payment to settlement date}}{\text{Number of days in coupon period}} \right)$$

Where

$AI$  = Accrued interest (\$);

$c$  = Semiannual coupon payment (\$).

Accrued interest is not computed for all bonds. No accrued interest is computed for bonds in default and income bonds. A bond that trades without accrued interest is said to be traded “flat.”

### ***Clean Price and Dirty Price***

The full or dirty price includes the accrued interest that the seller is entitled to site. The *clean price* or *flat price* is the dirty price of the bond minus the accrued interest; that is,

$$\text{Clean price} = \text{Dirty price} - \text{Accrued interest.}$$

The price that the buyer pays the seller is the dirty price. It is important to note that in calculation of the dirty price, the next coupon payment is a discounted value, but in calculation of accrued interest it is an undiscounted value. Because of this market practice, if a bond is selling at par and the settlement date is not a coupon date, the yield will be slightly less than the coupon rate. Only when the settlement date and coupon date coincide is the yield equal to the coupon rate for a bond selling at par.

In the U.S. market, the convention is to quote a bond’s clean or flat price. The buyer, however, pays the seller the dirty price. In some non-U.S. markets, the dirty price is quoted.

*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 6, pp. 86-87.*

## **119. Explain Cum-dividend and Ex-dividend**

When the buyer receives the next coupon, the bond is said to be traded *cum-dividend* (or *cum-coupon*), and the buyer pays the seller accrued interest. If the buyer forgoes the next coupon, the bond is said to be traded *ex-dividend* (or *ex-coupon*), and the seller pays the buyer accrued interest. In some markets (the U.S. is one) bonds are always traded cum-dividend. In other markets, bonds are traded ex-dividend for a certain period before the coupon date.

*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 6, pp. 87.*

## **120. What is Callable and Puttable Bonds? How can it be calculated?**

### ***Callable bond***

An issue may have a provision granting the issuer an option to buy back all or part of the issue prior to the stated maturity date. The right of the issuer to retire the issue prior to the stated maturity date is referred to as a *call option* and the bond is said to be a *callable bond*. If an issuer exercises this right, the issuer is said to “call the bond”. The price which the issuer must pay to retire the issue is referred to as the *call price*. Typically, there is not one call price but a call schedule which sets forth a call price based on when the issuer can exercise the call option.

When a bond is issued, typically the issuer may not call the bond for a number of years. That is, the issue is said to have a *deferred call*. The date at which the bond may first be called is referred to as the *first call date*.

Generally, the call schedule is such that the call price at the first call date is a premium over the par value and scaled down to the par value over time. The date at which the issue is first callable at par value is referred to as the *first par call date*.

When a bond is callable, an investor can calculate a yield to an assumed call date. The *yield to call* is the interest rate that makes the present value of the cash flows to the assumed call date equal to the dirty price of the bond. Mathematically, the yield to an assumed call date for a bond on which the next coupon payment will be due 6 months from now can be expressed as follows:

$$p = \frac{c}{(1+y)^1} + \frac{c}{(1+y)^2} + \frac{c}{(1+y)^3} + \dots + \frac{c}{(1+y)^{n^*}} + \frac{CP}{(1+y)^{n^*}}$$

Where

$p$  = Price (\$);

$c$  = Semiannual coupon interest (\$);

$y$  = one-half the yield to call (in decimal form);

$n^*$  = Number of periods until assumed call date (*number of years*  $\times$  2);

$CP$  = Call price at assumed call date from the call schedule.

For a semi-annual-pay bond, doubling the interest rate ( $y$ ) gives the yield to call.

Alternatively, the yield to call can be expressed as follows:

$$p = \sum_{i=1}^{n^*} \frac{C}{(1+y)^i} + \frac{CP}{(1+y)^{n^*}}$$

Two commonly used call dates are the first call date and the first par call date. To calculate the yield to the first call date, the value for  $CP$  used in the formula is the call price on the first call date as given in the call schedule. In computing the yield to the first par call date, the par value is used for  $CP$  in the formula.

### ***Puttable bond:***

An issue with a put provision grants the bondholder the right to sell the issue back to the issuer at par value on designated dates. Consequently, a bond with a put provision gives the bondholder the right to change the maturity of the bond. A bond with this provision is called a puttable bond. There may be one put price or a schedule of put prices. Most puttable bonds have just one put price.

For a puttable bond, a yield to put can be calculated for any possible put date. The formula for the yield to put for any assumed put date is

$$p = \frac{c}{(1+y)^1} + \frac{c}{(1+y)^2} + \frac{c}{(1+y)^3} + \dots + \frac{c}{(1+y)^{n^*}} + \frac{PP}{(1+y)^{n^*}}$$

Where:

$p$  = Price (\$);

$c$  = Semiannual coupon interest (\$);

$y$  = One-half the yield to call (in decimal form);

$n^*$  = Number of periods until assumed call date (*number of years*  $\times$  2);

$PP$  = put price at assumed put date.

Doubling  $y$  gives the yield to put on a bond-equivalent basis.

Alternatively, the yield to put can be expressed as follows:

$$p = \sum_{i=1}^{n^*} \frac{C}{(1+y)^i} + \frac{PP}{(1+y)^{n^*}}$$

*Fixed Income Mathematics*, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 7, pp. 102-106

## 121. What types of bond portfolio yields do you know?

There are two conventions that have been adopted by practitioners to calculate a portfolio yield:

- (1) Weighted-average portfolio yield
- (2) Portfolio internal rate of return.

### **Weighted-Average Portfolio Yield**

Probably the most common-and most flawed – method for calculating a portfolio yield is to calculate the weighted average of the yield of all the securities in the portfolio. The yield is weighted by the proportion of the portfolio that a security makes up. In general, if let

$W_i$  = Market value of security  $i$  relative to the total market value of the portfolio;

$y_i$  = Yield on security  $i$ ;

$K$  = Number of securities in the portfolio;

Then, the weighted-average portfolio yield is

$$Y_p = Y_1W_1 + Y_2W_2 + Y_3W_3 + \dots + Y_kW_k$$

Equivalently

$$Y_p = \sum_{i=1}^K Y_iW_i$$

### **Portfolio Internal Rate of Return**

Another measure used to calculate a portfolio yield is the internal rate of the portfolio's cash flow. It is computed by first determining the cash flows for all the securities in the portfolio, and then finding the interest rate that will make the present value of the cash flows equal to the market value of the portfolio.

Cash Flows for a three Bonds Portfolio

<i>Period Cash Flow Received</i>	Bond A(\$)	Bond B(\$)	Bond C(\$)	Portfolio (\$) A+B+C
1	350,000	1,050,000	900,000	2,300,000
2	350,000	1,050,000	900,000	2,300,000
3	350,000	1,050,000	900,000	2,300,000
4	350,000	1,050,000	900,000	2,300,000
5	350,000	1,050,000	900,000	2,300,000
6	350,000	1,050,000	30,900,000	32,300,000
7	350,000	1,050,000	-----	1,400,000
8	350,000	1,050,000	-----	1,400,000
9	350,000	1,050,000	-----	1,400,000
10	10,350,000	1,050,000	-----	11,400,000
11	-----	1,050,000	-----	1,050,000
12	-----	1,050,000	-----	1,050,000
13	-----	1,050,000	-----	1,050,000
14	-----	21,050,000	-----	21,050,000

Where

	Semi-annual Yield	Price (\$)
Bond A	5%	\$8,841,739.76
Bond B	6%	\$18,605,752.41
Bond C	7%	\$24,280,152.41
<b>Total Price or Initial investment</b>		<b>\$51,727,644.58</b>

$$0 = \text{Initial investment} + \sum_{i=1}^n \frac{C_{p_i}}{(1 + IRR_p)^i}$$

In this case portfolio IRR is 6.12% doubling and gives 12.24% which is the yield on the portfolio on a bond equivalent basis.

*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 7, pp. 106-108.*

## 122. How can the yield on investment be calculated for single cash flow investment? Multiple cash flow investment?

Consider a financial instrument that can be purchased for \$6,805.82 promises to pay \$10,000 in 5 years. The yield is the interest rates that will make \$6,805.82 grow to \$10,000 in 5 years. That is, we are looking for the value of  $y$  that will satisfy the following relationship:



$$\$10\,000 = \$6\,805,82(1 + y)^5$$

Solving it, we have  $y = 0,08$  or 8% and hence the yield on this investment is 8%.

The following formula is consistent with the above:

$$y = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1$$

Where

$n$  – Number of periods until the cash flow will be received

$FV$  – Future value of an investment

$PV$  – Initial investment

For multiple cash flow case, yield calculation becomes identical to the calculation of internal rate of return ( $IRR$ ) of an investment as you might be aware of the fact that  $IRR$  of a bond is its yield. To see that it is true, let's compare the *Bond Price* and  $IRR$  formulas:

$$0 = -C_0 + \sum_{i=1}^n \frac{C_i}{(1 + IRR)^i}$$

$$Bond\ Price = \sum_{i=1}^n \frac{C_i}{(1 + y)^i}, \text{ or } 0 = -Bond\ Price + \sum_{i=1}^n \frac{C_i}{(1 + y)^i}$$

As you can see from the above,  $IRR$  of an investment in a bond is its yield.

*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 5, pp. 58-59*

### **123. How are the Coupon rate and Yield Related to each other? What influence does their relationship have on price of bond?**

For a bond issue, the coupon rate and the term to maturity are fixed. Consequently, as yields in the marketplace change, the only variable that can change to compensate for the new yield required in the market is the price of the bond. If the required yield increases (decreases), the price of the bond decreases (increases).

Generally, a bond's coupon rate at the time of issuance is set at approximately the prevailing yield in the market. The price of the bond will then be approximately equal to its par value.

*Selling at par*

*When the coupon rate equals the required yield, then the price equals the par value.*

*When the price equals the par value, then the coupon rate equals the required yield.*

When yields in the marketplace rise above the coupon rate at a particular time, the price of the bond has to adjust so that the investor can realize additional interest income. This adjustment happens when the bond's price falls below the par value. The difference between

the par value and the price is a capital gain and represents a form of interest income to the investor to compensate for the coupon rate being lower than the required yield.

#### Selling at *Discount*

*When the coupon rate is less than the required yield, then the price is less than the par value.*

*When the price is less than the par value, then the coupon rate is less than the required yield.*

Finally, when the required yield in the market is below the coupon rate, the bond must sell above its par value. This occurs because investors who would have the opportunity to purchase the bond at par would be getting a coupon rate in excess of what the market requires. Because its yield is attractive, investors would bid up the price of the bond to a price that offers the required yield in the market.

#### Selling *premium*:

*When the coupon rate is higher than the required yield, then the price is higher than the par value.*

*When the price is higher than the par value, then the coupon rate is higher than the required yield*

*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 6, pp. 72-73*

### **124. What is the required yield for bond? How can it be calculated?**

The interest rate or discount rate that an investor wants from investing in a bond is called the required yield. The required yield is determined by investigating the yields offered on comparable bonds in the market. By comparable, we mean bonds of the same credit quality and the same maturity.

The required yield is typically specified as an annual interest rate. When the cash flows are semi-annual, the convention is to use one-half the annual interest rate as the periodic interest rate with which to discount the cash flows.

Given the cash flows of a bond and the required yield, we have all the necessary data to price the bond. The price of a bond is the present value of the cash flows, which can be determined by adding:

1. The present value of the semi-annual coupon payments.
2. The present value of the par or maturity value.

In general, the price of a bond can be computed using the formula:

$$P = \frac{c}{(1+y)^1} + \frac{c}{(1+y)^2} + \frac{c}{(1+y)^3} + \dots + \frac{c}{(1+y)^n} + \frac{M}{(1+y)^n}$$

Where

$P$  = Price (\$);

$c$  = Semi-annual coupon payment (\$);  
 $y$  = Periodic interest rate (required yield/2);  
 $n$  = Number of periods (number of years $\times$ 2);  
 $M$  = Maturity value (\$).

Since the semi-annual coupon payments are equivalent to an ordinary annuity, the present value of the coupon payments, that is, the present value of:

$$P = \frac{c}{(1+y)^1} + \frac{c}{(1+y)^2} + \frac{c}{(1+y)^3} + \dots + \frac{c}{(1+y)^n}$$

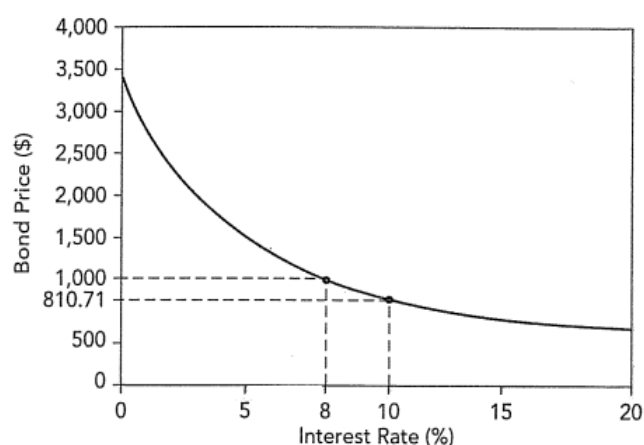
Can be expressed as:

$$P = c \left[ \frac{1 - \left[ \frac{1}{(1+y)^n} \right]}{y} \right]$$

*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 6, pp. 66-67*

## 125. Graph and explain the relationship between Bond Price and Required Yield

To demonstrate this method, we first need to review the relationship between a bond's price and its yield. In general, as a bond's price increases, yield decreases. This relationship is measured using the price value of a basis point (PVBP). By taking into account factors such as the bond's coupon rate and credit rating, the PVBP measures the degree to which a bond's price will change when there is a 0.01% change in interest rates. The charted relationship between bond price and required yield appears as a negative curve:



This is due to the fact that a bond's price will be higher when it pays a coupon that is higher than prevailing interest rates. As market interest rates increase, bond prices decrease.

*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 6, pp. 71-72*

### 126. What happens to Bond price when time reaches its maturity?

If the required yield is unchanged between the time a bond is purchased and the maturity date, what will happen to the price of the bond? For a bond selling at par value, the coupon rate is equal to the required yield. As the bond moves closer to maturity, the bond will continue to sell at par value. Thus, the price of a bond selling at par will remain at par as the bond moves toward the maturity date.

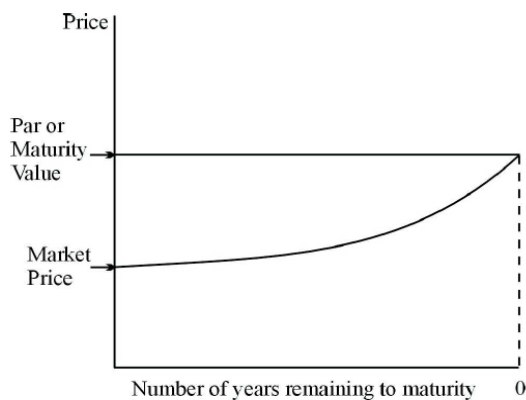
The price of a bond will not remain constant for a bond selling at a premium or a discount. This can be seen for a discount bond by comparing the price found in Illustration 1-1 to that found in Illustration 1-2. In both illustrations the bond has a 9% coupon rate and the required yield is 12% or 7%.

#### Illustration 1

Time Path of the Price of a Discount Bond: 20-Year, 9% Coupon, 12% Required Yield:

Years Remaining To Maturity	Present Value of Coupon Payments of \$45 at 6% (\$)	Present Value of Par Value at 6% (\$)	Price of Bond (\$)
20	677.08	97.22	774.30
18	657.94	122.74	780.68
16	633.78	154.96	788.74
14	603.28	195.63	798.91
12	564.77	256.98	811.75
10	516.15	311.80	827.95
8	454.77	393.65	848.42
6	377.27	496.97	874.24
4	279.44	627.41	906.85
2	155.93	792.09	948.02
1	82.50	890	972.50
0	0	1000	1000

Time Path of a Discount Bond Assuming No Change in Required Yield:

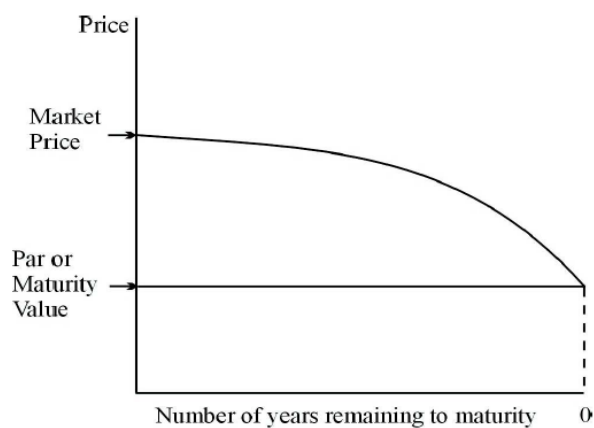


*Illustration 2*

Time Path of the Price of a Premium Bond: 20-Year, 9% Coupon, 7% Required Yield:

Years Remaining To Maturity	Present Value of Coupon Payments of \$45 at 3.5% (\$)	Present Value of Par Value at 3.5% (\$)	Price of Bond (\$)
20	960.98	252.57	1213.55
18	913.07	289.83	1202.90
16	858.10	332.59	1190.69
14	795.02	381.65	1176.67
12	722.63	437.96	1160.59
10	639.56	502.57	1142.13
8	544.24	576.71	1120.95
6	434.85	661.78	1096.63
4	309.33	759.41	1068.74
2	165.29	871.44	1036.73
1	85.49	933.51	1019.00
0	0.00	1000.00	1000.00

Time Path of a Premium Bond Assuming No Change in Required Yield:



*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 6, pp. 73-76*

### 127. What is the Yield to Maturity of a Bond?

The yield to maturity is the interest rate that will make the present value of the cash flows equal to the price (or initial investment). The yield to maturity is computed in the same way as IRR; the cash flows are those that the investor would realize by holding the bond to

maturity and reinvesting coupons at the same rate. For bond, the yield to maturity is computed by solving the following relationship for  $y$ :

$$P = \frac{c}{(1+y)^1} + \frac{c}{(1+y)^2} + \frac{c}{(1+y)^3} + \cdots + \frac{c}{(1+y)^n} + \frac{M}{(1+y)^n}$$

Where

- P = Price (\$);
- c = coupon interest (\$);
- y = yield to maturity;
- n = Number of periods;
- M = Maturity value (\$).

The yield to maturity considers not only the current coupon income but any capital gain or loss that the investor will realize by *holding the bond to maturity*. The yield to maturity also considers the timing of the cash flows.

*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 7, p. 94*

## 128. What are Zero-Coupon bonds? What is its Yield to Maturity?

Bonds that do not make any periodic coupon payments are called zero coupon bonds. Instead, the investor realizes interest by the amount of the difference between the maturity value and the purchase price.

The pricing of a zero-coupon bond is no different from the pricing of a coupon bond: its price is the present value of the expected cash flows. In the case of a zero-coupon bond, the only cash flow is the maturity value. Therefore, the formula for the price of a zero-coupon bond that matures  $N$  years from now is:

$$P = \frac{M}{(1+y)^n}$$

Where

- $P$  = Price (\$);
- $M$  = Maturity value (\$);
- $y$  = Periodic interest rate;
- $n$  = Number of periods;

A zero-coupon bond is characterized by a single cash flow resulting from the investment. Consequently, the following formula can be applied to compute the yield to maturity for a zero coupon bond:

$$y = (\text{Future value per dollar invested})^{1/n} - 1$$

Where

- $y$  = yield to maturity;

and future value per dollar invested is given by:

$$\text{Future value per dollar invested} = \frac{\text{Maturity value}}{\text{Price}}$$

*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 7, pp. 96-97*

### 129. What is Current Yield? How is it calculated? What interdependence it has with Yield to Maturity and Coupon Rate?

The current yield relates the *annual coupon* interest to the market price. The formula for the current yield is:

$$\text{Current yield} = \frac{\text{Annual dollar coupon interest}}{\text{Price}}$$

These relationships pertain among coupon rate, current yield, and yield to maturity:

Bond Selling at	Relationship
Par	Coupon rate = Current Yield = Yield to Maturity
Discount	Coupon rate < Current Yield < Yield to Maturity
Premium	Coupon rate > Current Yield > Yield to Maturity

*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 7, pp. 93, 99*

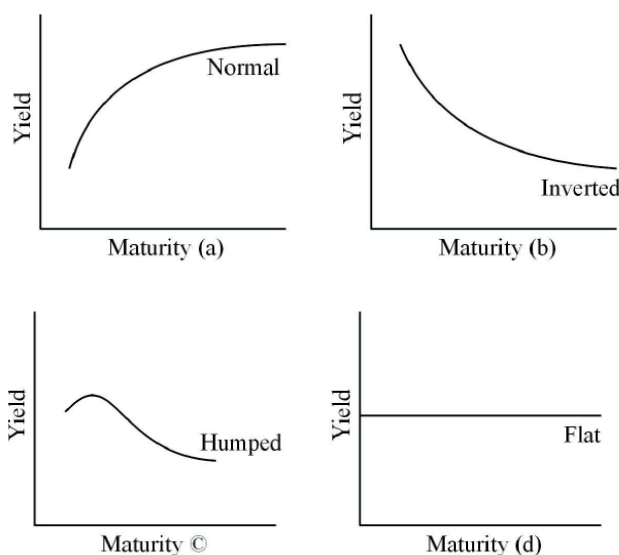
### 130. What is Yield Curve?

The graphical depiction of the relationship between the yield to maturity on securities of the same credit risk and different maturity is called the *yield curve*. The yield curve is constructed from the maturity and observed yield of Treasury securities because Treasuries reflect the pure effect of maturity alone on yield, given that market participants do not perceive government securities to have any credit risk. When market participants refer to the “yield curve,” they usually mean the Treasury yield curve.

There are four Hypothetical Yield Curves:

- *Upward sloping or normal yield curve* – When the yield increases with maturity.
- *Downward sloping or an inverted yield curve* – When yield decreases with maturity.
- *Humped yield curve* – When the yield curve initially is upward sloping, but after a certain maturity it becomes downward sloping.
- *Flat yield curve* – When yield curve is one where the yield is the same regardless of the maturity.

## Four Hypothetical Yield Curves:



*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 8, pp. 116-117.*

### 131. What is Spot Rate Curve?

The Treasury yield curve shows the relationship between the yield on Treasury securities (Treasury bills and coupon securities) and maturity. According to the pure expectations theory and arbitrage arguments, we can determine the theoretical relationship between the yield on zero-coupon Treasury securities and maturity. This relationship is called the *Treasury spot rate curve*. The yield on a zero-coupon instrument is called a *spot rate*.

To illustrate how to construct a theoretical spot rate curve from the yield offered on Treasury securities, use the 10 hypothetical Treasury securities.

The basic principle is that the value of a Treasury coupon security should be equal to the value of a package of zero-coupon Treasury securities. Consider first the 6-month Treasury bill in Exhibit 1. Because a Treasury bill is a zero-coupon instrument, its yield of 8% is equal to the spot rate. Similarly, for the 1-year Treasury, the yield of 8.3% is the 1-year spot rate. Given these two spot rates, we can compute the spot rate for a 1.5-year zero-coupon Treasury. The value or price of a 1.5-year zero-coupon Treasury should equal the present value of the three cash flows from the 1.5-year coupon Treasury where the yield used for discounting is the spot rate corresponding to the cash flow. Using \$100 as par, the cash flows for the 1.5-year coupon Treasury are as follows:

0.5 year	$0.0850 \times \$100 \times 0.5 = \$4.25$
1 year	$0.0850 \times \$100 \times 0.5 = \$4.25$
1.5 year	$0.0850 \times \$100 \times 0.5 + 100 = \$104.25$



The present value of the cash flows is then

$$\frac{4.25}{(1+z_1)} + \frac{4.25}{(1+z_2)^2} + \frac{104.25}{(1+z_3)^3}$$

*Exhibit 1*

Maturity (Years)	Coupon Rate	Yield to Maturity	Price(\$)
0.5	0.0000	0.080	96.15
1	0.0000	0.083	92.19
1.5	0.0850	0.089	99.45
2	0.0900	0.092	99.64
2.5	0.1100	0.094	103.49
3	0.0950	0.097	99.49
3.5	0.1000	0.100	100.00
4	0.1000	0.104	98.72
4.5	0.1150	0.106	103.16
5	0.0875	0.108	92.24

Where

$z_1 = \text{One-half the 6-month theoretical spot rate}$

$z_2 = \text{One-half the 1-year theoretical spot rate}$

$z_3 = \text{One-half the 1.5-year theoretical spot rate}$

Because the 6-month spot rate and 1-year spot rate are 8.0% and 8.3% respectively then

$$z_1 = \frac{0.08}{2} = 0.04 \text{ and } z_2 = \frac{0.083}{2} = 0.0415$$

Therefore the present value of the 1.5-year coupon Treasury security is

$$\frac{4.25}{(1+0.04)} + \frac{4.25}{(1+0.0415)^2} + \frac{104.25}{(1+z_3)^3}$$

As the price of the 1.5-year coupon Treasury Security is \$99.45, the following relationship must hold:

$$99.45 = \frac{4.25}{(1+0.04)} + \frac{4.25}{(1+0.0415)^2} + \frac{104.25}{(1+z_3)^3}$$

Solve for the theoretical 1.5-year spot as follows:

$$99.45 = 4.08654 + 3.91805 + \frac{104.25}{(1+z_3)^3}$$

$$91.4451 = \frac{104.25}{(1+z_3)^3}$$

$$(1+z_3)^3 = 1.140024$$

$$z_3 = 0.04465$$

Doubling this yield, obtain the bond-equivalent yield of 0.0893 or 8.93%, which is the theoretical 1.5-year spot rate.

In general, to compute the theoretical spot rate for the  $n$ th 6-month period, the following equation must be solved:

$$P_n = \frac{c^*}{(1+z_1)^1} + \frac{c^*}{(1+z_2)^2} + \frac{c^*}{(1+z_3)^3} + \dots + \frac{c^* + 100}{(1+z_n)^n}$$

Where

$P_n$  = Price of the coupon Treasury with  $n$  periods to maturity (per \$100 of par value)

$c^*$  = Semiannual coupon interest for the coupon Treasury with  $n$  periods to maturity per \$100 of par value and  $z_i$  for  $i = 1, 2, \dots, n-1$  are the theoretical spot rates that are known.

This expression can be written as follows:

$$P_n = \sum_{i=1}^{n-1} \frac{c^*}{(1+z_i)^i} + \frac{c^* + 100}{(1+z_n)^n}$$

Solving for  $z_n$  get:

$$z_n = \left[ \frac{c^* + 100}{P_n + \sum_{i=1}^{n-1} \frac{c^*}{(1+z_i)^i}} \right]^{1/n} - 1$$

Doubling  $z_n$  gives the theoretical spot rate on a bond-equivalent basis.

*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 8, pp. 117-121.*

### 132. What is Forward Rates? How can it be calculated?

The theoretical spot rate curve can be constructed from the yield curve. But there may be more information contained in the yield curve. Specifically, can use the yield curve to infer the market's expectations of future interest rates? Let's explore this possibility.

Suppose an investor with a 1-year investment horizon is considering two alternatives:

*Alternative 1:* Buy a 1-year Treasury bill.

*Alternative 2:* Buy a 6-month Treasury bill, and when it matures in 6 months buy another 6-month Treasury bill.

The investor will be indifferent between the two alternatives if they produce the same yield or the same number of dollars per dollar invested over the 1-year investment horizon.

The investor knows the spot rate on the 6-month Treasury bill and the 1-year Treasury bill, but not what yield will be available on a 6-month Treasury bill purchased 6 months from now. The yield on a 6-month Treasury bill 6 months from now is called the *forward rate*. Given the spot rate for the 6-month Treasury bill and the 1-year Treasury bill rate, it is possible to determine the forward rate on a 6-month Treasury bill that will make investors indifferent to the two alternatives.

By investing in the 1-year Treasury bill, the investor will receive the maturity value at the end of 1 year. Suppose that the maturity value of the 1-year Treasury bill is \$100. The price (cost) of the 1-year Treasury bill would be as follows:

$$\frac{100}{(1 + z_2)}$$

where  $z_2$  is one-half the bond-equivalent yield of the theoretical 1 year spot rate.

Suppose that an investor purchase a 6-month Treasury bill for  $P$  dollars. At the end of 6 months, the value of this investment would be

$$P(1 + z_1)$$

where  $z_1$  is one-half the bond-equivalent yield of the theoretical 6-month spot rate.

Let  $f$  be one-half the forward rate on a 6-month Treasury bill available 6 months from now. Then the future dollars available at the end of 1 year from the  $P$  dollars invested would be given by

$$P(1 + z_1)(1 + f)$$

Suppose that today want to know how many  $P$  dollars an investor must invest in order to get \$100 in 1 year. This can be found as follows:

$$P(1 + z_1)(1 + f) = 100$$

Solving for  $P$ , get

$$P = \frac{100}{(1 + z_1)(1 + f)}$$

The investor will be indifferent between the two alternatives if the same dollar investor is made and \$100 is received from both investments at the end of 1 year. That is, an investor will be indifferent if

$$\frac{100}{(1 + z_2)} = \frac{100}{(1 + z_1)(1 + f)}$$

Solving for  $f$ , get

$$f = \frac{(1 + z_2)^2}{(1 + z_1)} - 1$$

Doubling  $f$  gives the bond-equivalent yield for the 6-month forward rate. In general, the formula for the forward rate is:

$$f_n^t = \left[ \frac{(1 + z_{n+t})^{n+t}}{(1 + z_n)^n} \right]^{1/t} - 1$$

Where  $f_n^t$  = forward rates periods from now for  $t$  periods.

*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 8, pp. 127-133.*

### 133. How to value cash flows and bonds using forward rates?

Since spot rates and forward rates are related, then it makes no difference whether spot rates or forward rates are used to determine the present value of a cash flow. In general, the present value of a cash flow  $c$  in period  $r$  using forward rates is:

$$\frac{c}{(1 + z_1)(1 + f_1^1)(1 + f_2^1) \dots (1 + f_n^1)}$$

The general formula for using forward rates to value an  $n$ -period-maturity bond whose semiannual cash flow is denoted by  $c$  and maturity value is denoted by  $M$  is

$$\begin{aligned} Price = & \frac{c}{(1 + z_1)} + \frac{c}{(1 + z_1)(1 + f_1^1)} + \frac{c}{(1 + z_1)(1 + f_1^1)(1 + f_2^1)} + \dots \\ & + \frac{c + M}{(1 + z_1)(1 + f_1^1)(1 + f_2^1) \dots (1 + f_n^1)} \end{aligned}$$

Oftentimes, vendors of analytical systems will state how they value cash flows. Some state that spot rates are used while others state that forward rates are used. The results from the discounting process will be the same.

*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 8, pp. 135.*

### 134. Decompose total return of bond held to maturity and explain interest on interest component

The total return is the interest rates that will make the initial investment in the bond grow to the computed total future amount. To illustrate the computation, we first show how the total return is computed assuming that a bond is held to the maturity date.

The total return is then the interest rate that will make the amount invested in the bond equal to the total future amount available at the maturity date given that received coupons are reinvested at some reinvestment rate. More formally, the steps for computing the total return for a bond *held to maturity* are now given.

*Step 1.* Compute the total future amount that will be received from the investment.

That is the sum of (1) the total amount of received coupon payments, (2) interest on interest from reinvesting coupon payments at the assumed reinvestment rate, (3) and the par value. The coupon payments plus the interest on interest can be computed by using the formula:

$$\text{coupon interest plus interest on interest} = c \left[ \frac{(1+r)^n - 1}{r} \right]$$

Where

- $c$  = semi-annual coupon payment (\$);
- $r$  = semi-annual reinvestment rate;
- $n$  = number of semi-annual periods.

Generally, coupons are assumed to be reinvested at the rate equal to the yield to maturity of the bond, though the market might not offer such opportunity.

Thus, total future amount is computed as:

$$\text{Total future amount} = c \left[ \frac{(1+r)^n - 1}{r} \right] + \text{par value}$$

The total future amount of the bond sold prior to maturity at some time,  $k$ , includes only the sale price, not the entire par value. That is:

$$\text{Total future amount} = c \left[ \frac{(1+r)^k - 1}{r} \right] + \text{sale price}$$

*Step 2.* To obtain the semi-annual total return, use the following formula:

$$\left[ \frac{\text{total future amount}}{\text{price of bond}} \right]^{\frac{1}{n}} - 1$$

*Step 3.* Since interest is assumed to be paid semi-annually, double the interest rate found in Step 2. The resulting interest rate is the total return on a bond-equivalent basis. The total return can also be computed by compounding the semi-annual rate as follows:

$$(1 + \text{Semiannual total return})^2 - 1$$

The total return calculated in this manner is said to be on an *effective rate basis*.

*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 10, pp. 155-158.*

### 135. Decompose dollar return of bond held to maturity

An investor who purchases a bond can expect to receive a dollar return from one or more of the following sources:

1. The periodic interest payments made by the issuer (that is, the coupon interest payments);

2. Any capital gain (or capital loss) when the bond matures or is sold;
3. Income from reinvestment of the periodic interest payments (the interest-on-interest component).

The interest-on-interest component can represent a substantial portion of a bond's potential return.

For purposes of computing the coupon interest and interest on interest for a bond paying interest semi-annually, we can rewrite the formula for the future value of an annuity for computing the coupon interest plus interest on interest, as:

$$\text{coupon interest plus interest on interest} = c \left[ \frac{(1+r)^n - 1}{r} \right]$$

Where

- $c$  = semi-annual coupon payment;
- $r$  = semi-annual reinvestment rate;
- $n$  = number of semi-annual periods.

The total coupon interest is found by multiplying the semi-annual coupon interest by the number of periods, that is:

$$\text{total coupon interest} = n \times c$$

The interest-on-interest component then mathematically can be expressed as:

$$\text{interest on interest} = c \left[ \frac{(1+r)^n - 1}{r} \right] - n \times c$$

The final part of dollar return is the capital gain (loss) when the bond matures or is sold. it can be computed by the following formulae:

$$\text{capital gain (loss)} = \text{sales price (or par value)} - \text{purchase price}$$

And

$$\text{dollar return} = \text{coupon payments} + \text{interest on interest} + \text{capital gain (loss)}$$

*Fixed Income Mathematics, 4<sup>th</sup> ed., Frank J. Fabozz, McGraw-Hill; Chapter 9, pp. 140-142*

# DERIVATIVES

*Course book: Derivatives Markets, 2nd ed., Robert L. McDonald; Pearson, 2006*

## **136. What are Derivative Instruments? Why are they called “Derivatives”?**

Options, futures and swaps are examples of derivatives. A derivative is financial instrument (or more precisely, an agreement between two people) that has value determined by the price of something else. For example, a bushel of corn is not a derivative; it is a commodity with a value determined by the price of corn. However, you could enter into an agreement with a friend that says: if the price of a bushel of corn is less than \$3, the friend will pay you \$1. This is a derivative in the sense that you have an agreement with a value derived by the price of something else (corn, in this case), that’s why it is called derivative – its value is derived from the price of something else.

You might think: “that’s not a derivative; that’s just a bet on the price of corn.” So it is: derivatives can be thought of as bets on the price of something. But don’t automatically think the term “bet” is pejorative. Suppose your family grows corn and you friend’s family buys corn to mill into cornmeal. The bet provides insurance: you earn \$1 if your family’s corn sells for a low price; this supplements you income. Your family’s friend earns \$1 if the corn his family buys is expensive; this offsets the high cost of corn. Viewed in this light, the bet hedges you both against unfavorable outcomes. The contract has reduced risk for both of you.

Investors could also use this kind of contract simply to speculate on the price of corn. In this case the contract is not insurance. And that is a key point: *it is not the contract itself, but how it is used, and who uses it, that determines whether or not it is risk-reducing.* Context is everything.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 1, pp. 1-2*

## **137. Explain the concept of Forward Contract**

Suppose you wish to buy a share of stock. Doing so entails at least three separate steps: (1) setting the price to be paid, (2) transferring cash from the buyer to the seller, and (3) transferring the share from the seller to the buyer. With an outright purchase of stock, all three occur simultaneously. However, as a logical matter, a price could be set today and the transfer of shares and cash would occur at a specified date in the future.

This is in fact a definition of a forward contract: it sets today the terms at which you buy or sell an asset or commodity at a specific time in the future. A forward contract does the following:

- Specifies the quantity and exact type of the asset or commodity the seller must deliver
- Specifies delivery logistics, such as time, date, and place
- Specifies the price the buyer will pay at the time of delivery
- Obligates the seller to sell and the buyer to buy, subject to the above specifications

The time at which the contract settles is called the *expiration date*. The asset or commodity on which the forward contract is based is called the *underlying asset*. Apart from commissions and bid-ask spreads, a forward contract requires no initial payment or premium. The contractual forward price simply represents the price at which consenting adults agree today to transact in the future at which time the buyer pays the seller the forward price and the seller delivers the asset.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 2, p. 21*

### **138. What are Future Contracts? What is the difference between Forwards and Futures Contracts?**

Futures contracts are similar to forward contract in that they create an obligation to buy or sell at a predetermined price at a future date. Futures contracts are essentially exchange-traded forward contracts. Because futures are exchange-traded, they are standardized and have specified delivery dates, locations, and procedures. Each exchange has an associated *clearinghouse*. The role of the clearinghouse is to match the buys and sells that take place during the day, and to keep track of the obligations and payments required of the members of the clearinghouse, who are *clearing members*. After matching trades, the clearinghouse typically becomes the counterparty for each clearing member.

Although forwards and futures are similar in many respects, there are differences:

- Whereas forward contracts are settled at expiration, futures contracts are settled daily. The determination of who owes what to whom is called marking-to-market
- As a result of daily settlement, futures are liquid – it is possible to offset an obligation on a given date by entering into the opposite position
- Over-the-counter forward contract can be customized to suit the buyer or seller, whereas futures contracts are standardized
- Because of daily settlement, the nature of credit risk is different with the futures contract, in fact, futures contract are structures so as to minimize the effects of credit risk
- There are typically daily price limits in the futures markets (and on some stock exchanges as well). A *price limit* is a move in the futures price that triggers a temporarily halt in trading

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 5, p. 142*



**139. Explain the concept of Call / Put Option. List their characteristics and draw graph for long and short positions. What is the difference between American and European Options?**

Whereas a forward contract obligates the buyer (the holder of the long position) to pay the forward price at expiration, even if the value of the underlying asset at expiration is less than the forward price, a **call option** is a contract where the buyer has the right to buy, but not the obligation to buy. Here are some key terms used to describe options:

**Strike price:** the **strike price**, or **exercise price**, of a call option is what the buyer pays for the asset

**Exercise:** the **exercise** of a call option is the act of paying the strike price to receive the asset

**Expiration:** the **expiration** of the option is the date by which the option must either be exercised or it becomes worthless

**Exercise style:** the **exercise style** of the option governs the time at which exercise can occur. If exercise can occur only at expiration, option is said to be **European-style option**. If the buyer has the right to exercise at any time during the life of the option, it is an **American-style option**. If the buyer can only exercise during specified periods, but not for the entire life of the option, the option is a **Bermudan-style option**.

To summarize, a European call option gives the owner of the call the right, but not the obligation, to buy the underlying asset on the expiration date by paying the strike price. The buyer is not obligated to buy the underlying, and hence will only exercise the option if the payoff is greater than zero. The algebraic expression for the *payoff* to a purchased (long) call is therefore:

$$\text{Long call payoff} = \max[0, S_T - K]$$

Where  $S_T$  is spot price of the underlying asset at expiration and  $K$  is the exercise price. The expression  $\max[a, b]$  means take the greater of the two values  $a$  and  $b$ . In computing profit at expiration, suppose we defer the premium payment (the price of an option); then by the time of expiration we accrue interest on the premium. So, the option *profit* is computed as:

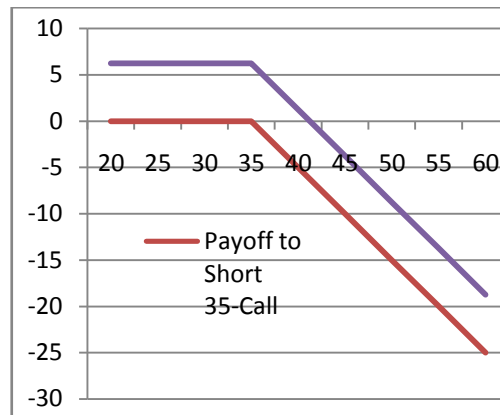
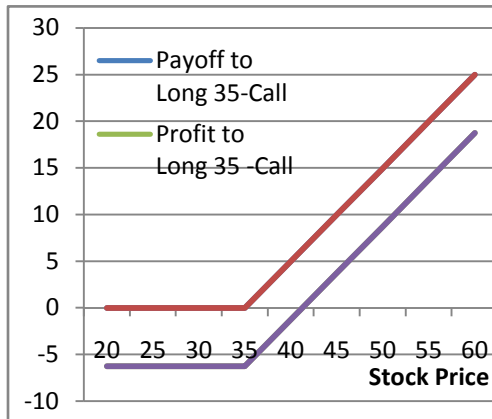
$$\text{Long call profit} = \max[0, S_T - K] - \text{future value of option premium}$$

The seller is said to be the **option writer**, or to have a short position in a call option. The payoff and profit to a written call are just the opposite of those for a purchased call:

$$\text{Short call payoff} = -\max[0, S_T - K]$$

$$\text{Short call profit} = -\max[0, S_T - K] + \text{future value of option premium}$$

Graphically, the payoff and profit calculations for long and short call option are:



Perhaps you wondered if there could also be a contract in which the *seller* could walk away if it is not in his or her interest to sell. The answer is yes. A **put option** is a contract where the seller has the right to sell, but not the obligation. The put option gives the put buyer the right to sell the underlying asset for the strike price. Thus, the *payoff* on the put option is:

$$\text{Long put payoff} = \max[0, K - S_T]$$

At the time the option is acquired, the put buyer pays the option premium to the put seller; we need to account for this in computing profit. If we borrow the premium amount, we must pay interest. The option *profit* is computed as:

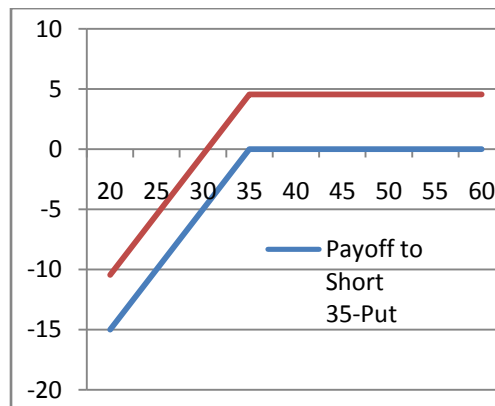
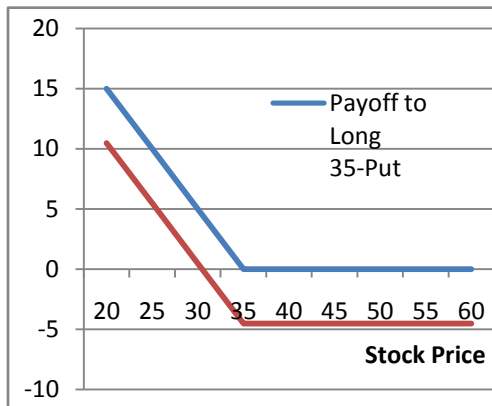
$$\text{Long put profit} = \max[0, K - S_T] - \text{future value of option premium}$$

The payoff and profit for a written put are the opposite:

$$\text{Short put payoff} = -\max[0, K - S_T]$$

$$\text{Short put profit} = -\max[0, K - S_T] + \text{future value of option premium}$$

Graphically, the payoff and profit calculations for long and short put option are:



## 140. List uses of Derivatives

**Risk management** Derivatives is a tool for companies and other users to reduce risks. The corn example above illustrates this in a simple way: The farmer—a seller of corn—enters into a contract which makes a payment when the price of corn is low. This contract reduces the risk of loss for the farmer, who we therefore say is hedging. It is common to think of derivatives as forbiddingly complex, but many derivatives are simple and familiar. Every form of insurance is a derivative, for example. Automobile insurance is a bet on whether you will have an accident. If you wrap your car around a tree, your insurance is valuable; if the car remains intact, it is not.

**Speculation** Derivatives can serve as investment vehicles. As you will see later in the book, derivatives can provide a way to make bets that are highly leveraged (that is, the potential gain or loss on the bet can be large relative to the initial cost of making the bet) and tailored to a specific view. For example, if you want to bet that the S&P500 stock index will be between 1 300 and 1 400 one year from today, derivatives can be constructed to let you do that.

**Reduced transaction costs** Sometimes derivatives provide a lower-cost way to effect a particular financial transaction. For example, the manager of a mutual fund may wish to sell stocks and buy bonds. Doing this entails paying fees to brokers and paying other trading costs, such as the bid-ask spread, which we will discuss later. It is possible to trade derivatives instead and achieve the same economic effect as if stocks had actually been sold and replaced by bonds. Using the derivative might result in lower transaction costs than actually selling stocks and buying bonds.

**Regulatory arbitrage** it is sometimes possible to circumvent regulatory restrictions, taxes, and accounting rules by trading derivatives. Derivatives are often used, for example, to achieve the economic sale of stock (receive cash and eliminate the risk of holding the stock) while still maintaining physical possession of the stock. This transaction may allow the owner to defer taxes on the sale of the stock, or retain voting rights, without the risk of holding the stock.

These are common reasons for using derivatives. The general point is that derivatives provide an alternative to a simple sale or purchase, and thus increase the range of possibilities for an investor or manager seeking to accomplish some goal.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 1, pp. 2-3*

## 141. What are Financial Engineering and Security Design?

One of the major ideas in derivatives—perhaps the major idea—is that it is generally possible to create a given payoff in multiple ways. The construction of a given financial product from other products is sometimes called financial engineering. The fact that this is possible has several implications.

First, since market-makers need to hedge their positions, this idea is central in understanding how market-making works. The market maker sells a contract to an end-user, and then creates an offsetting position that pays him if it is necessary to pay the customer. This creates a hedged position.

Second, the idea that a given contract can be replicated often suggests how it can be customized. The market-maker can, in effect, turn dials to change the risk, initial premium, and payment characteristics of a derivative. These changes permit the creation of a product that is more appropriate for a given situation.

Third, it is often possible to improve intuition about a given derivative by realizing that it is equivalent to something we already understand.

Finally, because there are multiple ways to create a payoff, the regulatory arbitrage discussed above can be difficult to stop. Distinctions existing in the tax code, or in regulations, may not be enforceable, since a particular security or derivative that is regulated or taxed may be easily replaced by one that is treated differently but has the same economic profile.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 1, pp. 3-4*

#### **142. List reasons for short-selling an asset?**

When we buy something, we are said to have a long position in that thing. For example, if we buy the stock of XYZ, we pay cash and receive the stock. Sometime later, we sell the stock and receive cash. This transaction is lending, in the sense that we pay money today and receive money back in the future. The rate of return we receive may not be known in advance (if the stock price goes up a lot, we get a high return; if the stock price goes down, we get a negative return), but it is a kind of loan nonetheless. The opposite of a long position is a short position. A short-sale of XYZ entails borrowing shares of XYZ and then selling them, receiving the cash. Sometime later, we buy back the XYZ stock, paying cash for it, and return it to the lender. A short-sale can be viewed, then, as just a way of borrowing money. When you borrow money from a bank, you receive money today and repay it later, paying a rate of interest set in advance. This is also what happens with a short-sale, except that you don't necessarily know the rate you pay to borrow.

There are at least three reasons to short-sell:

- **Speculation:** A short-sale, considered in essence, makes money if the price of the stock goes down. The idea is to first sell high and then buy low. (With a long position, the idea is to first buy low and then sell high.)
- **Financing:** A short-sale is a way to borrow money, and it is frequently used as a form of financing. This is very common in the bond market, for example.
- **Hedging:** You can undertake a short-sale to offset the risk of owning the stock or a derivative on the stock. This is frequently done by market-makers and traders.

These reasons are not mutually exclusive. For example, a market-maker might use a short-sale to simultaneously hedge and finance a position.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 1, pp. 12-13*

### 143. Explain the concept of insuring a long position

Put options are insurance against a fall in the price of an asset. Thus, if we own the S&R index, we can insure the position by buying an S&R put option. The purchase of a put option is also called a floor, because we are guaranteeing a minimum sale price for the value of the index. To examine this strategy, we want to look at the combined payoff of the index position and put. Now we add them together to see the net effect of holding both positions at the same time.

Table below summarizes the result of buying a 1000-strike put with 6 months to expiration, in conjunction with holding an index position with a current value of \$ 1000. The table computes the payoff for each position and sums them to obtain the total payoff.

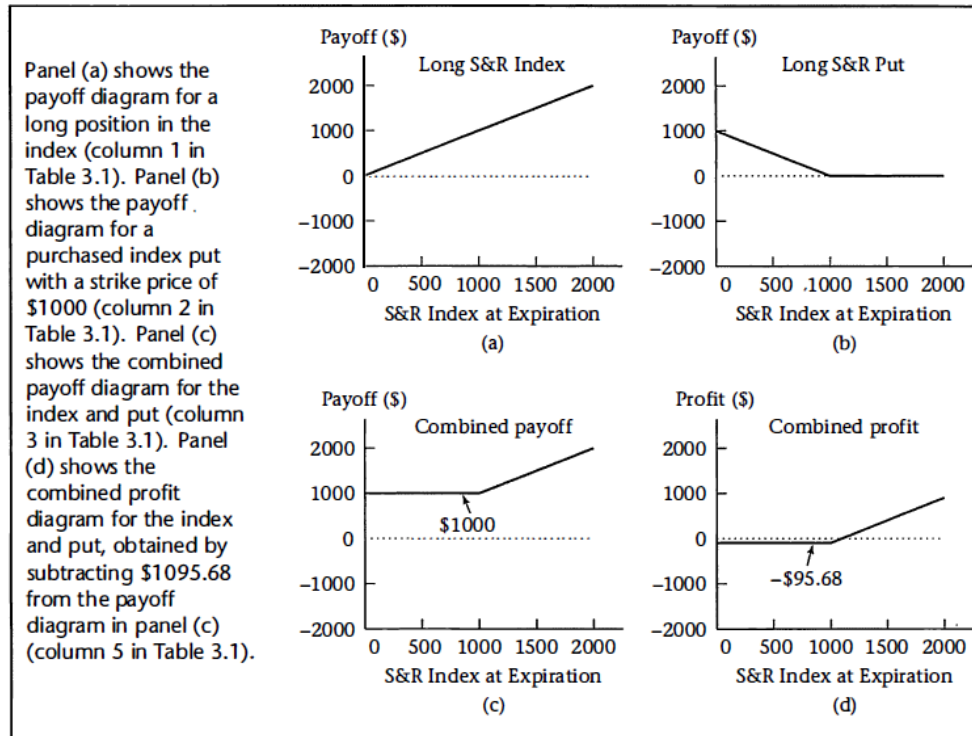
Payoff and profit at expiration from purchasing the S&R index and a 1000-strike put option. Payoff is the sum of the first two columns. Cost plus interest for the position is $(\$1000 + \$74.201) \times 1.02 = \$1095.68$ . Profit is payoff less \$1095.68.				
Payoff at Expiration		Payoff	-(Cost + Interest)	Profit
S&R Index	S&R Put			
\$900	\$100	\$1000	-\$1095.68	-\$95.68
950	50	1000	-1095.68	-95.68
1000	0	1000	-1095.68	-95.68
1050	0	1050	-1095.68	-45.68
1100	0	1100	-1095.68	4.32
1150	0	1150	-1095.68	54.32
1200	0	1200	-1095.68	104.32

The final column takes account of financing cost by subtracting cost plus interest from the payoff to obtain profit. "Cost" here means the initial cash required to establish the position. This is positive when payment is required and negative when cash is received. We could also have computed profit separately for the put and index. For example, if the index is \$900 at expiration, we have:

$$\underbrace{\$900 - (\$1000 \times 1.02)}_{\text{Profit on S\&R Index}} + \underbrace{\$100 - (\$74.201 \times 1.02)}_{\text{Profit on Put}} = -\$95.68$$

This gives the same result as the calculation performed in the Table above. The level of the floor is -\$95.68, which is the lowest possible profit. Figure below graphs the components

of the Table. Panels (c) and (d) show the payoff and profit for the combined index and put positions. The combined payoff graph in panel (c) is created by adding at each index price the value of the index and put positions; this is just like summing columns 1 and 2 in the Table.



*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 2, pp. 59-62*

#### 144. Explain the concept of insuring a short position

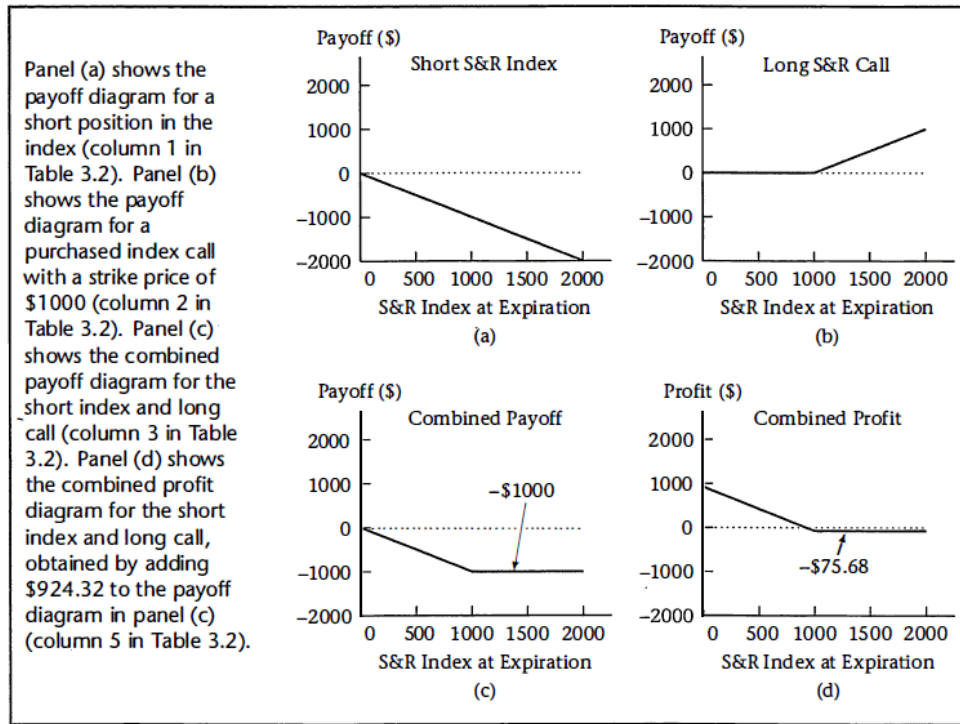
If we have a short position in the S&R index, we experience a loss when the index rises. We can insure a short position by purchasing a call option to protect against a higher price of repurchasing the index. Buying a call option is also called a cap.

Table below presents the payoff and profit for a short position in the index coupled with a purchased call option. Because we short the index, we earn interest on the short proceeds less the cost of the call option, giving -\$924.32 as the future value of the cost. Figure below graphs the columns of the Table. The payoff and profit diagrams resemble those of a purchased put. As with the insured index position, we have to be careful in dealing with cash flows. The payoff in panel (c) of the Figure is like that of a purchased put coupled with borrowing. In this case, the payoff diagram for shorting the index and buying a call is equivalent to that from buying a put and borrowing the present value of \$1000 (\$980.39). Since profit diagrams are unaffected by borrowing, however, the profit diagram in panel (d) is exactly the same as that for a purchased S&R index put. Not only does the insured short position look like a put, it has

the same loss as a purchased put if the price is above \$ 1000: \$75 .68, which is the future value of the \$74.201 putpremium.

Payoff and profit at expiration from short-selling the S&R index and buying a 1000 strike call option at a premium of \$93.809. The payoff is the sum of the first two columns. Cost plus interest for the position is  $(-\$1000 + \$93.809) \times 1.02 = -\$924.32$ . Profit is payoff plus \$924.32.

<b>Payoff at Expiration</b>				
Short S&R Index	S&R Call	Payoff	-(Cost + Interest)	Profit
-\$900	\$0	-\$900	\$924.32	\$24.32
-950	0	-950	924.32	-25.68
-1000	0	-1000	924.32	-75.68
-1050	50	-1000	924.32	-75.68
-1100	100	-1000	924.32	-75.68
-1150	150	-1000	924.32	-75.68
-1200	200	-1000	924.32	-75.68



### 145. List the strategies in which investors can be selling insurance

We can expect that some investors want to purchase insurance. However, for every insurance buyer there must be an insurance seller. In this section we examine strategies in which investors sell insurance.

It is possible, of course, for an investor to simply sell calls and puts. Often, however, investors also have a position in the asset when they sell insurance. Writing an option when there is a corresponding long position in the underlying asset is called covered writing, option overwriting, or selling a covered call. All three terms mean essentially the same thing. In contrast, naked writing occurs when the writer of an option does not have a position in the asset.

**Covered call writing**- if we own the S&R index and simultaneously sell a call option, we have written a covered call. A covered call will have limited profitability if the index increases, because an option writer is obligated to sell the index for the strike price. Should the index decrease, the loss on the index is offset by the premium earned from selling the call. A payoff with limited profit for price increases and potentially large losses for price decreases sounds like a written put.

Because the covered call looks like a written put, the maximum profit will be the same as with a written put. Suppose the index is \$1100 at expiration. The profit is:

$$\underbrace{\$1100 - (\$1000 \times 1.02)}_{\text{Profit on S\&R Index}} + \underbrace{(\$93.809 \times 1.02) - \$100}_{\text{Profit on Written Call}} = \$75.68$$

which is the future value of the premium received from writing a 1 000-strike put.

**Covered puts**- a covered put is achieved by writing a put against a short position on the index. The written put obligates you to buy the index-for a loss-if it goes down in price. Thus, for index prices below the strike price, the loss on the written put offsets the short stock. For index prices above the strike price, you lose on the short stock.

A position where you have a constant payoff below the strike and increasing losses above the strike sounds like a written call. In fact, shorting the index and writing a put produces a profit diagram that is exactly the same as for a written call.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 2, pp. 64-67*

### 146. Is it possible to create synthetic forward position with options?

#### Explain

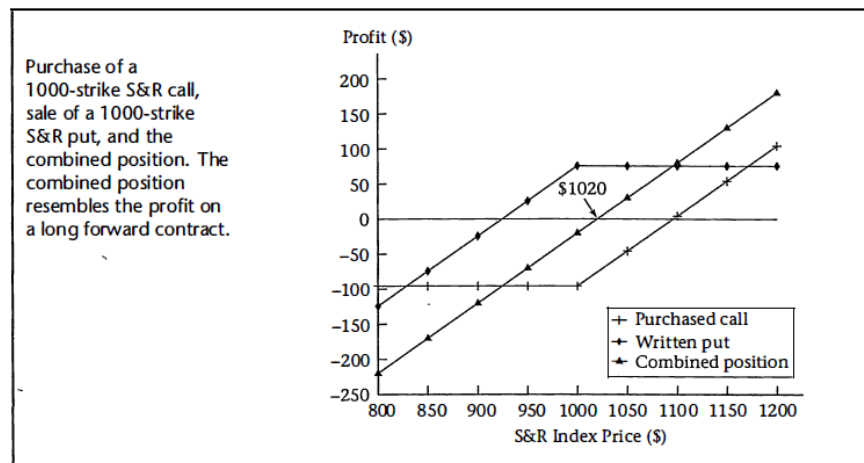
It is possible to mimic a long forward position on an asset by buying a call and selling a put, with each option having the same strike price and time to expiration. For example, we could buy the 1 000-strike S&R call and sell the 1 000-strike S&R put, each with 6 months to expiration. In 6 months we will be obliged to pay \$ 1 000 to buy the index, just as if we had entered into a forward contract.



For example, suppose the index in 6 months is at 900. We will not exercise the call, but we have written a put. The put buyer will exercise the right to sell the index for \$1000; therefore we are obligated to buy the index at \$1000. If instead the index is at \$1100, the put is not exercised, but we exercise the call, buying the index for \$1000. Thus, whether the index rises or falls, when the options expire we buy the index for the strike price of the options, \$1000.

The purchased call, written put, and combined positions are shown in Figure below. The purchase of a call and sale of a put creates a synthetic long forward contract, which has two minor differences from the actual forward:

1. The forward contract has a zero premium, while the synthetic forward requires that we pay the net option premium.
2. With the forward contract we pay the forward price, while with the synthetic forward we pay the strike price.



*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 2, pp. 64-67*

## 147. Define put-call parity relationship in options

The net cost of buying the index using options must equal the net cost of buying the index using a forward contract. If at time 0 we enter into a long forward position expiring at time  $T$ , we obligate ourselves to buying the index at the forward price,  $F_0^T$ . The present value of buying the index in the future is just the present value of the forward price,  $PV(F_0^T)$ .

If instead we buy a call and sell a put today to guarantee the purchase price for the index in the future, the present value of the cost is the net option premium for buying the call and selling the put,  $Call(K, T) - Put(K, T)$ , plus the present value of the strike price,  $PV(K)$ . (The notations " $Call(K, T)$ " and " $Put(K, T)$ " denote the premiums of options with strike price  $K$  and with  $T$  periods until expiration)

Equating the costs of the alternative ways to buy the index at time  $t$  gives us:

$$PV(F_0^T) = [Call(K, T) - Put(K, T)] + PV(K)$$

We can rewrite this as:

$$\text{Call}(K, T) - \text{Put}(K, T) = \text{PV}(F_{0,T} - K)$$

In words, the present value of the bargain element from buying the index at the strike price [the right-hand side of the equation] must be offset by the initial net option premium [the left-hand side of the equation]. Equation is known as put-call parity, and one of the most important relations in options.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 3, pp. 68-70*

## 148. What are Bull and Bear Spreads?

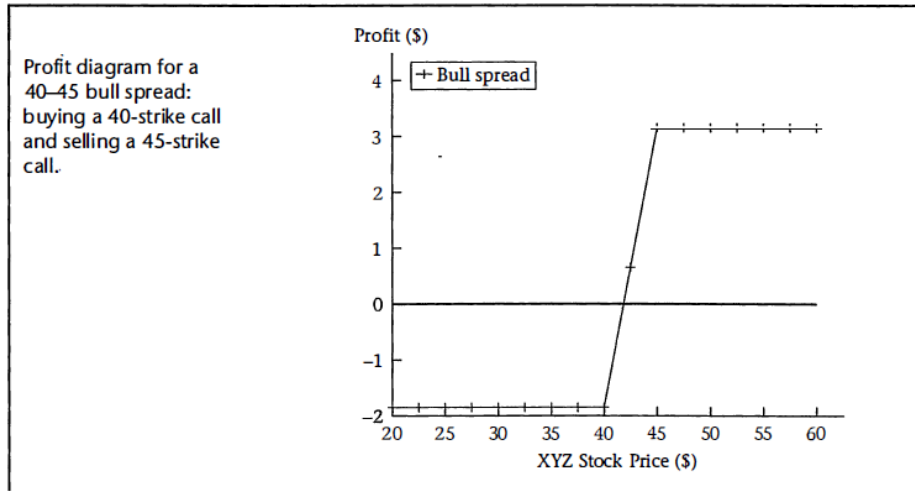
An option spread is a position consisting of only calls or only puts, in which some options are purchased and some written. Spreads are a common strategy.

Suppose you believe a stock will appreciate. You can enter into either a long forward contract or a call option to benefit from price increase. Long forward costs zero but expose you to the risk of loss, while call option cannot cause a negative payoff though it has a premium. On the other hand, spread strategies essentially serve as tools for lowering the cost of your strategy if you are willing to reduce your profit should the stock appreciate. You can do this by selling a call at a higher strike price. The owner of this second call buys appreciation above the higher strike price and pays you a premium. You achieve a lower cost by giving up some portion of profit. A position in which you buy a call and sell an otherwise identical call with a higher strike price is an example of a bull spread.

Bull spreads can also be constructed using puts. Perhaps surprisingly, you can achieve the same result either by buying a low-strike call and selling a high-strike call, or by buying a low-strike put and selling a high-strike put.

Spreads constructed with either calls or puts are sometimes called vertical spreads. The terminology stems from the way option prices are typically presented, with strikes arrayed vertically (as in Table below).

Profit at expiration from purchase of 40-strike call and sale of 45-strike call.				
Stock Price at Expiration	Purchased 40-Call	Written 45-Call	Premium Plus Interest	Total
\$35.0	\$0.0	\$0.0	-\$1.85	-\$1.85
37.5	0.0	0.0	-1.85	-1.85
40.0	0.0	0.0	-1.85	-1.85
42.5	2.5	0.0	-1.85	0.65
45.0	5.0	0.0	-1.85	3.15
47.5	7.5	-2.5	-1.85	3.15
50.0	10.0	-5.0	-1.85	3.15



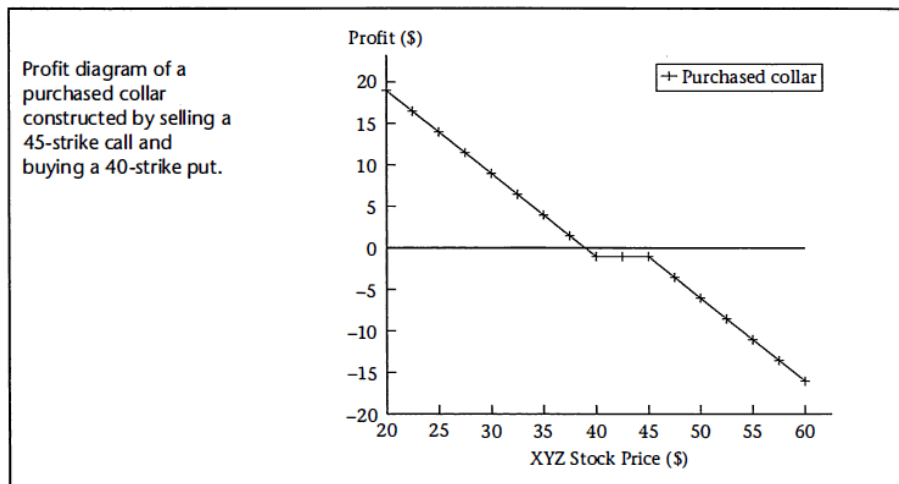
The opposite of a bull spread is a bear spread. Using the options from the above example, we could create a bear spread by selling the 40-strike call and buying the 45-strike call. The profit diagram would be exactly the opposite of the graph above.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 3, pp. 71-72*

#### 149. What is a collar?

A collar is the purchase of a put option and the sale of a call option with a higher strike price, with both options having the same underlying asset and having the same expiration date. If the position is reversed (sale of a put and purchase of a call), the collar is written. The collar width is the difference between the call and put strikes.

**Example:** Suppose we sell a 45-strike call with a \$0.97 premium and buy a 40-strike put with a \$1.99 premium. This collar is shown in Figure below. Because the purchased put has a higher premium than the written call, the position requires investment of \$1.02.



If you hold this book at a distance and squint at Figure 3.8, the collar resembles a short forward contract. Economically, it is like a short forward contract in that it is fundamentally a short position: The position benefits from price decreases in the underlying asset and suffers losses from price increases. A collar differs from a short forward contract in having a range between the strikes in which the expiration payoff is unaffected by changes in the value of the underlying asset.

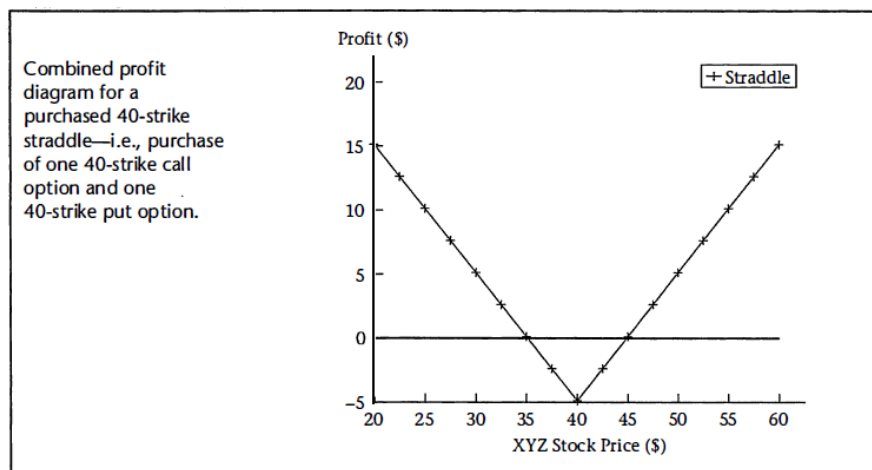
In practice collars are frequently used to implement insurance strategies—for example, by buying a collar when we own the stock. This position, which we will call a collared stock, entails buying the stock, buying a put, and selling a call. It is an insured position because we own the asset and buy a put. The sale of a call helps to pay for the purchase of the put. The collared stock looks like a bull spread; however, it arises from a different set of transactions. The bull spread is created by buying one option and selling another. The collared stock begins with a position in the underlying asset that is coupled with a collar.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 1, pp. 73-78*

### 150. What are the strategies for speculation on volatility?

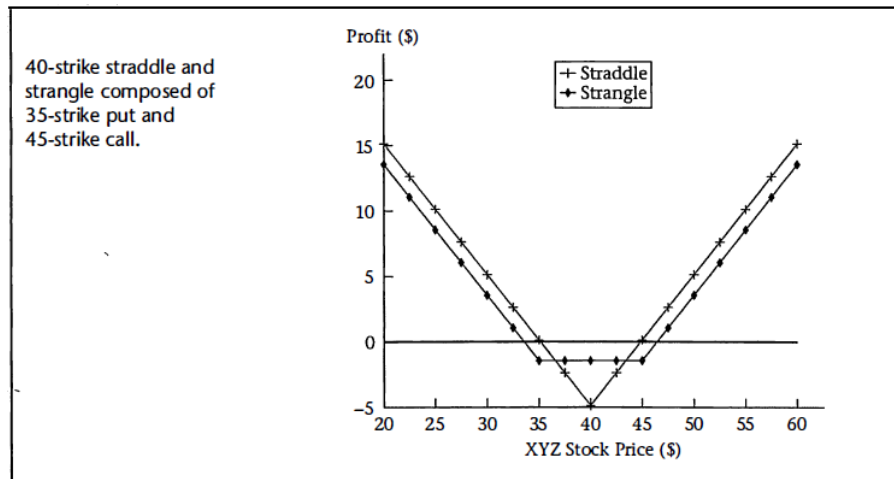
Besides directional positions like a bull spread or a collar which is a bet that the price of the underlying asset will increase, options can also be used to create positions that are non-directional with respect to the underlying asset. With a non-directional position, the holder does not care whether the stock goes up or down, but only how much it moves.

**Straddles** Consider the strategy of buying a call and a put with the same strike price and time to expiration: This strategy is called a straddle. The general idea of a straddle is simple: If the stock price rises, there will be a profit on the purchased call, and if the stock price declines there will be a profit on the purchased put. Thus, the advantage of a straddle is that it can profit from stock price moves in both directions. The disadvantage to a straddle is that it has a high premium because it requires purchasing two options. Figure below demonstrates that a straddle is a bet that volatility will be high: The buyer of an at-the-money straddle is hoping that the stock price will move but does not care about the direction of the move.



**Strangle** The disadvantage of a straddle is the high premium cost. To reduce the premium, you can buy out-of-the-money options rather than at-the-money options. Such a position is called a strangle. For example, consider buying a 35-strike put and a 45-strike call, for a total premium of \$1.41, with a future value of \$1.44. These transactions reduce your maximum loss if the options expire with the stock near \$40, but they also increase the stock-price move required for a profit.

Figure below shows the 40-strike straddle graphed against the 35-45 strangle. This comparison illustrates a key point: In comparing any two fairly priced option positions, there will always be a region where each outperforms the other. Indeed, this is necessary to have a fairly priced position.



*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 3, pp. 78-83*

### 151. Explain basic risk management strategies from the producer's perspective / the buyer's perspective?

Business, like life, is inherently risky. Firms convert inputs such as labor, raw materials, and machines into goods and services. A firm is profitable if the cost of what it produces exceeds the cost of the inputs. Prices can change, however, and what appears to be a profitable activity today may not be profitable tomorrow. Many instruments are available that permit firms to hedge various risks, ranging from commodity prices to weather. A firm that actively uses derivatives and other techniques to alter its risk and protect its profitability is engaging in risk management.

From the producer's perspective, risk management considers protection from price decline. Let's consider a gold mining company. Suppose the gold price is \$350/oz. If gold miners produce no gold, the firm loses its fixed cost, \$330/oz. If they do produce gold, the firm has a fixed cost of \$330/oz. and a variable cost of \$50/oz., and so they lose  $\$350 - (\$330 + \$50) = -\$30/\text{oz}$ . It is better to lose only \$30, so gold will continue to be mined even when net

income is negative. If the gold price were to fall below the variable cost of \$50, then it would make sense to stop producing.

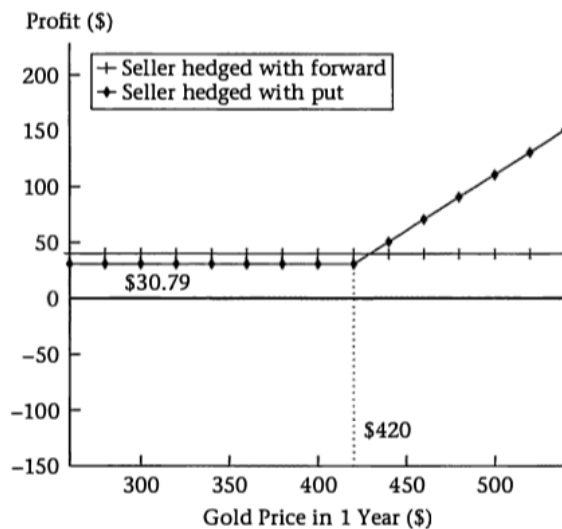
Gold mining company can lock in a price for gold in 1 year by entering into a short forward contract, agreeing today to sell its gold for delivery in 1 year. Profit calculations when gold miners are hedged by forward contract are summarized in below table.

Gold diggers's net income one year from today, hedged with a forward sale of gold.				
Gold Price in One Year	Fixed Cost	Variable Cost	Profit on Short Forward	Net Income on Hedged Position
\$350	-\$330	-\$50	\$70	\$40
\$400	-\$330	-\$50	\$20	\$40
\$450	-\$330	-\$50	-\$30	\$40
\$500	-\$330	-\$50	-\$80	\$40

A possible objection to hedging with a forward contract is that if gold prices do rise, gold miners will still receive only \$420/oz. There is no prospect for greater profit. Gold insurance with a put option provides a way to have higher profits at high gold prices while still being protected against low prices. Suppose that the market price for a 420-strike put is \$8.77/oz. This put provides a floor on the price. Since the put premium is paid 1 year prior to the option payoff, we must take into account interest cost when we compute profit in 1 year. The future value of the premium is  $\$8.77 \times 1.05 = \$9.21$ . Table below shows the result of buying this put:

Gold diggers's net income 1 year from today, hedged with a 420-strike put option.				
Gold Price in One Year	Fixed Cost	Variable Cost	Profit on Put Option	Net Income
\$350	-\$330	-\$50	\$60.79	\$30.79
\$400	-\$330	-\$50	\$10.79	\$30.79
\$450	-\$330	-\$50	-\$9.21	\$60.79
\$500	-\$330	-\$50	-\$9.21	\$110.79

Figure below compares the profit from the two protective strategies we have examined: Selling a forward contract and buying a put. As you would expect, neither strategy is clearly preferable; rather, there are trade-offs, with each contract outperforming the other for some range of prices:



From buyer's perspective risk management considers protection against price increase. For example, Auric Enterprises is a manufacturer of widgets, a product that uses gold as an input. We will suppose for simplicity that the price of gold is the only uncertainty Auric faces. In particular, we assume that:

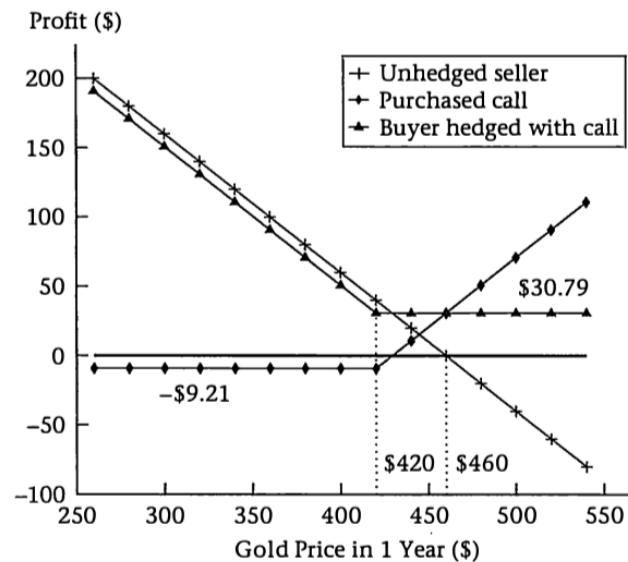
- Auric sells each widget for a fixed price of \$800, a price known in advance.
- The fixed cost per widget is \$340.
- The manufacture of each widget requires 1 oz. of gold as an input.
- The non-gold variable cost per widget is zero.
- The quantity of widgets to be sold is known in advance.

Because Auric makes a greater profit if the price of gold falls, Auric's gold position is implicitly short. The forward price is \$420 as before. Auric can lock in a profit by entering into a long forward contract. Auric thereby guarantees a profit of

$$\text{Profit} = \$800 - \$340 - \$420 = \$40$$

Note that whereas gold miners were selling in the forward market, Auric is buying in the forward market. Thus, gold miners and Auric are natural counterparties in an economic sense.

Rather than lock in a price unconditionally, Auric might like to pay \$420/oz. if the gold price is greater than \$420/oz. but pay the market price if it is less. Auric can accomplish this by buying a call option. As a future buyer, Auric is naturally short; hence, a call is insurance. Suppose the call has a premium of \$8.77/oz. The future value of the premium is  $\$8.77 \times 1.05 = \$9.21$ . Figure below compares the profit from the two protective strategies we have examined for Auric: buying a forward contract and buying a call option:



*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 4, pp. 91-108*

## 152. What is cross-hedging?

In the buyer's perspective example with Auric we assumed that widget prices are fixed. However, since gold is used to produce widgets, widget prices might vary with gold prices. If widget and gold prices vary one-for-one, Auric's profits would be independent of the price of gold and Auric would have no need to hedge.

More realistically, the price of widgets could change with the price of gold, but not one-for-one; other factors could affect widget prices as well. In this case, Auric might find it helpful to use gold derivatives to hedge the price of the widgets it sells as well as the price of the gold it buys. Using gold to hedge widgets would be an example of **cross hedging**: the use of a derivative on one asset to hedge another asset. Cross-hedging arises in many different contexts.

The hedging problem for Auric is to hedge the difference in the price of widgets and gold. Conceptually, we can think of hedging widgets and gold separately, and then combining those separate hedges into one net hedge. The ability to cross-hedge depends upon the correlation between the hedging instrument and the asset being hedged. We can determine the hedging amount as a regression coefficient. The same analysis is used with stock index futures contracts to cross-hedge a stock portfolio.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 4, pp. 114-116*



### 153. Describe alternative ways to buy a stock?

The purchase of a share of XYZ stock has three components: (1) fixing the price, (2) the buyer making payment to the seller, and (3) the seller transferring share ownership to the buyer. If we allow for the possibility that payment and physical receipt can occur at different times, say time  $0$  and time  $T$ , then once the price is fixed there are four logically possible purchasing arrangements:

1. **Outright purchase:** The typical way to think about buying stock. You simultaneously pay the stock price in cash and receive ownership of the stock.
2. **Fully leveraged purchase:** A purchase in which you borrow the entire purchase price of the security. Suppose you borrow the share price,  $S_0$ , and agree to repay the borrowed amount at time  $T$ . If the continuously compounded interest rate is  $r$ , at time  $T$  you would owe  $e^{rT}$  per dollar borrowed, or  $S_0e^{rT}$ .
3. **Prepaid forward contract:** An arrangement in which you pay for the stock today and receive the stock at an agreed-upon future date. The difference between a prepaid forward contract and an outright purchase is that with the former, you receive the stock at time  $T$ .
4. **Forward contract:** An arrangement in which you both pay for the stock and receive it at time  $T$ , with the time  $T$  price specified at time  $0$ .

Table below depicts these four possibilities. It is clear that you pay interest when you defer payment. The interesting question is how deferring the physical receipt of the stock affects the price; this deferral occurs with both the forward and prepaid forward contracts.

Four different ways to buy a share of stock that has price $S_0$ at time 0. At time 0 you agree to a price, which is paid either today or at time $T$ . The shares are received either at 0 or $T$ . The interest rate is $r$ .			
Description	Pay at Time	Receive Security at Time	Payment
Outright purchase	0	0	$S_0$ at time 0
Fully leveraged purchase	$T$	0	$S_0e^{rT}$ at time $T$
Prepaid forward contract	0	$T$	?
Forward contract	$T$	$T$	$? \times e^{rT}$

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 5, pp. 127-128*

### 154. Explain the concept of forward pricing. Write no-arbitrage bounds of forward prices. Give an interpretation of the forward pricing formula

Forwarding pricing is derived from analyzing prepaid forward prices and then adjusting for the only difference of timing of the stock. Thus, the forward price is just the future value of the prepaid forward.

A prepaid forward contract entails paying today to receive something – stocks, a foreign currency, or bonds – in the future. The sale of a prepaid forward contract permits the owner to sell an asset while retaining physical possession for a period of time. The prepaid forward price can be derived using three different methods: pricing by analogy, pricing by present value, and pricing by arbitrage.

Here are forward prices for stocks with discrete and continuous dividends:

**Discrete dividends:**

$$F_{0,T} = S_0 e^{rT} - \sum_{i=1}^n e^{r(T-t_i)} D_i$$

**Continuous dividends:**

$$F_{0,T} = S_0 e^{(r-\delta)T}$$

Tables below illustrate strategies used by market – makers and arbitrageurs to offset the risk of holding counter position with their customers.

Transactions and cash flows for a cash-and-carry: A market-maker is short a forward contract and long a synthetic forward contract.		
Transaction	Cash Flows	
	Time 0	Time T (expiration)
Buy tailed position in stock, paying $S_0 e^{-\delta T}$	$-S_0 e^{-\delta T}$	$+S_T$
Borrow $S_0 e^{-\delta T}$	$+S_0 e^{-\delta T}$	$-S_0 e^{(r-\delta)T}$
Short forward	0	$F_{0,T} - S_T$
<b>Total</b>	0	$F_{0,T} - S_0 e^{(r-\delta)T}$

Transactions and cash flows for a reverse cash-and-carry: A market-maker is long a forward contract and short a synthetic forward contract.		
Transaction	Cash Flows	
	Time 0	Time T (expiration)
Short tailed position in stock, receiving $S_0 e^{-\delta T}$	$+S_0 e^{-\delta T}$	$-S_T$
Lend $S_0 e^{-\delta T}$	$-S_0 e^{-\delta T}$	$+S_0 e^{(r-\delta)T}$
Long forward	0	$S_T - F_{0,T}$
<b>Total</b>	0	$S_0 e^{(r-\delta)T} - F_{0,T}$

A transaction in which you buy the underlying asset and short the offsetting forward contract is called a **cash-and-carry**. A cash-and-carry has no risk: You have an obligation to deliver the asset but also own the asset. The market-maker offsets the short forward position with a cash-and-carry. An arbitrage that involves buying the underlying asset and selling it

forward is called a **cash-and-carry arbitrage**. As you might guess, a **reverse cash-and-carry** entails short-selling the index and entering into a long forward position

Tables above demonstrate that an arbitrageur can make a costless profit if  $FT \neq S_0 e^{(r-\delta)T}$ . This analysis ignores transaction costs. In practice an arbitrageur will face trading fees, bid-ask spreads, different interest rates for borrowing and lending, and the possibility that buying or selling in large quantities will cause prices to change. The effect of such costs will be that, rather than there being a single no-arbitrage price, there will be a no-arbitrage bound: a lower price  $F^-$  and an upper price  $F^+$  such that arbitrage will not be profitable when the forward price is between these bounds.

Suppose that the stock and forward have bid and ask prices of  $S^b < S^a$  and  $F^b < F^a$ , a trader faces a cost  $k$  of transacting in the stock or forward, and the interest rates for borrowing and lending are  $r^b > r^l$ . Incorporating above in arbitrageur's strategies, we get no-arbitrage bounds of forward prices:

$$F_{0,T} > F^+ = (S_0^a + 2k)e^{r^b T}$$

$$F_{0,T} < F^- = (S_0^b - 2k)e^{r^l T}$$

The forward pricing formula for a stock index, equation  $S_0 e^{(r-\delta)T}$  depends on  $r - \delta$ , the difference between the risk-free rate and the dividend yield. This difference is called the cost of carry. Here is an interpretation of the forward pricing formula:

$$\text{Forward price} = \text{Spot price} + \underbrace{\text{Interest to carry the asset} - \text{Asset lease rate}}_{\text{Cost of carry}}$$

The forward contract, unlike the stock, requires no investment and makes no payouts and therefore has a zero cost of carry. One way to interpret the forward pricing formula is that, to the extent the forward contract saves our having to pay the cost of carry, we are willing to pay a higher price.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 5, pp. 128-141*

## 155. List main differences between forwards and futures

Futures contracts are essentially exchange-traded forward contracts. As with forwards, futures contracts represent a commitment to buy or sell an underlying asset at some future date. Because futures are exchange-traded, they are standardized and have specified delivery dates, locations, and procedures. Futures may be traded either electronically or in trading pits, with buyers and sellers shouting orders to one another (this is called **open outcry**). Each exchange has an associated **clearinghouse**. The role of the clearinghouse is to match the buys and sells that take place during the day, and to keep track of the obligations and payments required of the members of the clearinghouse, who are called *clearing members*. After matching trades, the clearinghouse typically becomes the counterparty for each clearing member.

Although forwards and futures are similar in many respects, there are differences.

- Whereas forward contracts are settled at expiration, futures contracts are settled daily. The determination of who owes what to whom is called marking-to-market. Frequent marking-to-market and settlement of a futures contract can lead to pricing differences between the futures and an otherwise identical forward.
- As a result of daily settlement, futures contracts are liquid – it is possible to offset an obligation on a given date by entering into the opposite position. For example, if you are long the September S&P 500 futures contract, you can cancel your obligation to buy by entering into an offsetting obligation to sell the September S&P 500 contract. If you use the same broker to buy and to sell, your obligation is officially cancelled.
- Over-the-counter forward contracts can be customized to suit the buyer or seller, whereas futures contracts are standardized. For example, available futures contracts may permit delivery of 250 units of a particular index in March or June. A forward contract could specify April delivery of 300 units of the index.
- Because of daily settlement, the nature of credit risk is different with the futures contract. In fact, futures contracts are structured so as to minimize the effects of credit risk through **initial margin** and **maintenance margin** requirements from counterparties.
- There are typically daily price limits in futures markets (and on some stock exchanges as well). A **price limit** is a move in the futures price that triggers a temporary halt in trading. For example, there is an initial 5% limit on *down* moves in the S&P 500 futures contract. An offer to sell exceeding this limit can trigger a temporary trading halt, after which time a 10% price limit is in effect. If that is exceeded, there are subsequent 15% and 20% limits. The rules can be complicated, but it is important to be aware that such rules exist.

Figure below shows a quotation for the S&P 500 index futures contract with its specifications:

Specifications for the S&P 500 index futures contract.	Underlying	S&P 500 index
	Where traded	Chicago Mercantile Exchange
	Size	\$250 × S&P 500 index
	Months	Mar, Jun, Sep, Dec
	Trading ends	Business day prior to determination of settlement price
	Settlement	Cash-settled, based upon opening price of S&P 500 on third Friday of expiration month

### 156. What are margins and marking-to-market?

Suppose the futures price is 1100 and you wish to acquire a \$2.2 million position in the S&P 500 index. The notional value of one contract is  $\$250 \times 1100 = \$275,000$ ; this represents the amount you are agreeing to pay at expiration per futures contract. To go long \$2.2 million of the index, you would enter into  $\$2.2 \text{ million} / \$0.275 \text{ million} = 8$  long futures contracts. The notional value of 8 contracts is  $8 \times \$250 \times 1100 = \$2,000 \times 1100 = \$2.2 \text{ million}$ .

A broker executes your buy order. For every buyer there is a seller, which means that one or more investors must be found who simultaneously agree to sell forward the same number of units of the index. The total number of open positions (buy/sell pairs) is called the open interest of the contract.

Both buyers and sellers are required to post a performance bond with the broker to ensure that they can cover a specified loss on the position. This deposit, which can earn interest, is called **margin** and is intended to protect the counterparty against your failure to meet your obligations. The margin is a performance bond, not a premium. Hence, futures contracts are costless (not counting, of course, commissions and the bid-ask spread).

To understand the role of margin, suppose that there is 10% margin and weekly settlement (in practice, settlement is daily). The margin on futures contracts with a notional value of \$2.2 million is \$220,000. If the S&P 500 futures price drops by 1, to 1099, we lose \$2000 on our futures position. The reason is that 8 long contracts obligate us to pay  $\$2000 \times 1100$  to buy 2000 units of the index which we could now sell for only  $\$2000 \times 1099$ . Thus, we lose  $(1099-1100) \times \$2000 = -\$2000$ .

We have a choice of either paying this loss directly; or allowing it to be taken out of the margin balance. It doesn't matter which we do since we can recover the unused margin balance plus interest at any time by selling our position.

The decline in the margin balance means the broker has significantly less protection should we default. For this reason, participants are required to maintain the margin at a minimum level, called the maintenance margin. This is often set at 70% to 80% of the initial margin level. Shall the margin balance declines below maintenance, we would have to post additional margin. The broker would make a margin call, requesting additional margin. If we failed to post additional margin, the broker would close the position by selling 2000 units of the index, and return to us the remaining margin. In practice, marking-to-market and settling up are performed at least daily.

Since margin you post is the broker's protection against your default, a major determinant of margin levels is the volatility of the underlying asset. The minimum margin on the S&P 500 contract has generally been less than the 10% that we assume above. In August 2004, for example, the minimum margin on the S&P 500 futures contract was about 6% of the notional value of the contract.

### 157. Explain asset allocation use of index futures

An index futures contract is economically like borrowing to buy the index. Why use an index futures contract if you can synthesize one? One answer is that index futures can permit trading the index at a lower transaction cost than actually trading a basket of the stocks that make up the index. If you are taking a temporary position in the index, either for investing or hedging, the transaction cost saving could be significant.

Asset allocation strategies involve switching investments among asset classes, such as stocks, money market instruments, and bonds. Trading the individual securities, such as the stocks in an index can be expensive. The practical implication of synthetic positions is that a portfolio manager can invest in a stock index without holding stocks, commodities without holding physical commodities, and so on.

As an example of asset allocation, suppose that we have an investment in the S&P 500 index and we wish to temporarily invest in T – bills instead of the index. Instead of selling all 500 stocks and investing in T – bills, we can simply keep our stock portfolio and take a short forward position in the S&P 500 index. This converts our cash investment in the index into a cash-and-carry, creating a synthetic T – bill. When we wish to revert to investing in stocks, we simply offset the forward position.

To illustrate this, suppose that the current index price,  $S_0$ , is \$100, and the effective 1-year risk-free rate is 10%. The forward price is therefore \$110. Suppose that in 1 year, the index price could be either \$80 or \$130. If we sell the index and invest in T – bills, we will have \$110 in 1 year. Table below shows that if, instead of selling, we keep the stock and short the forward contract, we earn a 10% return no matter what happens to the value of the stock. In this example 10% is the rate of return implied by the forward premium. If there is no arbitrage, this return will be equal to the risk-free rate.

Effect of owning the stock and selling forward, assuming that $S_0 = \$100$ and $F_{0,1} = \$110$ .			
Transaction	Cash Flows		
	Today	1 year, $S_1 = \$80$	1 year, $S_1 = \$130$
Own stock @ \$100	-\$100	\$80	\$130
Short forward @ \$110	0	\$110 – \$80	\$110 – \$130
<b>Total</b>	-\$100	\$110	\$110

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 5, pp. 150-151*

### 158. Describe pricing of currency forwards

Currency futures and forwards are widely used to hedge against changes in exchange rates. The pricing of currency contracts is a straightforward application of the principles used

in pricing prepaid forwards on stocks. Many corporations use currency futures and forwards for short-term hedging. An importer of consumer electronics, for example, may have an obligation to pay the manufacturer ¥150 million 90 days in the future. The dollar revenues from selling these products are likely known in the short run, so the importer bears pure exchange risk due to the payable being fixed in yen. By buying ¥150 million forward 90 days, the importer locks in a dollar price to pay for the yen, which will then be delivered to the manufacturer.

Suppose that 1 year from today you want to have ¥1. A prepaid forward allows you to pay dollars today to acquire ¥1 in 1 year. What is the prepaid forward price? Suppose the yen-denominated interest rate is  $r_y$  and the exchange rate today ( $\$/¥$ ) is  $x_0$ . We can work backward. If we want ¥1 in 1 year, we must have  $e^{-r_y}$  in yen today. To obtain that many yen today, we must exchange  $x_0 e^{-r_y}$  dollars into yen.

Thus, the prepaid forward price for a yen is:

$$F_{0,T}^P = x_0 e^{-r_y T}$$

*The economic principle governing the pricing of a prepaid forward on currency is the same as that for a prepaid forward on stock.* By deferring delivery of the underlying asset, you lose income. In the case of currency, if you received the currency immediately, you could buy a bond denominated in that currency and earn interest. The prepaid forward price reflects the loss of interest from deferring delivery, just as the prepaid forward price for stock reflects the loss of dividend income.

The prepaid forward price is the *dollar* cost of obtaining 1 yen in the future. Thus, to obtain the forward price, compute the future value using the dollar-denominated interest rate,  $r$ :

$$F_{0,T} = x_0 e^{(r-r_y)T}$$

The forward currency rate will exceed the current exchange rate when the domestic risk-free rate is higher than the foreign risk-free rate.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 5, pp. 154-156*

### 159. What is the lease rate for a commodity?

Here is how a lender will think about a commodity loan: "If I lend the commodity, I am giving up possession of a unit worth  $S_0$ . At time  $T$ , I will receive a unit worth  $S_T$ . I am effectively making an investment of  $S_0$  in order to receive the random amount  $S_T$ ."

How would you analyze this investment? Suppose that  $\alpha$  is the expected return on a stock that has the same risk as the commodity;  $\alpha$  is therefore the appropriate discount rate for the cash flow  $S_T$ . The NPV of the investment is:

$$\text{NPV} = E_0(S_T) e^{-\alpha T} - S_0$$

Suppose that we expect the commodity price to increase at the rate  $g$ , so that:

$$E_0(S_T) = S_0 e^{gT}$$

Then the NPV of the commodity loan, without payments, is:

$$\text{NPV} = S_0 e^{(g-\alpha)T} - S_0$$

If  $g < \alpha$ , the commodity loan has a negative NPV. However, suppose the lender demands that the borrower return  $e^{(\alpha-g)T}$  units of the commodity for each unit borrowed. If one unit is loaned,  $e^{(\alpha-g)T}$  units will be returned. This is like a continuous proportional lease payment of  $\alpha - g$  to the lender. Thus, the lease rate is the difference between the commodity discount rate and the expected growth rate of the commodity price, or:

$$\delta = \alpha - g$$

With this payment, the NPV of a commodity loan is:

$$\text{NPV} = S_0 e^{(\alpha-g)T} e^{(g-\alpha)T} - S_0 = 0$$

Now the commodity loan is a fair deal for the lender. The commodity lender must be compensated by the borrower for the opportunity cost associated with lending.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 6, pp. 178-179*

## 160. How are implied forward rates calculated?

The implied forward rate is an implicit rate that can be earned from year 1 to year 2 that must be consistent with the other two rates. Suppose we could today guarantee a rate we could earn from year 1 to year 2. We know that \$1 invested for 1 year earns  $[1 + r_0(0,1)]$  and \$1 invested for 2 years earns  $[1 + r_0(0,2)]^2$ . Thus, the time 0 forward rate from year 1 to year 2,  $r_0(1,2)$ , should satisfy:

$$[1 + r_0(0,1)][1 + r_0(1,2)] = [1 + r_0(0,2)]^2$$

or

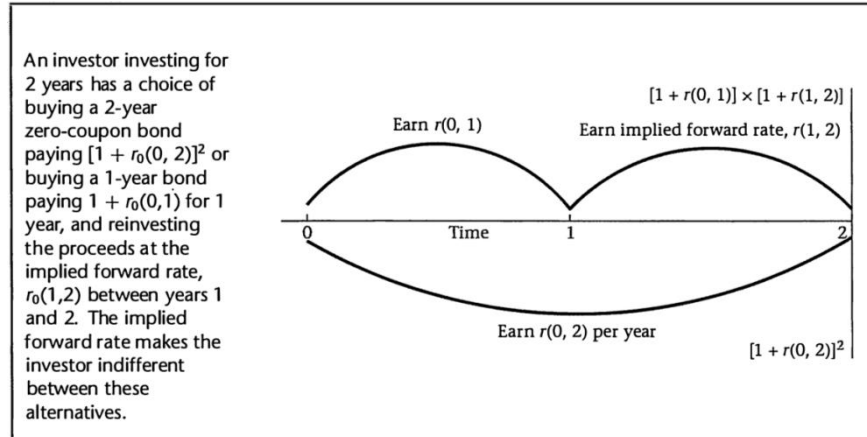
$$1 + r_0(1,2) = \frac{[1 + r_0(0,2)]^2}{1 + r_0(0,1)}$$

In general, we have:

$$[1 + r_0(t_1, t_2)]^{t_2 - t_1} = \frac{[1 + r_0(0, t_2)]^{t_2}}{[1 + r_0(0, t_1)]^{t_1}} = \frac{P(0, t_1)}{P(0, t_2)}$$

Figure below shows graphically how the implied forward rate is related to 1- and 2-year yields:





*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 7, pp. 208-210*

### 161. How coupon rate of par bond is calculated?

We can also compute the par coupon rate at which a bond will be priced at par. To describe a coupon bond, we need to know the date at which the bond is being priced, the start and end date of the bond payments, the number and amount of the payments, and the amount of principal.

We will let  $B_t(t_1, t_2, c, n)$  denote the time  $t$  price of a bond that is issued at  $t_1$ , matures at  $t_2$ , pays a coupon of  $c$  per dollar of maturity payment, and makes  $n$  evenly spaced payments over the life of the bond, beginning at time  $t_1 + (t_2 - t_1) / n$ . We will assume the maturity payment is \$1. If the maturity payment is different than \$1, we can just multiply all payments by that amount.

Since the price of a bond is the present value of its payments, at issuance time  $t$  the price of a bond maturing at  $T$  must satisfy:

$$B_t(t, T, c, n) = \sum_{i=1}^n c P_t(t, t_i) + P_t(t, T)$$

Where  $t_i = t + I(T - t)/n$ , with  $i$  being the index in the summation. Now, we can solve for the coupon as:

$$c = \frac{B_t(t, T, c, n) - P_t(t, T)}{\sum_{i=1}^n P_t(t, t_i)}$$

A par bond has  $B_t = 1$ , so the coupon on a par bond is given by:

$$c = \frac{1 - P_t(t, T)}{\sum_{i=1}^n P_t(t, t_i)}$$

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 7, pp. 210-211*

### 162. Define and give an example of forward rate agreement (FRA)/ synthetic FRAs are created?

Let's consider the problem of a borrower who wishes to hedge against increases in the cost of borrowing. We consider a firm expecting to borrow \$100 million for 91 days, beginning 120 days from today, in June. This is the borrowing date. The loan will be repaid in September on the loan repayment date. In the examples we will suppose that the effective quarterly interest rate at that time can be either 1.5% or 2%, and that the implied June 91-day forward rate (the rate from June to September) is 1.8%. Here is the risk faced by the borrower, assuming no hedging:

	120 days	211 days	
		$r_{\text{quarterly}} = 1.5\%$	$r_{\text{quarterly}} = 2\%$
Borrow \$100m	+100m	-101.5m	-102.0m

Depending upon the interest rate, there is a variation of \$0.5m in the borrowing cost. A forward rate agreement (FRA) is an over-the-counter contract that guarantees a borrowing or lending rate on a given notional principal amount. FRAs can be settled either at the initiation or maturity of the borrowing or lending transaction. If settled at maturity, we will say the FRA is settled in arrears. In the example above, the FRA could be settled on day 120, the point at which the borrowing rate becomes known and the borrowing takes place, or settled in arrears on day 211, when the loan is repaid.

FRAs are a forward contract based on the interest rate and as such does not entail the actual lending of money. Rather, the borrower who enters an FRA is paid if a reference rate is above the FRA rate, and the borrower pays if the reference rate is below the FRA rate. The actual borrowing is conducted by the borrower independently of the FRA. We will suppose that the reference rate used in the FRA is the same as the actual borrowing cost of the borrower.

**FRA settlement in arrears:** First consider what happens if the FRA is settled in September, on day 211, the loan repayment day. In that case, the payment to the borrower should be:

$$(r_{\text{quarterly}} - r_{\text{FRA}}) \times \text{notional principal}$$

Thus, if the borrowing rate is 1.5%, the payment under the FRA should be:

$$(0.015 - 0.018) \times \$100m = -\$300,000$$

Since the rate is lower than the FRA rate, the borrower pays the FRA counterparty.

Similarly, if the borrowing rate turns out to be 2.0%, the payment under the FRA should be:

$$(0.02 - 0.018) \times \$100m = \$200,000$$

Settling the FRA in arrears is simple and seems like the obvious way for the contract to work. However, settlement can also occur at the time of borrowing.

**FRA settlement in advance:** If the FRA is settled in June, at the time the money is borrowed, payments will be less than when settled in arrears because the borrower has time to earn interest on the FRA settlement. In practice, therefore, the FRA settlement is tailed by the reference rate prevailing on the settlement (borrowing) date. (Tailing in this context means that we reduce the payment to reflect the interest earned between June and September.) Thus, the payment for a borrower is:

$$\text{Notional principal} \times \frac{(r_{\text{quarterly}} - r_{\text{FRA}})}{1 + r_{\text{quarterly}}}$$

If  $r_{\text{quarterly}} = 1.5\%$ , the payment in June is:

$$\frac{-\$300,000}{1 + 0.015} = -\$295,566.50$$

By definition, the future value of this is  $-\$300,000$ . In order to make this payment, the borrower can borrow an extra  $\$295,566.50$ , which results in an extra  $\$300,000$  loan payment in September. If on the other hand  $r_{\text{quarterly}} = 2.0\%$ , the payment is:

$$\frac{\$200,000}{1 + 0.02} = \$196,078.43$$

The borrower can invest this amount, which gives  $\$200,000$  in September, an amount that offsets the extra borrowing cost.

If the forward rate agreement covers a borrowing period other than 91 days, we simply use the appropriate rate instead of the 91-day rate in the above calculations.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 7, pp. 214-215*

### 163. List specifications of Eurodollar futures

Eurodollar futures contracts are similar to FRAs in that they can be used to guarantee a borrowing rate. There are subtle differences between FRAs and Eurodollar contracts; however these differences are important to understand. Figure below describes the Eurodollar contract based on a  $\$1$  million 3month deposit earning LIBOR (the London Interbank Offer Rate), which is the average borrowing rate faced by large international London banks. Suppose that current LIBOR is 1.5% over 3 months. By convention, this is annualized by multiplying by 4, so the quoted LIBOR rate is 6%. Assuming a bank borrows  $\$1$  million for 3 months, a change in annualized LIBOR of 0.01% (one basis point) would raise its borrowing cost by  $0.0001/4 \times \$1 \text{ million} = \$25$ .

The Eurodollar futures price at expiration of the contract is:

$$100 - \text{Annualized 3-month LIBOR}$$

Thus, if LIBOR is 6% at maturity of the Eurodollar futures contract, the final futures price will be  $100 - 6 = 94$ . It is important to understand that the Eurodollar contract settles

based on current LIBOR, the interest rate quoted for the *next* 3 months. Thus, for example, the price of the contract that expires in June reflects the 3-month interest rate between June and September.

Specifications for the Eurodollar futures contract.	Where traded	Chicago Mercantile Exchange
	Size	3-month Eurodollar time deposit, \$1 million principal
	Months	Mar, Jun, Sep, Dec, out 10 years, plus 2 serial months and spot month
	Trading ends	5 A.M. (11 A.M. London) on the second London bank business day immediately preceding the third Wednesday of the contract month.
	Delivery Settlement	Cash settlement 100 – British Banker's Association Futures Interest Settlement Rate for 3-Month Eurodollar Interbank Time Deposits. (This is a 3-month rate annualized by multiplying by 360/90.)

Like most money-market interest rates, LIBOR is quoted assuming a 360-day year. Thus, the annualized 91-day rate,  $r_{91}$ , can be extracted from the futures price,  $F$ , by computing the 90-day rate and multiplying by 91/90. The quarterly effective rate is then computed by dividing the result by 4:

$$r_{91} = (100 - F) \times \frac{1}{100} \times \frac{1}{4} \times \frac{91}{90}$$

To use Eurodollars for hedging against borrowing rate, one shall enter into short Eurodollar contracts that will result in following payoff at expiration:

$$\text{short eurodollar futures payoff} = [F - (100 - r_{LIBOR})] \times 100 \times \$25$$

Where

$F$  – Eurodollar futures price

$r_{LIBOR}$  – current LIBOR rate

Similarly, for hedging the lending rate, one might enter into long Eurodollar futures contracts and have the payoff at expiration of:

$$\text{long eurodollar futures payoff} = [(100 - r_{LIBOR}) - F] \times 100 \times \$25$$

It is highly important to note, that the Eurodollar futures price is a construct, not the price of an asset. In this sense Eurodollar futures are different from any other futures contracts.

LIBOR is quoted in currencies other than dollars, and comparable rates are quoted in different locations. In addition to LIBOR, there are PIBOR (Paris), TIBOR (Tokyo), and Euribor (the European Banking Federation).

Finally, you might be wondering why we are discussing LIBOR rather than rates on Treasury bills. Business and bank borrowing rates move more in tandem with LIBOR than with the government's borrowing rate. Thus, these borrowers use the Eurodollar futures contract to hedge. LIBOR is also a better measure of the cost of funds for a market-maker, so LIBOR is typically used to price forward contracts.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 5, pp. 158-160*

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 7, pp. 218-223*

### 164. What is the market value of a swap? Give the general formula for swap price. Define swap's implicit loan balance

A **swap** is a contract calling for an exchange of payments over time. One party makes a payment to the other depending upon whether a price turns out to be greater or less than a reference price that is specified in the swap contract. A swap thus provides a means to hedge a stream of risky payments. By entering into an oil swap, for example, an oil buyer confronting a stream of uncertain oil payments can lock in a fixed price for oil over a period of time. The swap payments would be based on the difference between a fixed price for oil and a market price that varies over time. From this description, you can see that there is a relationship between swaps and forward contracts. In fact, a forward contract is a single-payment swap.

To illustrate the general calculations for determining the swap rate, suppose there are  $n$  swap settlements, occurring on dates  $t_i$ ,  $i = 1, \dots, n$ . The implied forward interest rate from date  $t_{i-1}$  to date  $t_i$ , known at date 0, is  $r_0(t_{i-1}, t_i)$ . [We will treat  $r_0(t_{i-1}, t_i)$  as not having been annualized.; i.e., it is the return earned from  $t_{i-1}$  to  $t_i$ ]. The price of a zero-coupon bond maturing on date  $t_i$  is  $P(0, t_i)$ .

The market-maker can hedge the floating – rate payments using forward rate agreements. The requirement that the hedged swap have zero net present value is:

$$\sum_{i=1}^n P(0, t_i)[R - r_0(t_{i-1}, t_i)] = 0$$

Where there are  $n$  payments on dates  $t_1, t_2, \dots, t_n$ . The cash flows  $R - r_0(t_{i-1}, t_i)$  can also be obtained by buying a fixed-rate bond paying  $R$  and borrowing at the floating rate. After solving for  $R$  and rearranging, we get:

$$R = \frac{1 - P_0(0, t_n)}{\sum_{i=1}^n P_0(0, t_i)}$$

The conclusion is that the swap rate is the coupon rate on a par coupon bond. This result is intuitive since a firm that swaps from floating-rate to fixed-rate exposure ends up with the economic equivalent of a fixed-rate bond.

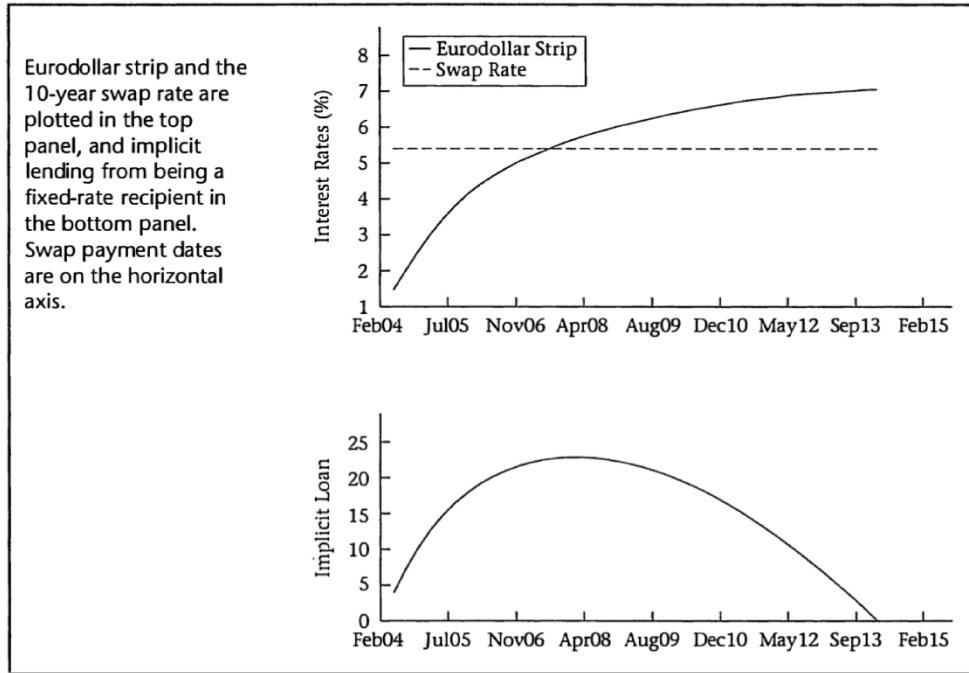
When the buyer first enters the swap, its market value is zero, meaning that either party could enter or exit the swap without having to pay anything to the other party (apart from commissions and bid-ask spreads). The forward contracts and forward rate agreement have zero value, so the swap does as well. Once the swap is struck, however, its market value will generally no longer be zero, for two reasons. First, the forward prices for oil and interest rates will change over time. New swaps would no longer have a fixed price; hence, one party will owe money to the other should one party wish to exit or unwind the swap.

Second, even if oil and interest rate forward prices do not change, the value of the swap will remain zero only *until the first swap payment is made*. Once the first swap payment is made, the buyer has overpaid by some amount relative to the forward curve, and hence, in order to exit the swap, the counterparty would have to pay the oil buyer this amount. Thus, even if prices do not change, the market value of swaps can change over time due to the implicit borrowing and lending caused by fixed swap price structure compared to forward strip.

An interest rate swap behaves much like the oil swap. At inception, the swap has zero value to both parties. If interest rates change, the present value of the fixed payments and, hence, the swap rate will change. The market value of the swap is the difference in the present value of payments between the old swap rate and the new swap rate. For example, consider the 3-year swap in table below. If interest rates rise after the swap is entered into, the value of the existing 6.9548% swap will fall for the party receiving the fixed payment.

Cash flows faced by a floating-rate borrower who enters into a 3-year swap with a fixed rate of 6.9548%.			
Year	Floating-Rate Debt Payment	Net Swap Payment	Net
1	-6%	6% - 6.9548%	-6.9548%
2	$-\tilde{r}_2$	$\tilde{r}_2 - 6.9548\%$	-6.9548%
3	$-\tilde{r}_3$	$\tilde{r}_3 - 6.9548\%$	-6.9548%

Even in the absence of interest rate changes, however, the swap in the table changes value over time. Once the first swap payment is made, the swap acquires negative value for the market-maker (relative to the use of forwards) because in the second year the market-maker will make net cash payment. Similarly, the swap will have positive value for the borrower (again relative to the use of forwards) after the first payment is made. In order to smooth payments, the borrower pays "too much" (relative to the forward curve) in the first year and receives a refund in the second year. *The swap is equivalent to entering into forward contracts and undertaking some additional borrowing and lending.* The implicit loan balance in the swap is illustrated in the figure below:



*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 8, pp. 247-261*

### 165. Explain early exercise for American options

When might we want to exercise an option prior to expiration? An important result is that an American call option on a non-dividend-paying stock should never be exercised prior to expiration. You may, however, rationally exercise an American-style put option prior to expiration.

**Early exercise for calls:** We can demonstrate that an American-style call option on a non-dividend-paying stock should never be exercised prior to expiration. Early exercise is not optimal if the price of an American call prior to expiration satisfies:

$$C_{\text{Amer}}(S_t, K, T - t) > S_t - K$$

If this inequality holds, you would lose money by early-exercising (receiving  $S_t - K$ ) as opposed to selling the option and receiving more. We will use put – call parity to demonstrate that early exercise is not rational. If the option expires at  $T$ , parity implies that:

$$C_{\text{Eur}}(S_t, K, T) = \underbrace{S_t - K}_{\text{Exercise value}} + \underbrace{P_{\text{Eur}}(S_t, K, T - t)}_{\text{Insurance against } S_T < K} + \underbrace{K(1 - e^{-r(T-t)})}_{\text{Time value of money on } K} > S_t - K$$

Since the put price and the time value of money on the strike are both positive, this equation establishes that the European call option premium on a non-dividend-paying stock always is at least as great as  $S_t - K$ . We also know that American option always costs at least as much as European option, hence, we have:

$$C_{\text{Amer}} \geq C_{\text{Eur}} > S_t - K$$

So, we would lose money exercising an American call prior to expiration, as opposed to selling the option.

Early-exercising has three effects. First, we throw away the implicit put protection should the stock later move below the strike price. Second, we accelerate the payment of the strike price. A third effect is the possible loss from deferring receipt of the stock. However, when there are no dividends, we lose nothing by waiting to take physical possession of the stock.

**Early exercise for puts:** When the underlying stock pays no dividend, a call will not be early-exercised, but a put might be. To see that early exercise for a put can make economic sense, suppose a company is bankrupt and the stock price falls to zero. Then a put that would not be exercised until expiration will be worth  $PV_{t,T}(K)$ . If we could early-exercise, we would receive  $K$ , if the interest rate is positive, then  $K > PV(K)$ . Therefore, early exercise would be optimal in order to receive the strike price earlier.

We can also use a parity argument to understand this. The put will never be exercised as long as  $P > K - S$ . Parity for the put implies:

$$P(S_t, K, T - t) = C(S_t, K, T - t) - S_t + PV_{t,T}(K)$$

$$P > K - S$$

$$C(S_t, K, T - t) - S_t + PV_{t,T}(K) > K - S_t$$

Or

$$C(S_t, K, T - t) > K - PV_{t,T}(K)$$

If the call is sufficiently valueless (as in the above example of a bankrupt company), parity cannot rule out early exercise. This does not mean that we *will* early-exercise; it simply means that we cannot rule it out.

We can summarize this discussion of early exercise. When we exercise an option, we receive something (the stock with a call, the strike price with a put). A necessary condition for early exercise is that we prefer to receive this something sooner rather than later. For calls, dividends on the stock are a reason to want to receive the stock earlier. For puts, interest on the strike is a reason to want to receive the strike price earlier. Thus, dividends and interest play similar roles in the two analyses of early exercise.



### 166. Formulate replicating portfolio principle for one-period binomial option pricing model

The binomial option pricing model assumes that, over a period of time, the price of the underlying asset can move only up or down by a specified amount, that is, the asset price follows a binomial distribution. Given this assumption, it is possible to determine a no-arbitrage price for the option. Surprisingly, this approach, which appears at first glance to be overly simplistic, can be used to price options, and it conveys much of the intuition underlying more complex (and seemingly more realistic) option pricing models. It is hard to overstate the value of thoroughly understanding the binomial approach to pricing options.

Binomial pricing achieves its simplicity by making a very strong assumption about the stock price: At any point in time, the stock price can change to either an up value or a down value. In-between, greater, or lesser values are not permitted. The restriction to two possible prices is why the method is called "binomial." The appeal of binomial pricing is that it displays the logic of option pricing in a simple setting, using only algebra to price options.

We have two instruments to use in replicating a call option: shares of stock and a position in bonds (i.e., borrowing or lending). To find the replicating portfolio, we need to find a combination of stock and bonds such that the portfolio mimics the option.

To be specific, we wish to find a portfolio consisting of  $\Delta$  shares of stock and a dollar amount  $B$  in lending, such that the portfolio imitates the option whether the stock rises or falls. We will suppose that the stock has a continuous dividend yield of  $\delta$ , which we reinvest in the stock. Thus, if you buy one share at time  $t$ , at time  $t + h$  you will have  $e^{\delta h}$  shares. The up and down movements of the stock price reflect the *ex-dividend* price.

We can write the stock price as  $u S_0$  when the stock goes up and as  $d S_0$  when the price goes down. We can represent the stock price tree as shown below. In this tree  $u$  is interpreted as one plus the rate of capital gain on the stock if it goes up, and  $d$  is one plus the rate of capital loss if it goes down. (If there are dividends, the total return is the capital gain or loss, plus the dividend.)

Let  $C_u$  and  $C_d$  represent the value of the option when the stock goes up or down, respectively. The tree for the stock implies a corresponding tree for the value of the option shown below as well:



If the length of a period is  $h$ , the interest factor per period is  $e^{r^h}$ . The problem is to solve for  $\Delta$  and  $B$  such that our portfolio of  $\Delta$  shares and  $B$  in lending duplicates the option payoff. The value of the replicating portfolio at time  $h$ , with stock price  $S_h$ , is

$$\Delta S_h + e^{r^h} B$$

At the prices  $S_h = dS$  and  $S_h = uS$ , a successful replicating portfolio will satisfy:

$$(\Delta \times dS \times e^{\delta h}) + (B \times e^{rh}) = C_d$$

$$(\Delta \times uS \times e^{\delta h}) + (B \times e^{rh}) = C_u$$

This is two equations in the two unknown's  $\Delta$  and  $B$ . Solving for  $\Delta$  and  $B$  gives:

$$\Delta = e^{-\delta h} \frac{C_u - C_d}{S(u - d)}$$

$$B = e^{-rh} \frac{uC_d - dC_u}{u - d}$$

Note that when there are dividends, the formula adjusts the number of shares in the replicating portfolio,  $\Delta$ , to offset the dividend income.

Given the expressions for  $\Delta$  and  $B$ , we can derive a simple formula for the value of the option. The cost of creating the option is the net cash required to buy the shares and bonds. Thus, the cost of the option is  $\Delta S + B$ . Finally, we have:

$$\Delta S + B = e^{-rh} \left( C_u \frac{e^{(r-\delta)h} - d}{u - d} + C_d \frac{u - e^{(r-\delta)h}}{u - d} \right)$$

The assumed stock price movements,  $u$  and  $d$ , should not give rise to arbitrage opportunities. In particular, we require that:

$$u > e^{(r-\delta)h} > d$$

Although probabilities are not needed for pricing the option, there is a probabilistic interpretation of equation for cost of an option. Notice that in equation the terms  $(e^{(r-\delta)h} - d)/(u - d)$  and  $(u - e^{(r-\delta)h})/(u - d)$  sum to 1 and are both positive (this follows from no-arbitrage condition stated above). Thus, we can interpret these terms as probabilities. Let

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d}$$

Option pricing equation then can be rewritten as:

$$C = e^{-rh} [p^* C_u + (1 - p^*) C_d]$$

This expression has the appearance of a discounted expected value. It is peculiar, though, because we are discounting at the risk-free rate, even though the risk of the option is at least as great as the risk of the stock (a call option is a leveraged position in the stock since  $B < 0$ ). In addition, there is no reason to think that  $p^*$  is the true probability that the stock will go up; in general it is not. We will call  $p^*$  the **risk-neutral probability** of an increase in the stock price.

### 167. How to use binomial option pricing for American put options?

The binomial method easily accommodates put options also, as well as other derivatives. We compute put option prices using the same stock price tree and in almost the same way as call option prices; the only difference with a European put option occurs at expiration: Instead of computing the price as  $\max(0, S - K)$ , we use  $\max(O, K - S)$ .

In this case of the American option we should also check whether early exercise is optimal or not. The value of the option if it is left "alive" (i.e., unexercised) is given by the value of holding it for another period, equation for put option will be:

$$e^{-rh} [P(uS, K, t + h)p^* + P(dS, K, t + h)(1 - p^*)]$$

The value of the option if it is exercised is given by  $\max(O, S - K)$  if it is a call and  $\max(O, K - S)$  if it is a put. Thus, for an American put, the value of the option at a node is given by:

$$P(S, K, t) = \max(K - S, e^{-rh} [P(uS, K, t + h)p^* + P(dS, K, t + h)(1 - p^*)])$$

Where

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d}$$

In general, the valuation of American options proceeds with checking at each node for early exercise. If the value of the option is greater when exercised, we assign that value to the node. Otherwise, we assign the value of the option unexercised. We work backward through the tree and the greater value of the option at each node ripples back through the tree resulting in at least as much value as for European style option.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 10, pp. 329-330*

### 168. Explain the concept of risk-neutral pricing

The binomial option pricing formula can be written as:

$$C = e^{-rh} [p^*C_u + (1 - p^*)C_d]$$

Where

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d}$$

We labeled  $p^*$  the risk-neutral probability that the stock will go up. The equation has the appearance of a discounted expected value, where the expected value calculation uses  $p^*$  and discounting is done at the risk-free rate.

It is common in finance to emphasize that investors are risk averse. To see what risk aversion means, suppose you are offered either (a) \$1000, or (b) \$2000 with probability 0.5, and \$0 with probability 0.5. A **risk-averse** investor prefers (a), since alternative (b) is risky and has the same expected value as (a). This kind of investor will require a premium to bear risk when expected values are equal

A **risk-neutral** investor, on the other hand, is indifferent between a sure thing and a risky bet with an expected payoff equal to the value of the sure thing. A risk-neutral investor, for example, will be equally happy with alternative (a) or (b).

Before proceeding, we need to emphasize that at no point are we assuming that investors are risk-neutral. Having said this, let's consider what an imaginary world populated by risk-neutral investors would be like. In such a world, investors care only about expected returns and not about riskiness. Assets would have no risk premium since investors would be willing to hold assets with an expected return equal to the risk-free rate.

In this hypothetical risk-neutral world, we can solve for the probability of the stock going up,  $p^*$ , such that the stock is expected to earn the risk-free rate. In the binomial model we assume that the stock can go up to  $uS$  or down to  $dS$ . If the stock is to earn the risk-free return on average, then the probability that the stock will go up,  $p^*$ , must satisfy:

$$p^*uSe^{\delta h} + (1 - p^*)dSe^{\delta h} = e^{r h} S$$

Solving for  $p^*$  gives:

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d}$$

This is exactly the definition of  $p^*$  in option pricing equation. This is why we refer to  $p^*$  as the *risk-neutral probability that the stock price will go up*. It is the probability that the stock price would increase in a risk-neutral world.

Not only would the risk-neutral probability be used in a risk neutral world, but also all discounting would take place at the risk-free rate. Thus, the option pricing formula can be said to price options as if investors are risk-neutral. At the risk of being repetitious, we are not assuming that investors are actually risk-neutral, and we are not assuming that risky assets are actually expected to earn the risk-free rate of return. Rather, *risk-neutral pricing* is an interpretation of the formulas above. Those formulas in turn arise from finding the cost of the portfolio that replicates the option payoff.

Interestingly, this interpretation of the option-pricing procedure has great practical importance; risk-neutral pricing can sometimes be used where other pricing methods are too difficult.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 10, pp. 320-321*  
*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 11, pp. 346-347*

### 169. Write formulas for constructing Cox-Ross-Rubinstein Binomial Tree

The best – known way to construct a binomial tree is that in Cox et al. (1979), in which the tree is constructed as

$$\begin{aligned}u &= e^{\sigma\sqrt{h}} \\d &= e^{-\sigma\sqrt{h}}\end{aligned}$$

The Cox-Ross-Rubinstein approach is often used in practice. A problem with this approach, however, is that if  $h$  is large or  $\sigma$  is small, it is possible that  $e^{rh} > e^{\sigma h}$ , in which case the binomial tree violates the restriction of no – arbitrage condition. In real applications  $h$  would be small, so this problem does not occur.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 11, pp. 359*

### 170. List properties of continuously compounded returns

Here is a summary of the important properties of continuously compounded returns:

**The logarithmic function computes returns from prices** Let  $S_t$  and  $S_{t+h}$  be stock prices at times  $t$  and  $t+h$ . The continuously compounded return between  $t$  and  $t+h$ ,  $r_{t,t+h}$  is then:

$$r_{t,t+h} = \ln(S_{t+h}/S_t)$$

**The exponential function computes prices from returns** if we know the continuously compounded return, we can obtain  $S_{t+h}$  by exponentiation of both sides of equation above. This gives:

$$S_{t+h} = S_t e^{r_{t,t+h}}$$

**Continuously compounded returns are additive** Suppose we have continuously compounded returns over a number of periods – for example,  $r_{t,t+h}$ ,  $r_{t+h,t+2h}$ , etc. The continuously compounded return over a long period is the *sum* of continuously compounded returns over the shorter periods, i.e.

$$r_{t,t+nh} = \sum_{i=1}^n r_{t+(i-1)h,t+ih}$$

**Continuously compounded returns can be less than -100%** a continuously compounded return that is a large negative number still gives a positive stock price. The reason is that  $e^r$  is positive for any  $r$ . Thus, if the log of the stock price follows a random walk, the stock price cannot become negative.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 11, pp. 353-354*

### 171. Explain estimation of the volatility of underlying asset

In practice we need to figure out what parameters to use in the binomial model. The most important decision is the value we assign to  $\sigma$ , which we cannot observe directly. One possibility is to measure  $\sigma$  by computing the standard deviation of continuously compounded historical returns. Volatility computed from historical stock returns is **historical volatility**.

Table below lists 3 weeks of Wednesday closing prices for the S&P 500 composite index and for IBM, along with the standard deviation of the continuously compounded returns, computed using the *StDev* function in Excel.

Over the 13-week period in the table, the weekly standard deviation was 0.0309 and 0.0365 for the S&P 500 index and IBM, respectively. These are weekly standard deviations since they are computed from weekly returns; they therefore measure the variability in weekly returns. We obtain annualized standard deviations by multiplying the weekly standard deviations by  $\sqrt{52}$ , giving annual standard deviations of 22.32% for the S&P 500 index and 26.32% for IBM.

Weekly prices and continuously compounded returns for the S&P 500 index and IBM, from 3/5/03 to 5/28/03.				
Date	S&P 500		IBM	
	Price	$\ln(S_t/S_{t-1})$	Price	$\ln(S_t/S_{t-1})$
03/05/03	829.85	—	77.73	—
03/12/03	804.19	-0.0314	75.18	-0.0334
03/19/03	874.02	0.0833	82.00	0.0868
03/26/03	869.95	-0.0047	81.55	-0.0055
04/02/03	880.90	0.0125	81.46	-0.0011
04/09/03	865.99	-0.0171	78.71	-0.0343
04/16/03	879.91	0.0159	82.88	0.0516
04/23/03	919.02	0.0435	85.75	0.0340
04/30/03	916.92	-0.0023	84.90	-0.0100
05/07/03	929.62	0.0138	86.68	0.0207
05/14/03	939.28	0.0103	88.70	0.0230
05/21/03	923.42	-0.0170	86.18	-0.0288
05/28/03	953.22	0.0318	87.57	0.0160
Std. deviation		0.0309	—	0.0365
Std. deviation $\times \sqrt{52}$		0.2232	—	0.2632

The procedure outlined above is a reasonable way to estimate volatility when continuously compounded returns are independent and identically distributed. However, if returns are not independent – as with some commodities, for example – volatility estimation becomes more complicated. If a high price of oil today leads to decreased demand and increased supply, we would expect prices in the future to come down. In this case, the volatility over  $T$  years will be less than  $\sigma\sqrt{T}$ , reflecting the tendency of prices to revert from

extreme values. Extra care is required with volatility if the random walk model is not a plausible economic model of the asset's price behavior.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 11, pp. 360-361*

## 172. Write The Black-Scholes formula. List assumption of The Black-Scholes formula

To introduce the Black-Scholes formula, we first revise the binomial model. When computing a binomial option price, we can vary the number of binomial steps, holding fixed the time to expiration. Changing the number of steps changes the option price, but once the number of steps becomes great enough we appear to approach a limiting value for the price. We can't literally have infinity of steps in a binomial tree, but it is possible to show that as the number of steps approaches infinity, the option price is given by the Black-Scholes formula. Thus, the Black-Scholes formula is a limiting case of the binomial formula for the price of a European option.

The Black-Scholes formula for a European call and put options on a stock that pays dividends at the continuous rate  $\delta$  is:

$$C(S, K, \sigma, r, T, \delta) = Se^{-\delta T} N(d_1) - Ke^{-rT} N(d_2)$$

$$P(S, K, \sigma, r, T, \delta) = Ke^{-rT} N(-d_2) - Se^{-\delta T} N(-d_1)$$

Where

$$d_1 = \frac{\ln(S/K) + (r - \delta + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

As with the binomial model, there are six inputs to the Black-Scholes formula:  $S$ , the current price of the stock;  $K$ , the strike price of the option;  $\sigma$ , the volatility of the stock;  $r$  the continuously compounded risk-free interest rate;  $T$ , the time to expiration; and  $\delta$ , the dividend yield on the stock.

$N(x)$  in the Black-Scholes formula is the cumulative normal distribution function, which is the probability that a number randomly drawn from a standard normal distribution (i.e., a normal distribution with mean 0 and variance 1) will be less than  $x$ . Most spreadsheets have a built-in function for computing  $N(x)$ . In Excel, the function is *NormSDist*.

Two of the inputs ( $K$  and  $T$ ) describe characteristics of the option contract. The others describe the stock ( $S$ ,  $\sigma$ , and  $\delta$ ) and the discount rate for a risk-free investment ( $r$ ). All of the inputs are self-explanatory with the exception of volatility, which is the standard deviation of the rate of return on the stock – a measure of the uncertainty about the future return on the stock.

Derivations of the Black-Scholes formula make a number of assumptions that can be sorted into two groups: assumptions about how the stock price is distributed, and assumptions

about the economic environment. For the version of the formula we have presented, assumptions about the distribution of the stock price include the following:

- Continuously compounded returns on the stock are normally distributed and independent over time (we assume there are no "jumps" in the stock price)
- The volatility of continuously compounded returns is known and constant
- Future dividends are known, either as a dollar amount or as a fixed dividend yield

Assumptions about the economic environment include these:

- The risk-free rate is known and constant
- There are no transaction costs or taxes
- It is possible to short-sell costlessly and to borrow at the risk-free rate

Many of these assumptions can easily be relaxed. For example, with a small change in the formula, we can permit the volatility and interest rate to vary over time in a known way.

As a practical matter, the first set of assumptions – those about the stock price distribution – are the most crucial. Most academic and practitioner research on option pricing concentrates on relaxing these assumptions. They will also be our focus when we discuss empirical evidence. You should keep in mind that almost *any* valuation procedure, including ordinary discounted cash flow, is based on assumptions that appear strong; the interesting question is how well the procedure works in practice.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 12, pp. 375-379*

### **173. Define option Greeks and Greek measures for portfolios**

Option Greeks are formulas that express the change in the option price when an input to the formula changes, taking as fixed all the other inputs. Specifically, the Greeks are mathematical derivatives of the option price formula with respect to the inputs. One important use of Greek measures is to assess risk exposure. For example, a market – making bank with a portfolio of options would want to understand its exposure to stock price changes, interest rates, volatility, etc. An options investor would like to know how interest rate changes and volatility changes affect profit and loss.

Keep in mind that the Greek measures by assumption change only one input at a time. In real life, we would expect interest rates and stock prices, for example, to change together. The Greeks answer the question, what happens when *one and only one* input changes? Here are list of some Greeks and their definitions:

- **Delta** ( $\Delta$ ) measures the option price change when the stock price increases by \$1.
- **Gamma** ( $\Gamma$ ) measures the change in delta when the stock price increases by \$1.
- **Vega** measures the change in the option price when there is an increase in volatility of one percentage point.



- **Theta** ( $\theta$ ) measures the change in the option price when there is a decrease in the time to maturity of 1 day.
- **Rho** ( $\rho$ ) measures the change in the option price when there is an increase in the interest rate of 1 percentage point (100 basis points).
- **Psi** ( $\psi$ ) measures the change in the option price when there is an increase in the continuous dividend yield of 1 percentage point (100 basis points).

A useful mnemonic device for remembering some of these is that "vega" and "volatility" share the same first letter, as do "theta" and "time." Also "r" is often used to denote the interest rate and is the first letter in "rho."

*The Greek measure of a portfolio is the sum of the Greeks of the individual portfolio components.* This relationship is important because it means that the risk of complicated option positions is easy to evaluate. For a portfolio containing  $n$  options with a single underlying stock, where the quantity of each option is given by  $\omega_i$ , we have:

$$\Delta_{\text{portfolio}} = \sum_{i=1}^n \omega_i \Delta_i$$

The same relation holds true for the other Greeks as well.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 12, pp. 382-389*

#### 174. Describe option elasticity

An option is an alternative to investing in the stock. Delta tells us the dollar risk of the option relative to the stock: If the stock price changes by \$1, by how much does the option price change? The option elasticity, by comparison, tells us the risk of the option relative to the stock in percentage terms: If the stock price changes by 1%, what is the percentage change in the value of the option?

**Dollar risk of option:** If the stock price changes by  $\epsilon$ , the change in the option price is:

$$\begin{aligned} \text{Change in option price} &= \text{Change in stock price} \times \text{option delta} \\ &= \epsilon \times \Delta \end{aligned}$$

**Percentage risk of option:** The **option elasticity** computes the percentage change in the option price relative to the percentage change in the stock price. The percentage change in the stock price is simply  $\epsilon / S$ . The percentage change in the option price is the dollar change in the option price,  $\epsilon \Delta$ , divided by the option price,  $C$ :

$$\frac{\epsilon \Delta}{C}$$

The option elasticity, denoted by  $\Omega$ , is the ratio of these two:

$$\Omega \equiv \frac{\% \text{ change in option price}}{\% \text{ change in stock price}} = \frac{\frac{\epsilon \Delta}{C}}{\frac{\epsilon}{S}} = \frac{S \Delta}{C}$$

The elasticity tells us the percentage change in the option for a 1% change in the stock. It is effectively a measure of the leverage implicit in the option. Elasticity can be used to compute option volatility and the risk premium of an option.

**Option Volatility:** The volatility of an option is the elasticity times the volatility of the stock:

$$\sigma_{\text{option}} = \sigma_{\text{stock}} \times |\Omega|$$

Where  $|\Omega|$  is the absolute value of  $\Omega$ .

**The risk premium of an option:** Since elasticity measures the percentage sensitivity of the option relative to the stock, it tells us how the risk premium of the option compares to that of the stock.

At a point in time, the option is equivalent to a position in the stock and in bonds; hence, the return on the option is a weighted average of the return on the stock and the risk-free rate. Let  $\alpha$  denote the expected rate of return on the stock,  $\gamma$  the expected return on the option, and  $r$  the risk-free rate. We have:

$$\gamma = \frac{\Delta S}{C(S)} \alpha + \left(1 - \frac{\Delta S}{C(S)}\right) r$$

Since  $\Delta S / C(S)$  is elasticity, this can be rewritten as:

$$\gamma - r = (\alpha - r) \times \Omega$$

Thus, the risk premium on the option equals the risk premium on the stock times  $\Omega$ .

Using our earlier facts about elasticity, we conclude that if the stock has a positive risk premium, then a call always has an expected return at least as great as the stock and that, other things equal, the expected return on an option goes down as the stock price goes up. In terms of the capital asset pricing model, we would say that the option beta goes down as the option becomes more in-the-money. For puts, we conclude that the put always has an expected return less than that of the stock.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 12, pp. 389-395*

### 175. Draw profit diagrams before maturity for a call option

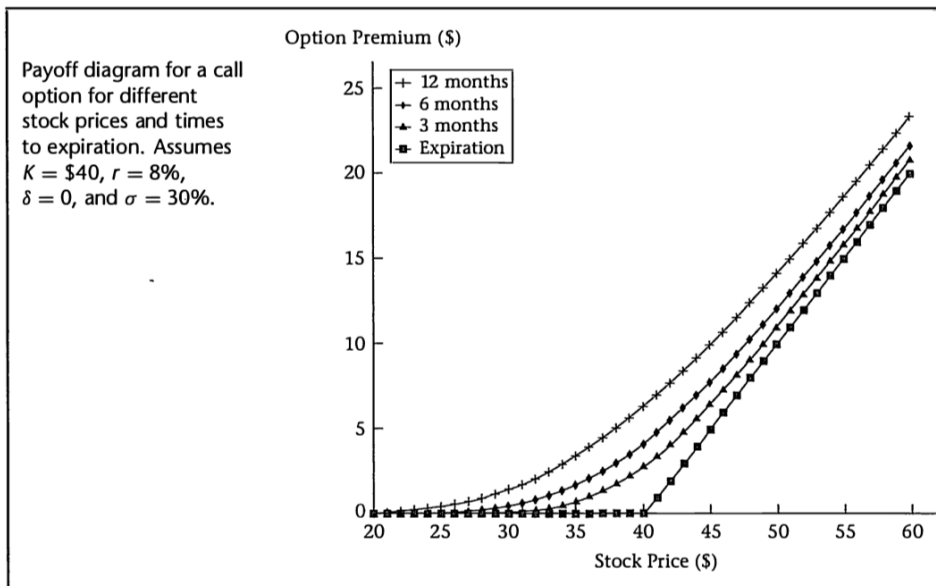
In order to evaluate investment strategies using options, we would like to be able to answer questions such as: If the stock price in 1 week is \$5 greater than it is today, what will be the change in the price of a call option? What is the profit diagram for an option position in

which the options have different times to expiration? To do this we need to use an option pricing formula.

Consider the purchase of a call option. Just as with expiring options, we can ask what the value of the option is at a particular point in time and for a particular stock price. The table below shows the Black-Scholes value of a call option for five different stock prices at four different times to expiration. By varying the stock price for a given time to expiration, keeping everything else the same, we are able to graph the value of the call.

Value of 40-strike call option at different stock prices and times to expiration. Assumes $r = 8\%$ , $\sigma = 30\%$ , $\delta = 0$ .				
Stock Price (\$)	Time to Expiration			
	12 Months	6 Months	3 Months	0 (Expiration)
36	3.90	2.08	1.00	0
38	5.02	3.02	1.75	0
40	6.28	4.16	2.78	0
42	7.67	5.47	4.07	2
44	9.15	6.95	5.58	4

The figure below plots Black-Scholes call option prices for stock prices ranging from \$20 to \$60, including the values in the table above. Notice that the value of the option prior to expiration is a smoothed version of the value of the option at expiration.



### 176. How to compute implied volatility of an underlying asset?

Volatility is unobservable, and though option prices, particularly for near-the-money options, can be quite sensitive to volatility. Thus, choosing a volatility to use in pricing an option is difficult but also quite important.

One approach to obtaining volatility is to use the history of returns to compute historical volatility. A problem with historical volatility is that history is not a reliable guide to the future: Markets have quiet and turbulent periods and predictable events such as Federal Reserve Board Open Market Committee meetings can create periods of greater than normal uncertainty. There are sophisticated statistical models designed to improve upon simple volatility estimates, but no matter what you do, you cannot count on history to provide you with a reliable estimate of future volatility.

In many cases we can observe option prices for an asset. We can then invert the question: Instead of asking what volatility we should use to price an option, we can compute an option's **implied volatility**, which is the volatility that would explain the observed option price. Assuming that we observe the stock price  $S$ , strike price  $K$ , interest rate  $r$ , dividend yield  $\delta$ , and time to expiration  $T$ , the implied call volatility is the  $\hat{\sigma}$  that solves:

$$\text{Market option price} = C(S, K, \hat{\sigma}, r, T, \delta)$$

By definition, if we use implied volatility to price an option, we obtain the market price of the option. Thus, we cannot use implied volatility to assess whether an option price is correct, but implied volatility does tell us the market's assessment of volatility.

Computing an implied volatility requires that we (1) observe a market price for an option and (2) have an option pricing model with which to infer volatility. Equation above cannot be solved directly for the implied volatility,  $\hat{\sigma}$ , so it is necessary to use an iterative procedure to solve the equation. Any pricing model can be used to calculate an implied volatility, but Black-Scholes implied volatilities are frequently used as benchmarks.

Table below lists ask prices of calls and puts on the S&P 500 index, along with implied volatilities computed using the Black-Scholes formula. These S&P options are European style, so the Black-Scholes model is appropriate. Notice that, although the implied volatilities in the table are not all equal, they are all in a range between 13% and 16%. We could describe the general level of S&P option prices by saying that the options are trading at about a 15% volatility level. There are typically numerous options on a given asset; implied volatility can be used to succinctly describe the general level of option prices for a given underlying asset.

When you graph implied volatility against the strike price, the resulting line can take different shapes, often described as "smiles," "frowns," and "smirks". This systematic change in implied volatility across strike prices occurs generally for different underlying assets and is called **volatility skew**.

When examining implied volatilities, it is helpful to keep put-call parity in mind. If options are European, then puts and calls with the same strike and time to expiration must have the same implied volatility. This is true because prices of European puts and calls must satisfy the parity relationship or else there is an arbitrage opportunity. Although call and put

volatilities are not exactly equal in the table above, they are close enough that parity arbitrage would not be profitable after transaction costs are taken into account.

Implied volatilities for S&P 500 options, 10/28/2004. Option prices (ask) from <a href="http://www.cboe.com">www.cboe.com</a> ; assumes $S = \$1127.44$ , $\delta = 1.85\%$ , $r = 2\%$ .					
Strike (\$)	Expiration	Call Price (\$)	Implied Volatility	Put Price (\$)	Implied Volatility
1100	11/20/2004	34.80	0.1630	6.80	0.1575
1125	11/20/2004	17.10	0.1434	14.70	0.1447
1150	11/20/2004	5.80	0.1284	29.20	0.1389
1100	12/18/2004	41.70	0.1559	13.80	0.1539
1125	12/18/2004	24.50	0.1396	22.50	0.1436
1150	12/18/2004	13.00	0.1336	35.50	0.1351
1100	1/22/2005	49.10	0.1567	20.40	0.1518
1125	1/22/2005	33.00	0.1463	29.40	0.1427
1150	1/22/2005	20.00	0.1363	41.50	0.1337

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 12, pp. 400-402*

### 177. List and define types of exotic options

By altering the terms of standard contracts like options, futures, and swaps, you obtain a "nonstandard" or "exotic" option. Exotic options can provide precise tailoring of risk exposures, and they permit investment strategies difficult or costly to realize with standard options and securities. Some basic kinds of exotic options include Asian, barrier, compound, gap, and exchange options.

**Asian Options:** An **Asian option** has a payoff that is based on the average price over some period of time. An Asian option is an example of a **path-dependent option**, which means that the value of the option at expiration depends upon the path by which the stock arrived at its final price.

The payoff at maturity can be computed using the average stock price either as the price of the underlying asset or as the strike price. When the average is used as the asset price, the option is called an average price option. When the average is used as the strike price, the option is called an average strike option. Here are the eight variants of options based on the geometric and arithmetic average:

$$\text{Arithmetic average price call} = \max[0, A(T) - K]$$

$$\text{Geometric average price call} = \max[0, G(T) - K]$$

$$\text{Arithmetic average price put} = \max[K - A(T), 0]$$

$$\text{Geometric average price put} = \max[K - G(T)]$$

$$\text{Arithmetic average strike call} = \max[0, S_T - A(T)]$$

$$\text{Geometric average strike call} = \max[0, S_T - G(T)]$$

$$\text{Arithmetic average strike put} = \max[0, A(T) - S_T]$$

$$\text{Geometric average strike put} = \max[0, G(T) - S_T]$$

The terms "average price" and "average strike" refer to whether the average is used in place of the asset price or the strike price.

**Barrier Options:** A barrier option is an option with a payoff depending upon whether, over the life of the option, the price of the underlying asset reaches a specified level, called the barrier. Barrier puts and calls either come into existence or go out of existence the first time the asset price reaches the barrier. If they are in existence at expiration, they are equivalent to ordinary puts and calls.

Since barrier puts and calls never pay more than standard puts and calls, they are no more expensive than standard puts and calls. Barrier options are another example of a path-dependent option.

Barrier options are widely used in practice. One appeal of barrier options may be their lower premiums, although the lower premium of course reflects a lower average payoff at expiration. There are three basic kinds of barrier options:

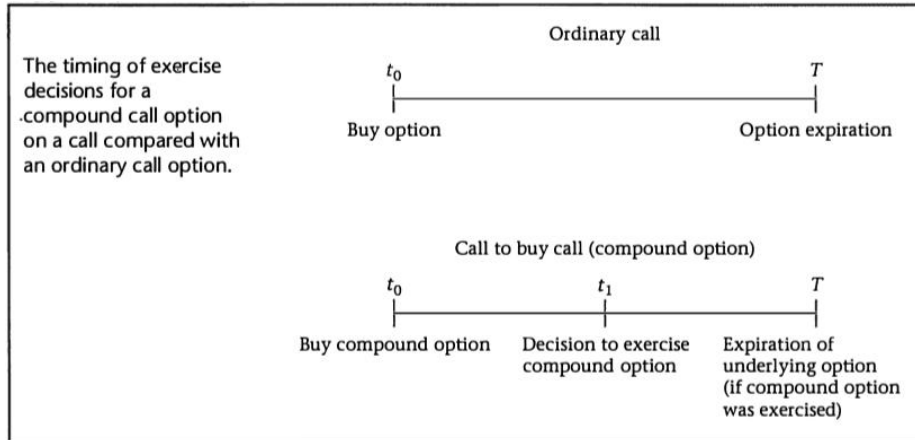
1. *Knock-out options:* These go out of existence (are "knocked-out") if the asset price reaches the barrier. If the price of the underlying asset has to fall to reach the barrier, the option is a down-and-out. If the price of the underlying asset has to rise to reach the barrier, the option is an up-and-out.
2. *Knock-in options:* These come into existence (are "knocked-in") if the barrier is touched. If the price of the underlying asset has to fall to reach the barrier, the option is a down-and-in. If the asset price has to rise to reach the barrier, it is an up-and-in.
3. *Rebate options:* These make a fixed payment if the asset price reaches the barrier. The payment can occur either at the time the barrier is reached, or at the time the option expires, in which case it is a deferred rebate. Rebate options can be either "up rebates" or "down rebates," depending on whether the barrier is above or below the current price.

The important parity relation for barrier options is:

$$\text{"Knock-in" option} + \text{"Knock-out" option} = \text{Ordinary option}$$

**Compound Options:** A compound option is an option to buy an option. If you think of an ordinary option as an asset – analogous to a stock – then a compound option is similar to an ordinary option. Compound options are a little more complicated than ordinary options because there are two strikes and two expirations, one each for the underlying option and for the compound option.

Figure below compares the timing of the exercise decisions for *CallonCall* compound option with the exercise decision for an ordinary call expiring at time  $T$ .

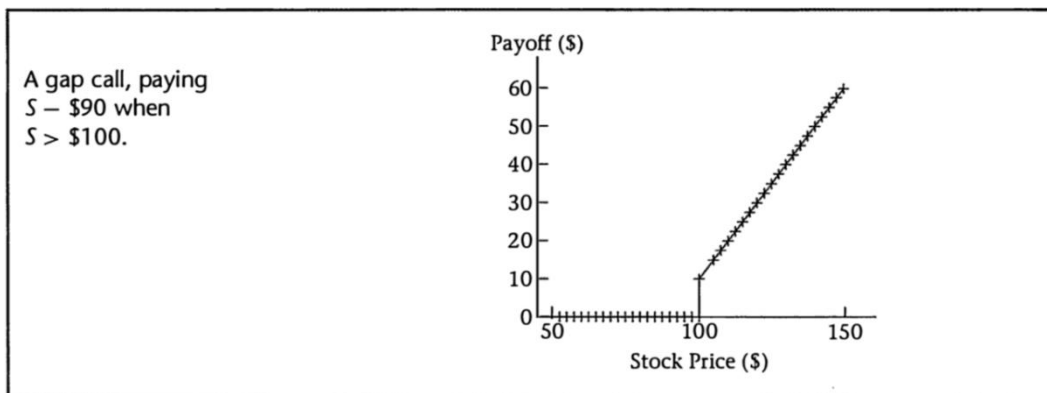


The four examples of compound options are:

- An option to buy a call (*CallOnCall*)
- An option to sell a call (*PutOnCall*)
- An option to buy a put (*CallOnPut*), and
- An option to sell a put (*PutOnPut*)

**Gap Options:** A call option pays  $S - K$  when  $S > K$ . The strike price,  $K$ , here serves to determine both the range of stock prices where the option makes a payoff (when  $S > K$ ) and also the size of the payoff ( $S - K$ ). However, we could imagine separating these two functions of the strike price. Consider an option that pays  $S - 90$  when  $S > 100$ . Note that there is a difference between the prices that govern when there is a payoff (\$100) and the price used to determine the size of the payoff (\$90). This difference creates a discontinuity – or gap – in the payoff diagram, which is why the option is called a **gap option**.

Figure below shows a gap call option with payoff  $S - 90$  when  $S > 100$ . The gap in the payoff occurs when the option payoff jumps from \$0 to \$10 as a result of the stock price changing from \$99.99 to \$100.01.



**Exchange Options:** Exchange options are one kind of executives' compensation. Executive stock options are sometimes constructed so that the strike price of the option is the price of an

index, rather than a fixed cash amount. The idea is to have an option that pays off only when the company outperforms competitors, rather than one that pays off simply because all stock prices have gone up. As a hypothetical example of this, suppose Bill Gates, chairman of Microsoft, is given compensation options that pay off only if Microsoft outperforms Google. He will exercise these options if and only if the share price of Microsoft,  $S_{MSFT}$ , exceeds the share price of Google,  $S_{GOOG}$ , i.e.,  $S_{MSFT} > S_{GOOG}$ . From Gates's perspective, this is a call option, with the payoff:

$$\max(0, S_{MSFT} - S_{GOOG})$$

Now consider the compensation option for Eric Schmidt, CEO of Google. He will receive a compensation option that pays off only if Google outperforms Microsoft, i.e.,

$$\max(0, S_{GOOG} - S_{MSFT})$$

This is a call from Schmidt's perspective. Here is the interesting twist: Schmidt's Google call looks to Gates like a Microsoft put! And Gates's Microsoft call looks to Schmidt like a Google put. Either option can be viewed as a put or call; it is simply a matter of perspective. *The distinction between a put and a call in this example depends upon what we label the underlying asset and what we label as the strike asset.*

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 9, pp. 288-289*  
*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 14, pp. 443-461*

## 178. Define Modigliani-Miller Theorem

The starting point for any discussion of modern financial engineering is the analysis of Franco Modigliani and Merton Miller (Modigliani and Miller, 1958). Before their work, financial analysts would puzzle over how to compare the values of firms with similar operating characteristics but different financial characteristics. Modigliani and Miller realized that different financing decisions (for example, the choice of the firm's debt-to-equity ratio) may carve up the firm's cash flows in different ways, but if the total cash flows paid to all claimants is unchanged, the total value of all claims would remain the same. They showed that if firms differing only in financial policy differed in market value, profitable arbitrage would exist. Using their famous analogy, the price of whole milk should equal the total prices of the skim milk and butterfat that can be derived from that milk.

The Modigliani-Miller analysis requires numerous assumptions: For example, there are no taxes, no transaction costs, no bankruptcy costs, and no private information. Nevertheless, the basic Modigliani-Miller result provided clarity for a confusing issue, and it created a starting point for thinking about the effects of taxes, transaction costs, and the like, revolutionizing finance.

All of the no – arbitrage pricing arguments embody the Modigliani-Miller spirit. For example, we could synthetically create a forward contract using options, a call option using a forward contract, bonds, and a put, and so forth. An option could also be synthetically created



from a position in the stock and borrowing or lending. If prices of actual claims differ from their synthetic equivalents, arbitrage is possible.

Financial engineering is an application of the Modigliani-Miller idea. We can combine claims such as stocks, bonds, forwards, and options and assemble them to create new claims. The price for this new security is the sum of the pieces combined to create it. When we create a new instrument in this fashion, as in the Modigliani Miller analysis, value is neither created nor destroyed. Thus, financial engineering has no value in a pure Modigliani-Miller world. However, in real life, the new instrument may have different tax, regulatory, or accounting characteristics, or may provide a way for the issuer or buyer to obtain a particular payoff at lower transaction costs than the alternatives. Financial engineering thus provides a way to create instruments that meet specific needs of investors and issuers.

As a starting point, you can ask the following questions when you confront new financial instruments:

- What is the payoff of the instrument?
- Is it possible to synthetically create the same payoffs using some combination of assets, bonds, and options?
- Who might issue or buy such an instrument?
- What problem does the instrument solve?

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 15, pp. 473-474*

### 179. How debt and equity can be viewed as options?

Firms often issue securities that have derivative components. For example, firms issue options to employees for financing, and convertible debt is a bond coupled with a call option. However, even simple securities, such as ordinary debt and equity, can be viewed as derivatives.

Consider a firm with the following very simple capital structure. The firm has non – dividend – paying equity outstanding, along with a single zero – coupon debt issue. Represent the time  $t$  values of the assets of the firm, the debt, and the equity as  $A_t$ ,  $B_t$ , and  $E_t$ . The debt matures at time  $T$  and has maturity value  $B$ .

The value of the debt and equity at time  $T$  will depend upon the value of the firm's assets. Equity – holders are the legal owners of the firm; in order for them to have unambiguous possession of the firm's assets, they must pay the debt – holders  $B$  at time  $T$ . Therefore, the value of the equity will be:

$$E_T = \begin{cases} A_T - B, & \text{if } A_T > B \\ 0, & \text{if } A_T < B \end{cases}$$

This is analogous to the payoff to a call option with the assets of the firm as the underlying asset and  $B$  as the strike price:

$$E_T = \max(0, A_T - B)$$

Because equity-holders control the firm, bondholders receive the smallest payment to which they are legally entitled. If the firm is bankrupt – i.e., if  $A_T < B$  – the bondholders receive  $A_T$ . If the firm is solvent – i.e., if  $A_T > B$  – the bondholders receive  $B$ . Thus the value of the debt is:

$$B_T = \min(A_T, B)$$

This can be rewritten as:

$$\begin{aligned} B_T &= A_T + \min(0, \bar{B} - A_T) \\ &= A_T - \max(0, A_T - \bar{B}) \end{aligned}$$

Equation above says that the bondholders own the firm, but have written a call option to the equity – holders. A different way to rewrite original equation for the debt value of the firm is:

$$\begin{aligned} B_T &= \bar{B} + \min(0, A_T - \bar{B}) \\ &= \bar{B} - \max(0, \bar{B} - A_T) \end{aligned}$$

The interpretation of above equation is that the bondholders own risk – free debt with a payoff equal to  $B$ , but have written a put option on the assets with strike price  $B$ .

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 16, pp. 503-506*

## 180. How to calculate expected return on debt and equity

We can compute the expected return on both debt and equity using the concept of option elasticity. Recall that the elasticity of an option tells us the relationship between the expected return on the underlying asset and that on the option. So, we can compute the expected return on equity as:

$$r_E = r + (r_A - r) \times \Omega_E$$

Where

$r_A$  is the expected return on assets,

$r$  is the risk-free rate, and

$\Omega_E$  is the elasticity of the equity

With

$$\Omega_E = \frac{A_I \Delta_E}{E_I}$$

Where

$\Delta_E$  is the option delta

We can compute the expected return on debt using the debt elasticity,  $\Omega_B$ :

$$r_B = r + (r_A - r) \times \Omega_B$$

The elasticity calculation is slightly more involved for debt than for equity. Since we compute debt value as  $B_t = A_t - E_t$ . The elasticity of debt is a weighted average of the asset and equity elasticity:

$$\Omega_B = \frac{A_t}{A_t - E_t} \Omega_A - \frac{E_t}{A_t - E_t} \Omega_E$$

Using above equations you can verify that if you owned a proportional interest in the debt and equity of the firm, the expected return on your portfolio would be the expected return on the assets of the firm:

$$(\% \text{Equity} \times r_E) + (\% \text{Debt} \times r_B) = r_A$$

It bears emphasizing that this relationship requires that  $r_B$  represent the expected return on debt, not the yield to maturity.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 16, pp. 503-512*

### 181. Write a lognormal model of stock prices

A random variable,  $y$ , is said to be lognormally distributed if  $\ln(y)$  is normally distributed. Put another way, if  $x$  is normally distributed,  $y$  is lognormal if it can be written in either of two equivalent ways:

$$\ln(y) = x \quad \text{or} \quad y = e^x$$

How do we implement lognormality as a model for the stock price? If the stock price  $S_t$  is lognormal, we can write:

$$\frac{S_t}{S_0} = e^x$$

Where  $x$  is the continuously compounded return from 0 to  $t$ , is normally distributed. We want to find a specification for  $x$  that provides a useful way to think about stock prices.

Let the continuously compounded return from time  $t$  to some later times be  $R_{(t,s)}$ . Suppose we have times  $t_0 < t_1 < t_2$ . By the definition of the continuously compounded return, we have:

$$\begin{aligned} S_{t_1} &= S_{t_0} e^{R(t_0, t_1)} \\ S_{t_2} &= S_{t_1} e^{R(t_1, t_2)} \end{aligned}$$

The stock price at  $t_2$  can therefore be expressed as

$$\begin{aligned} S_{t_2} &= S_{t_1} e^{R(t_1, t_2)} \\ &= S_{t_0} e^{R(t_0, t_1)} e^{R(t_1, t_2)} \\ &= S_{t_0} e^{R(t_0, t_1) + R(t_1, t_2)} \end{aligned}$$

Thus, the continuously compounded return from  $t_0$  to  $t_2$ ,  $R(t_0, t_2)$ , is the sum of the continuously compounded returns over the shorter periods:

$$R(t_0, t_2) = R(t_0, t_1) + R(t_1, t_2)$$

As we saw together with the assumption that returns are independent and identically distributed over time, implies that the mean and variance of returns over different horizons are proportional to the length of the horizon. Take the period of time from 0 to  $T$  and carve it up into  $n$  intervals of length  $h$ , where  $h = T / n$ . We can then write the continuously compounded return from 0 to  $T$  as the sum of the  $n$  returns over the shorter periods:

$$\begin{aligned} R(0, T) &= R(0, h) + R(h, 2h) + \dots + R[(n-1)h, T] \\ &= \sum_{i=1}^n R[(i-1)h, ih] \end{aligned}$$

Let

$$E(R[(i-1)h, ih]) = \alpha_h \text{ and } \text{Var}(R[(i-1)h, ih]) = \sigma_h^2$$

Then over the entire period, the mean and variance are:

$$\begin{aligned} E[R(0, T)] &= n\alpha_h \\ \text{Var}[R(0, T)] &= n\sigma_h^2 \end{aligned}$$

Thus, if returns are independent and identically distributed, the mean and variance of the continuously compounded returns are proportional to time.

Now we have enough background to present an explicit lognormal model of the stock price. Generally, let  $t$  be denominated in years and  $\alpha$  and  $\sigma$  be the annual mean and standard deviation, with  $\delta$  the annual dividend yield on the stock. We will assume that the continuously compounded capital gain from 0 to  $t$ ,  $\ln(S_t / S_0)$ , is normally distributed with mean  $(\alpha - \delta - 0.5\sigma^2)t$  and variance  $\sigma^2 t$ .

This gives us two equivalent ways to write an expression for the stock price. First, recall that we can convert a standard normal random variable,  $z$ , into one with an arbitrary mean or variance by multiplying by the standard deviation and adding the mean. We can write:

$$\ln(S_t / S_0) = (\alpha - \delta - \frac{1}{2}\sigma^2)t + \sigma\sqrt{t}z$$

Second, we can exponentiate above to obtain an expression for the stock price:

$$S_t = S_0 e^{(\alpha - \delta - \frac{1}{2}\sigma^2)t + \sigma\sqrt{t}z}$$

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 18, pp. 587-598*

## 182. Write stock price probabilities under lognormal distribution

If  $S_t$  is lognormally distributed, we can use this fact to compute a number of probabilities and expectations. For example, we can compute the probability that an option will expire in the money, and, given that it expires in the money, the expected stock price.

If the stock price today is  $S_0$ , what is the probability that  $S_t < K$ , where  $K$  is some arbitrary number? Note that  $S_t < K$  exactly when  $\ln(S_t) < \ln(K)$ . Since  $\ln(S)$  is normally distributed, we have:

$$\ln(S_t/S_0) \sim \mathcal{N}[(\alpha - \delta - 0.5\sigma^2)t, \sigma^2 t]$$

Or, equivalently,

$$\ln(S_t) \sim \mathcal{N}[\ln(S_0) + (\alpha - \delta - 0.5\sigma^2)t, \sigma^2 t]$$

We can create a standard normal number random variable,  $z$ , by subtracting the mean and dividing by the standard deviation:

$$z = \frac{\ln(S_t) - \ln(S_0) - (\alpha - \delta - 0.5\sigma^2)t}{\sigma\sqrt{t}}$$

We have  $\text{Prob}(S_t < K) = \text{Prob}[\ln(S_t) < \ln(K)]$ . Subtracting the mean from both  $\ln(S_t)$  and  $\ln(K)$  and dividing by standard deviation, we obtain:

$$\begin{aligned} \text{Prob}(S_t < K) &= \\ \text{Prob} \left[ \frac{\ln(S_t) - \ln(S_0) - (\alpha - \delta - 0.5\sigma^2)t}{\sigma\sqrt{t}} < \frac{\ln(K) - \ln(S_0) - (\alpha - \delta - 0.5\sigma^2)t}{\sigma\sqrt{t}} \right] \end{aligned}$$

Since the left-hand side is a standard normal random variable, the probability that  $S_t < K$  is:

$$\text{Prob}(S_t < K) = \text{Prob} \left[ z < \frac{\ln(K) - \ln(S_0) - (\alpha - \delta - 0.5\sigma^2)t}{\sigma\sqrt{t}} \right]$$

Since  $z \sim N(0, 1)$ ,  $\text{Prob}(S_t < K)$  is:

$$\text{Prob}(S_t < K) = N \left[ \frac{\ln(K) - \ln(S_0) - (\alpha - \delta - 0.5\sigma^2)t}{\sigma\sqrt{t}} \right]$$

This can also be written as:

$$\text{Prob}(S_t < K) = N(-\hat{d}_2)$$

Where  $\hat{d}_2$  is the standard Black – Scholes argument with the risk-free rate,  $r$ , replaced with the actual expected return on the stock,  $\alpha$ . We can also perform the complementary calculation. We have  $\text{Prob}(S_t > K) = 1 - \text{Prob}(S_t < K)$ , so:

$$\text{Prob}(S_t > K) = N(\hat{d}_2)$$

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 18, pp. 599-600*

### 183. Write conditional expected stock prices under lognormal stock prices distribution

Given that an option expires in the money, what is the expected stock price? The answer to this question is the conditional expected stock price. For a put with strike price  $K$ , we want

to calculate  $E(S_t | S_t < K)$ , the expected stock price conditional on  $S_t < K$ . To compute this expectation, we need to take into account only the portion of the probability density representing stock prices below  $K$ .

The partial expectation of  $S_t$ , conditional on  $S_t < K$ , is:

$$\begin{aligned} \int_0^K S_t g(S_t; S_0) dS_t &= S_0 e^{(\alpha-\delta)t} N\left(\frac{\ln(K) - [\ln(S_0) + (\alpha - \delta + 0.5\sigma^2)t]}{\sigma\sqrt{t}}\right) \\ &= S_0 e^{(\alpha-\delta)t} N(-\hat{d}_1) \end{aligned}$$

Where  $g(S_t; S_0)$  is the probability density of  $S_t$  conditional on  $S_0$ , and  $\hat{d}_1$  is the Black – Scholes  $d_1$  equation with  $\alpha$  replacing  $r$ .

The probability that  $S_t < K$  is  $N(-d_2)$ . Thus, the expectation of  $S_t$  conditional on  $S_t < K$  is:

$$E(S_t | S_t < K) = S e^{(\alpha-\delta)t} \frac{N(-\hat{d}_1)}{N(-\hat{d}_2)}$$

For a call, we are interested in the expected price conditional on  $S > K$ . The partial expectation of  $S_t$  conditional on  $S_t > K$  is:

$$\begin{aligned} \int_K^\infty S_t g(S_t; S_0) dS_t &= S e^{(\alpha-\delta)t} N\left(\frac{\ln(S_0) - \ln(K) + (\alpha - \delta + 0.5\sigma^2)t}{\sigma\sqrt{t}}\right) \\ &= S_0 e^{(\alpha-\delta)t} N(\hat{d}_1) \end{aligned}$$

As before, except for the fact that it contains the expected rate of return on the stock,  $\alpha$ , instead of the risk – free rate, the second term is just the Black-Scholes expression,  $N(d_1)$ . The conditional expectation is:

$$E(S_t | S_t > K) = S e^{(\alpha-\delta)t} \frac{N(\hat{d}_1)}{N(\hat{d}_2)}$$

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 18, pp. 602-604*

## 184. Derive The Black-Scholes formula

Using equations for calculating  $\text{Prob}(S_t < K)$ ,  $\text{Prob}(S_t > K)$ , the expectation of  $S_t$  conditional on  $S_t < K$  and expectation of  $S_t$  conditional on  $S_t > K$ , we can heuristically derive the Black – Scholes formula. Recall that the Black-Scholes formula can be derived by assuming risk-neutrality. In this case, the expected return on stocks,  $\alpha$ , will equal  $r$ , the risk – free rate. If we let  $g^*$  denote the risk – neutral lognormal probability density,  $E^*$  denote the expectation taken with respect to risk-neutral probabilities, and  $\text{Prob}^*$  denote these probabilities, the price of a European call option on a non – dividend paying stock will be:

$$\begin{aligned}
 C(S, K, \sigma, r, t, \delta) &= e^{-rt} \int_K^{\infty} (S_t - K) g^*(S_t; S_0) dS_t \\
 &= e^{-rt} E^*(S - K | S > K) \times \text{Prob}^*(S > K)
 \end{aligned}$$

This can be rewritten as:

$$\begin{aligned}
 C(S, K, \sigma, r, t, \delta) &= e^{-rt} E^*(S | S > K) \times \text{Prob}^*(S > K) \\
 &\quad - e^{-rt} E^*(K | S > K) \times \text{Prob}^*(S > K)
 \end{aligned}$$

Substituting  $\alpha$  with  $r$ , this becomes:

$$C(S, K, \sigma, r, t, \delta) = e^{-\delta t} S N(d_1) - K e^{-rt} N(d_2)$$

This is the Black – Scholes formula.

Similarly, the formula for a European put option on a non – dividend paying stock is derived by computing:

$$P(S, K, \sigma, r, t, \delta) = e^{-rt} E^*(K - S | K > S) \times \text{Prob}^*(K > S)$$

This can be rewritten as:

$$\begin{aligned}
 P(S, K, \sigma, r, t, \delta) &= e^{-rt} E^*(K | K > S) \times \text{Prob}^*(K > S) \\
 &\quad - e^{-rt} E^*(S | K > S) \times \text{Prob}^*(K > S)
 \end{aligned}$$

Substituting  $\alpha$  with  $r$ , this becomes the Black – Scholes formula:

$$P(S, K, \sigma, r, t, \delta) = K e^{-rt} N(-d_2) - e^{-\delta t} S N(-d_1)$$

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 18, pp. 604-605*

## 185. Explain Monte Carlo Valuation of a European Call

A lognormal stock price can be written as:

$$S_T = S_0 e^{(\alpha - \delta - \frac{1}{2}\sigma^2)T + \sigma\sqrt{T}Z}$$

Suppose we wish to draw random stock prices for 2 years from today. From lognormal stock price equation the stock price is driven by the normally distributed random variable  $Z$ . Set  $T = 2$ ,  $\alpha = 0.10$ ,  $\delta = 0$ , and  $\sigma = 0.30$ . If we then randomly draw a set of standard normal  $Z$ 's and substitute the results into lognormal stock price, the result is a random set of lognormally distributed  $S_2$ 's. The continuously compounded mean return will be 20% (10% per year) and the continuously compounded standard deviation of  $\ln(S_2)$  will be  $0.3 \times \sqrt{2} = 42.43\%$ .

In Monte Carlo valuation, we perform similar calculation to that above for lognormal stock price. The option payoff at time  $T$  is a function of the stock price,  $S_T$ . Representing this payoff as  $V(S_T, T)$ . The time – 0 Monte Carlo prices,  $V(S_0, 0)$  is then:

$$V(S_0, 0) = \frac{1}{n} e^{-rT} \sum_{i=1}^n V(S_T^i, T)$$

Where  $S_1^i, \dots, S_T^i$  are  $n$  randomly drawn time –  $T$  stock prices. For the case of a call option, for example,  $V(S_T^i, T) = \max(0, S_T^i - K)$ .

As an illustration of Monte Carlo techniques, let's consider pricing of a European call option. We assume that the underlying stock follows lognormal distribution with  $\alpha = r$ . We generate random standard normal variables,  $Z$ , substitute them into equation for lognormal stock price and generate many random future stock prices. Each  $Z$  creates one trial. Suppose we compute  $N$  trials. For each trial,  $i$ , we compute the value of a call as:

$$\max(0, S_T^i - K) = \max\left(0, S_0 e^{(r-\delta-0.5\sigma^2)T + \sigma\sqrt{T}Z_i} - K\right); \quad i = 1, \dots, N$$

Average the resulting values:

$$\frac{1}{N} \sum_{i=1}^N \max(0, S_T^i - K)$$

This expression gives us an estimate of the expected option payoff at time  $T$ . We discount the average payoff back at the risk-free rate in order to get an estimate of the option value, getting:

$$\bar{C} = e^{-rT} \frac{1}{N} \sum_{i=1}^N \max(0, S_T^i - K)$$

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 19, pp. 623-626*

## 186. Explain Control Variate Method for Monte Carlo Valuation

"Naive" Monte Carlo is often used term for describing simplest simulation without making an attempt to reduce the variance of the simulated answer for a given number of trials. There are a number of methods to achieve faster Monte Carlo valuations.

Control Variate Method is use to increase Monte Carlo accuracy. The idea underlying this method is to estimate the error on each trial by using the price of a related option that does have a pricing formula. The error estimate obtained from this control price can be used to improve the accuracy of the Monte Carlo price on each trial.

To be specific, we use simulation to estimate the arithmetic price,  $A'$ , and the geometric price,  $G'$ . Let  $G$  and  $A$  represent the true geometric and arithmetic prices. The error for the Monte Carlo estimate of the geometric price is  $(G - G')$ . We want to use this error to improve our estimate of the arithmetic price. Consider calculating:



$$A^* = \bar{A} + (G - \bar{G})$$

This is a control variate estimate. Since Monte Carlo provides an unbiased estimate,  $E(G') = G$ . Hence,  $E(A^*) = E(A) = A$ . Moreover, the variance of  $A^*$  is:

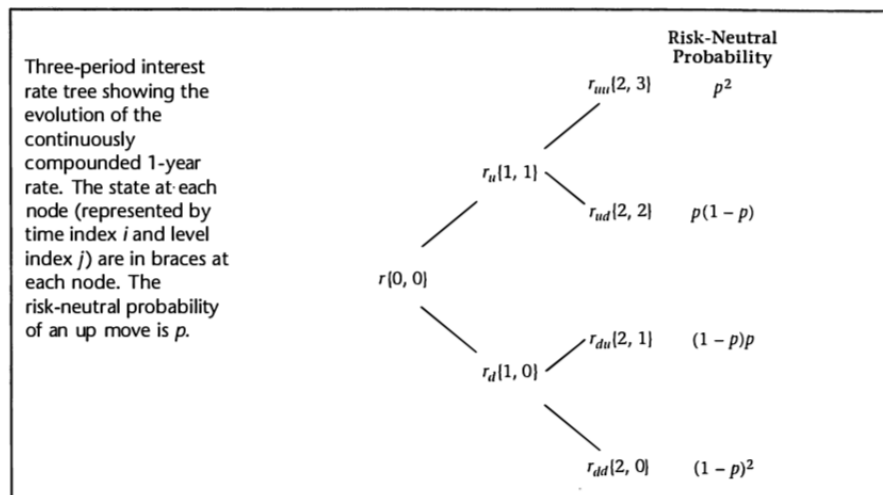
$$\text{Var}(A^*) = \text{Var}(\bar{A}) + \text{Var}(\bar{G}) - 2\text{Cov}(\bar{A}, \bar{G})$$

As long as the estimate  $G'$  is highly correlated with the estimate  $A'$ , the variance of the estimate  $A^*$  can be less than the variance of  $A'$ . In practice, the variance reduction from the control variate method can be dramatic.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 19, pp. 624-626*

### 187. Construct a binomial interest rate model

Binomial interest rate models permit the interest rate to move randomly over time. One approach is to model the short-term rate, where the definition of short-term is,  $h$ , the length of the binomial period. Let's assume  $h=1$ . To construct a binomial tree of the 1-year rate, note that we can observe today's 1-year rate. We assume the 1-year rate moves up or down the second year, and again the third year. This behavior gives us the tree in Figure below, which is drawn so that it need not recombine.



The notation used for interest rate trees is  $r_{t_0}(t, T)$  as the forward interest rate at time  $t_0$  for time  $t$  to time  $T$ . This notation accounts for the fact that at a point in time, there is a set of forward interest rates at different future times ( $t$ ) and covering different times to maturity ( $T-t$ ). When  $t_0 = t$ ,  $r_t(t, T)$  is the set of current spot interest rates for different times to maturity.

At time 0 we can determine a bond price on the binomial tree in much the same way we determined option prices in a binomial stock-price tree. The one-period bond price at any

time is determined by discounting at the current one – period rate, which is given at each node:

$$P_i(i, i + 1; j) = e^{-r_i(i, i+1; j)h}$$

We can value a two – period bond by discounting the expected one – period bond price, one period hence. At any node we can value an  $n$  – period zero – coupon bond by proceeding in this way recursively. Beginning in period  $i + n$ , we value one – period bonds, then in period  $i + n - 1$  we have two – period bond values, and so forth.

For the one-period bond we have:

$$P_0(0, 1; 0) = e^{-rh}$$

The two – year bond is priced by working backward along the tree. In the second period, the price of the bond is \$1. One year from today, the bond will have the price  $e^{-ru}$  with probability  $p$  or  $e^{-rd}$  with probability  $1 - p$ . The price of the bond is therefore:

$$\begin{aligned} P_0(0, 2; 0) &= e^{-rh} [pe^{-ru} + (1 - p)e^{-rd}] \\ &= e^{-rh} [pP_1(1, 2; 1) + (1 - p)P_1(1, 2; 0)] \end{aligned}$$

Thus, we can price the 2 – year bond using either the interest rate tree or the implied bond prices. Finally, the 3 – year bond is again priced by traversing the entire tree:

$$\begin{aligned} P_0(0, 3; 0) &= e^{-r} [pe^{-ru} (pe^{-ru} + (1 - p)e^{-rd}) \\ &\quad + (1 - p)e^{-rd} (pe^{-rd} + (1 - p)e^{-rd})] \end{aligned}$$

The 3 – year bond calculation can be written differently. By collecting terms in above equation, we can rewrite it as:

$$\begin{aligned} P_0(0, 3; 0) &= p^2 e^{-(r+ru+ru^2)} + p(1 - p)e^{-(r+ru+ru^2)} \\ &\quad + (1 - p)pe^{-(r+rd+rd^2)} + (1 - p)^2 e^{-(r+rd+rd^2)} \end{aligned}$$

This version of the equation makes clear that we can value the bond by considering separately each *path* the interest rate can take. Each path implies a realized discount factor. We then compute the expected discount factor, using risk – neutral probabilities. Denoting this expectation as  $E^*$ , the value of the zero – coupon bond is:

$$E^* (e^{-(r_0+r_1+r_2)h})$$

More generally, letting  $r_i$  represent the time –  $i$  rate, we have:

$$E^* (e^{-\sum_{i=0}^n r_i h})$$

*All bond valuation models implicitly calculate this equation.*

### 188. Explain meaning of calibration

At any point in time we can observe the yield curve and the volatilities of bond options. Thus far we have ignored the important practical question of whether a particular interest rate model fits these data. For example, for any interest rate model, we can ask whether it correctly prices zero-coupon bonds (in which case it will correctly price forwards and swaps) and selected options. Matching a model to fit the data is called *calibration*.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 24, pp. 798-808*

### 189. Explain bond ratings and its transition

Bond ratings provide a measure of the credit risk for specific bonds. Such ratings, which are provided by third parties, attempt to measure the likelihood that a company will default on a bond. In the United States, the Securities and Exchange Commission (SEC) identifies specific credit – rating firms as Nationally Recognized Statistical Rating Organizations (NRSROs).

Moody's rates bonds using the designations Aaa, Aa, A, Baa, Ba, B, Caa, Ca, and C. Within each ratings category, bonds may be further rated as a 1, 2, or 3, with 1 denoting the highest quality within a category. Standard and Poor's and Fitch have a similar rating system, using the designations AAA, AA, A, BBB, BB, B, CCC, CC, C.

The market distinguishes between "investment grade" (a rating of Baa/BBB or above) and "below-investment grade" or "speculative grade" (a rating below Baa/BBB) bond. Some investors are permitted to hold only investment grade bonds, and some contracts have triggers based upon whether a company's bond rating is investment grade.

A company that goes bankrupt will typically have had ratings downgrades prior to bankruptcy. By looking at the frequency with which bonds experience a ratings change, it is possible to estimate the ultimate bankruptcy probability. A change in ratings is called a **ratings transition**.

The table below is a *ratings transition matrix*, reporting the probability that a firm in a given ratings category will switch to another ratings category over the course of a year. Firms rated Aaa, Aa, or A, all have about an 89% chance of retaining their rating over a one – year horizon, and almost no chance of suffering a default over that time. They do, however, have some chance of experiencing a downgrade, after which bankruptcy becomes likelier: The default probability increases as the rating decreases.

Moody's average 1-year credit ratings transition matrix, 1970 to 2004. "WR" stands for "withdrawn rating."

Rating	Count	Aaa	Aa	A	Baa	Ba	B	Caa-C	Default	WR
Aaa	3,179	89.48	7.05	0.75	0.00	0.03	0.00	0.00	0.00	2.69
Aa	11,310	1.07	88.41	7.35	0.25	0.07	0.01	0	0	2.83
A	22,981	0.05	2.32	88.97	4.85	0.46	0.12	0.01	0.02	3.19
Baa	18,368	0.05	0.23	5.03	84.5	4.6	0.74	0.15	0.16	4.54
Ba	12,702	0.01	0.04	0.46	5.28	78.88	6.48	0.5	1.16	7.19
B	10,794	0.01	0.03	0.12	0.4	6.18	77.45	2.93	6.03	6.85
Caa-C	2,091	0	0	0	0.52	1.57	4	62.68	23.12	8.11

*Source: Hamilton et al. (2005).*

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 26, pp. 841-852*

# BANK RISK THEORY

*Course book: Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008*

## 190. What is the Interest Rate Risk?

In mismatching the maturities of assets and liabilities as part of their asset-transformation function, FIs potentially expose themselves to **interest rate risk**.

If FI has liabilities with maturity less than assets, it is said to be “short-funded”. After liabilities reach its maturity, FI will need to refinance them in order to match cash flows with assets. However as future is uncertain, there is always a risk, however, that interest rates will change. If interest rates were to rise and the FI will borrow at higher interest and profits will decline or become negative. Thus, as a result, when an FI holds longer-term assets relative to liabilities, it potentially exposes itself to **refinancing risk**. This is the risk that the cost of rolling over or reborrowing funds could be more than the return earned on asset investments.

Alternatively, if FI has assets with maturity less than its liabilities, FI is said to be “long-funded”. After assets reach its maturity, FI will need to reinvest them. However as future is uncertain, rates at which FI will need to reinvest them may fall, thus it faces **reinvestment risk**.

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 8, pp 195-196*

## 191. What is duration?

Duration is a more complete measure of an asset or liability’s interest rate sensitivity than is maturity because duration takes into account the time of arrival (or payment) of all cash flows as well as the asset’s (or liability’s) maturity.

Technically speaking, duration is the weighted-average time to maturity on the loan using the relative present values of the cash flows as weights. On a time value of money basis, duration measures the period of time required to recover the initial investment on the loan. Any cash flows received prior to the loan’s duration reflect recovery of the initial investment, while cash flows received after the period of the loan’s duration and before its maturity are the profits, or return, earned by the FI.

You can calculate the duration (or Macaulay’s duration<sup>4</sup>) for any fixed-income security using the following general formula:

$$D = \frac{\sum_{t=1}^N CF_t * DF_t * t}{\sum_{t=1}^N CF_t * DF_t} = \frac{\sum_{t=1}^N PV_t * t}{\sum_{t=1}^N PV_t}$$

A key assumption of the simple Macaulay duration model is that the yield curve or the term structure of interest rates is flat and that when rates change, the yield curve shifts in a parallel fashion. Further, the simple duration equation assumes that the issuer of a security or the borrower of a loan pays the interest and principal as promised, that is, the equation assumes no default risk.

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 9, pp 222-225*

## 192. What is the Economic Meaning of Duration? What is Modified duration

In addition to being a measure of the average life, in a cash flow sense, of an asset or liability, duration is also a direct measure of the interest rate sensitivity, or elasticity, of an asset or liability. In other words, the larger the numerical value of  $D$ , the more sensitive the price of that asset or liability is to changes or shocks in interest rates.

Consider the following equation showing that the current price of a bond that pays interest annually is equal to the present value of the coupons and principal payment on the bond:

$$P = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \dots + \frac{C+F}{(1+r)^N}$$

We want to find out how the price of the bond ( $P$ ) changes when yields ( $R$ ) rise. We know that bond prices fall, but we want to derive a direct measure of the size of this fall

$$\begin{aligned} \frac{dP}{dr} &= \frac{-C}{(1+r)^2} + \frac{-2C}{(1+r)^3} + \frac{-3C}{(1+r)^4} + \dots + \frac{-N(C+F)}{(1+r)^{N+1}} \\ \frac{dP}{dr} &= -\frac{1}{1+r} \left[ \frac{C}{1+r} + \frac{2C}{(1+r)^2} + \frac{3C}{(1+r)^3} + \dots + \frac{N(C+F)}{(1+r)^N} \right] \\ D &= \frac{\frac{1 \cdot C}{1+r} + \frac{2 \cdot C}{(1+r)^2} + \frac{3 \cdot C}{(1+r)^3} + \dots + \frac{N \cdot (C+F)}{(1+r)^N}}{P} \end{aligned}$$

$$\begin{aligned} D * P &= \frac{1 \cdot C}{1+r} + \frac{2 \cdot C}{(1+r)^2} + \frac{3 \cdot C}{(1+r)^3} + \dots + \frac{N \cdot (C+F)}{(1+r)^N} \\ \frac{dP}{dr} &= -\frac{1}{1+r} [D * P] \end{aligned}$$

Rearranging we obtain:

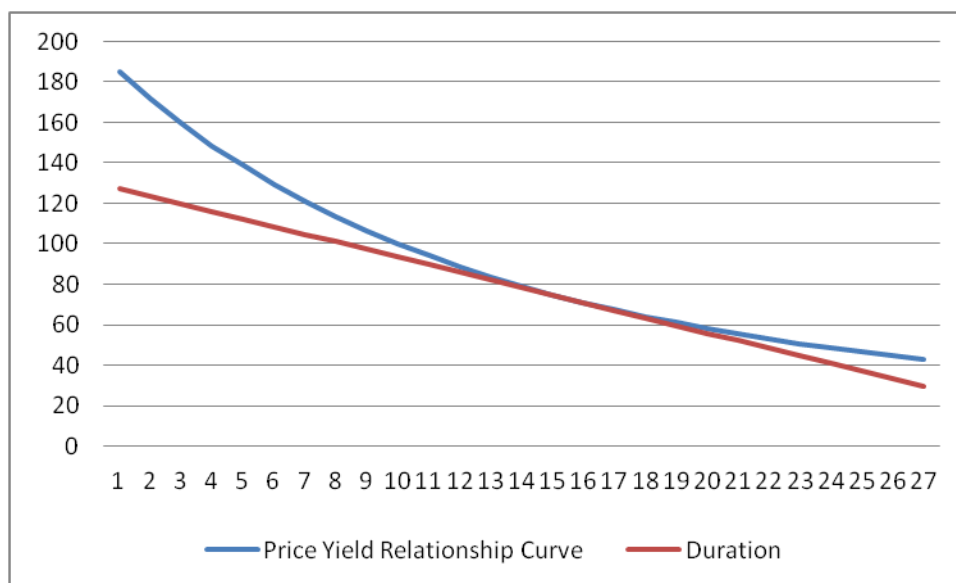
$$D = \frac{\frac{dP}{dr}}{\frac{P}{1+r}}$$

The economic interpretation of equation is that the number  $D$  is the interest elasticity, or sensitivity, of the security's price to small interest rate changes. That is,  $D$  describes the percentage price fall of the bond for any given (present value) increase in required interest rates or yields.

The duration equation can be rearranged, combining  $D$  and  $(1 + R)$  into a single variable  $D/(1 + R)$ , to produce what practitioners call modified duration (MD). For annual compounding of interest:

$$\frac{dP}{P} = -MD * dr$$

This form is more intuitive because we multiply MD by the simple change in interest rates rather than the discounted change in interest rates as in the general duration equation. Next, we use duration to measure the interest sensitivity of an asset or liability



*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 9, pp 230-233*

### 193. How can Duration model evaluate overall interest rate risk level?

The duration model can also evaluate the overall interest rate exposure for an FI, that is, measure the duration gap on its balance sheet.

To estimate the overall duration gap of an FI, we determine first the duration of an FI's asset portfolio (A) and the duration of its liability portfolio (L).

From the balance sheet:

$$A = L + E$$

and

$$\Delta A = \Delta L + \Delta E$$

That is, when interest rates change, the change in the FI's equity or net worth (E) is equal to the difference between the change in the market values of assets and liabilities on each side of the balance sheet. Since  $\Delta E = \Delta A - \Delta L$ , we need to determine how  $\Delta A$  and  $\Delta L$ —the changes in the market values of assets and liabilities on the balance sheet—are related to duration.

From the duration model (assuming annual compounding of interest):

$$\frac{\Delta A}{A} = -D_A * \frac{\Delta r}{1 + r}$$

$$\frac{\Delta L}{L} = -D_L * \frac{\Delta r}{1 + r}$$

Rearranging we obtain:

$$\Delta A = -D_A * A * \frac{\Delta r}{1 + r}$$

$$\Delta L = -D_L * L * \frac{\Delta r}{1 + r}$$

We can substitute these two expressions into the equation  $\Delta E = \Delta A - \Delta L$ .

Rearranging and combining this equation results in a measure of the change in the market value of equity on an FI's balance sheet for a change in interest rates:

$$\Delta E = -(D_A - kD_L) * A * \frac{\Delta r}{1 + r}$$

Where  $k = L/A$  is a measure of the FI's leverage, that is, the amount of borrowed funds or liabilities rather than owners' equity used to fund its asset portfolio. The effect of interest rate changes on the market value of an FI's equity or net worth ( $\Delta E$ ) breaks down into three effects:

- The leverage adjusted duration gap =  $[D_A - D_L k]$ . This gap is measured in years and reflects the degree of duration mismatch in an FI's balance sheet. Specifically, the larger this gap is in absolute terms, the more exposed the FI is to interest rate shocks.
- The size of the FI. The term  $A$  measures the size of the FI's assets. The larger the scale of the FI, the larger the dollar size of the potential net worth exposure from any given interest rate shock.
- The size of the interest rate shock =  $\Delta R/(1 + R)$ . The larger the shock, the greater the FI's exposure.

Given this, we express the exposure of the net worth of the FI as:

$$\Delta E = [\text{Leverage adjusted duration gap}] * \text{Asset size} * \text{Interest rate shock}$$

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 9, pp 238-240*



## 194. What is the Market Risk?

Market risk is the risk that the value of an investment will decrease due to moves in market factors such as interest rates, market volatility, and market liquidity. It cannot be eliminated through diversification, though it can be hedged against.

Conceptually, an FI's trading portfolio can be differentiated from its investment portfolio on the basis of time horizon and liquidity. The trading portfolio contains assets, liabilities, and derivative contracts that can be quickly bought or sold on organized financial markets (such as long and short positions in bonds, commodities, foreign exchange, equity securities, interest rate swaps, and options).

The investment portfolio (or, in the case of banks, the so-called banking book) contains assets and liabilities that are relatively illiquid and held for longer holding periods (such as consumer and commercial loans, retail deposits, and branches). Income from trading activities is increasingly replacing income from traditional FI activities of deposit taking and lending. The resulting earnings uncertainty, or market risk, can be measured over periods as short as a day or as long as a year.

Market risk is typically measured using a Value at Risk methodology. Value at risk is well established as a risk management technique. Moreover, market risk can be defined in absolute terms as a dollar exposure amount or as a relative amount against some benchmark.

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 10, pp. 266-268*

## 195. Describe Risk Metric Model for Fixed Income Securities

The ultimate objective of market risk measurement models can best be seen from the following quote by Dennis Weatherstone, former chairman of J. P. Morgan (JPM), now J. P. Morgan Chase: "At close of business each day tell me what the market risks are across all businesses and locations." In a nutshell, the chairman of J. P. Morgan wants a single dollar number at 4:15 pm New York time that tells him J. P. Morgan's market risk exposure the next day—especially if that day turns out to be an extremely "bad" day.

This is nontrivial, given the extent of JPM's trading business. When JPM developed its RiskMetrics Model in 1994 it had 14 active trading locations with 120 independent units trading fixed-income securities, foreign exchange, commodities, derivatives, emerging-market securities, and proprietary assets, with a total daily volume exceeding \$50 billion. This scale and variety of activities is typical of the major money centre banks, large overseas banks (e.g., Deutsche Bank and Barclays), and major insurance companies and investment banks.

Essentially, the FI is concerned with how to preserve equity if market conditions move adversely tomorrow; that is:

**Market risk Estimated = potential loss under adverse circumstances**

More specifically, the market risk is measured in terms of the FI's daily earnings at risk (DEAR) and has three components:

**DEAR = Dollar market value of position x Price sensitivity x Potential adverse move**

**DEAR = Dollar market value of position x Price volatility**

If the FI manager wants to know the potential exposure the FI faces should interest rates move against the FI as the result of an adverse or reasonably bad market move the next day. How much the FI will lose depends on the bond's price volatility. From the duration model we know that:

**Price volatility = Price sensitivity x Potential adverse move = (MD) x Adverse daily yield move**

**Modified Duration (MD) = Duration / (1+r)**

Therefore

**DEAR for Bonds = Dollar market value of position x (MD) x Adverse daily yield move**

$$\text{VaR} = \text{Dear} \sqrt{N}$$

Where N is number of days

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 10; pp.271-272*

## 196. Define Risk Metric Model for Foreign Exchange

If the FI wants to calculate the daily earnings at risk from this position (i.e., the risk exposure on this position should the next day be a bad day in the FX markets)

The first step is to calculate the dollar value of the position:

**Dollar equivalent value of position = (FX position) x (\$ per unit of foreign currency)**

Next step is to estimate volatility, or standard deviation of past spot exchange rates. However, FI might be interested in adverse changes that can happen only once in 20 days. Statistically speaking, if changes in exchange rates are historically "normally" distributed, the exchange rate must change in the adverse direction by  $1.65\sigma$ , for this change to be viewed as likely to occur only 1 day in every 20 days

**FX Volatility =  $1.65\sigma$**

As a result:

**DEAR = Dollar equivalent value of position x FX Volatility**

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 10; pp.272-273*

### 197. Describe Risk Metric Model for Stocks

Many large FIs also take positions in equities. As is well known from the Capital Asset Pricing Model (CAPM), there are two types of risk to an equity position in an individual stock  $i$ :

**Total Risk = Systematic Risk + Unsystematic Risk**

$$\sigma_{it}^2 = \beta_i^2 \sigma_{mt}^2 + \sigma_{eit}^2$$

Systematic risk reflects the co-movement of that stock with the market portfolio reflected by the stock's beta ( $\beta$ ) and the volatility of the market portfolio ( $\sigma_{mt}^2$ ), while unsystematic risk is specific to the firm itself ( $\sigma_{eit}^2$ )

In a very well diversified portfolio, unsystematic risk ( $\sigma_{eit}^2$ ) can be largely diversified away (i.e., will equal zero), leaving behind systematic (undiversifiable) market risk ( $\beta_i^2 \sigma_{mt}^2$ ). If the FI's trading portfolio follows (replicates) the returns on the stock market index, the  $\beta$  of that portfolio will be 1, since the movement of returns on the FI's portfolio will be one to one with the market, and the standard deviation of the portfolio,  $\sigma_{it}$ , will be equal to the standard deviation of the stock market index,  $\sigma_{mt}$ .

For portfolio consisting of stocks

**DEAR = Dollar value of position x  $\beta$  x Stock Market Volatility ( $\sigma_{mt}$ )**

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 10; pp.273-274*

### 198. Describe Market Risk Model for Aggregated Portfolio

For calculating DEAR of the aggregate portfolio consisting of Bonds, Stocks and FX, one must estimate Dollar Value of Positions, volatility for each of the components, and covariance or correlation among the fixed-income, FX, and equity trading positions. In particular, some of these asset shocks (adverse moves) may be negatively correlated. As is well known from modern portfolio theory, negative correlations among asset shocks will reduce the degree of portfolio risk.

$$DEAR = \left( \begin{aligned} & [(DEAR_A)^2 + (DEAR_B)^2 + (DEAR_C)^2]^{1/2} \\ & + (2 * \rho_{AB} * DEAR_A * DEAR_B) \\ & + (2 * \rho_{AC} * DEAR_A * DEAR_C) \\ & + (2 * \rho_{BC} * DEAR_B * DEAR_C) \end{aligned} \right)$$

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 10; pp. 274-275*

### **199. Describe Historic Simulation Approach for measuring risk**

A major criticism of RiskMetrics is the need to assume a symmetric (normal) distribution for all asset returns. Clearly, for some assets, such as options and short-term securities (bonds), this is highly questionable. For example, the most an investor can lose if he or she buys a call option on equity is the call premium; however, the investor's potential upside returns are unlimited. In a statistical sense, the returns on call options are not normal since they exhibit a positive skew. Because of these and other considerations discussed below, the large majority of FIs that have developed market risk models have employed a historic or back simulation approach.

The advantages of this approach are that (1) it is simple, (2) it does not require that asset returns be normally distributed, and (3) it does not require that the correlations or standard deviations of asset returns be calculated. The essential idea is to take the current market portfolio of assets (FX, bonds, equities, etc.) and revalue them on the basis of the actual prices (returns) that existed on those assets yesterday, the day before that, and so on. Frequently, the FI will calculate the market or value risk of its current portfolio on the basis of prices (returns) that existed for those assets on each of the last 500 days. It will then calculate the 5 percent worst case—the portfolio value that has the 25th lowest value out of 500. That is, on only 25 days out of 500, or 5 percent of the time, would the value of the portfolio fall below this number based on recent historic experience of exchange rate changes, equity price changes, interest rate changes, and so on.

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 10; pp. 277-281*

### **200. What is Credit Risk?**

Credit risk is an investor's risk of loss arising from a borrower who does not make payments as promised. Such an event is called a default. Other terms for credit risk are default risk and counterparty risk.

Credit quality problems, in the worst case, can cause an FI to become insolvent or can result in such a significant drain on capital and net worth that they adversely affect its growth prospects and ability to compete with other domestic and international FIs. However, credit risk does not apply only to traditional areas of lending and bond investing. As banks and other FIs have expanded into credit guarantees and other off-balance-sheet activities, new types of credit risk exposure have arisen, causing concern among managers and regulators. Thus, credit risk analysis is now important for a whole variety of contractual agreements between FIs and counterparties.

To calibrate the default risk exposure of credit and investment decisions as well as to assess the credit risk exposure in off-balance-sheet contractual arrangements such as loan

commitments, an FI manager needs to measure the probability of borrower default. The ability to do this depends largely on the amount of information the FI has about the borrower. At the retail level, much of the information needs to be collected internally or purchased from external credit agencies. At the wholesale level, these information sources are bolstered by publicly available information, such as certified accounting statements, stock and bond prices, and analysts' reports. Thus, for a publicly traded company, more information is produced and is available to an FI than is available for a small, single-proprietor corner store. The availability of more information, along with the lower average cost of collecting such information, allows FIs to use more sophisticated and usually more quantitative methods in assessing default probabilities for large borrowers compared with small borrowers. However, advances in technology and information collection are making quantitative assessments of even smaller borrowers increasingly feasible and less costly.

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 11, pp. 295-305*

## 201. Describe Credit Scoring Model

Credit scoring models are quantitative models that use observed borrower characteristics either to calculate a score representing the applicant's probability of default or to sort borrowers into different default risk classes. By selecting and combining different economic and financial borrower characteristics, an FI manager may be able to:

- Numerically establish which factors are important in explaining default risk.
- Evaluate the relative degree or importance of these factors.
- Improve the pricing of default risk.
- Be better able to screen out bad loan applicants.
- Be in a better position to calculate any reserves needed to meet expected future loan losses.

The primary benefit from credit scoring is that credit lenders can more accurately predict a borrower's performance without having to use more resources.

To use credit scoring models, the manager must identify objective economic and financial measures of risk for any particular class of borrower. For consumer debt, the objective characteristics in a credit scoring model might include income, assets, age, occupation, and location. For commercial debt, cash flow information and financial ratios such as the debt–equity ratio are usually key factors. After data are identified, a statistical technique quantifies, or scores, the default risk probability or default risk classification.

Credit scoring models include these three broad types: (1) linear probability models, (2) logit models, and (3) linear discriminant analysis.

The linear probability model uses past data, such as financial ratios, as inputs into a model to explain repayment experience on old loans. The relative importance of the factors used in explaining past repayment performance then forecasts repayment probabilities on new

loans. That is, factors explaining past repayment performance can be used for assessing  $p$ , the probability of repayment discussed earlier in this chapter (a key input in setting the credit premium on a loan or determining the amount to be lent) and the probability of default (PD). Briefly, we divide old loans ( $i$ ) into two observational groups: those that defaulted ( $PD_i = 1$ ) and those that did not default ( $PD_i = 0$ ). Then we relate these observations by linear regression to a set of  $j$  causal variables ( $X_{ij}$ ) that reflect quantitative information about the  $i^{\text{th}}$  borrower, such as leverage or earnings. We estimate the model by linear regression of this form:

$$PD_i = \sum_{j=1}^N \beta_j X_{ij} + \text{Error}$$

Where  $\beta_j$  is the estimated importance of the  $j^{\text{th}}$  variable (e.g., leverage) in explaining past repayment experience.

If we then take these estimated  $\beta_j$ s and multiply them by the observed  $X_{ij}$  for a prospective borrower, we can derive an expected value of  $PD_i$  for the prospective borrower. That value can be interpreted as the probability of default for the borrower:  $E(PD_i) = (1 - p_i)$  expected probability of default, where  $p_i$  is the probability of repayment on the loan.

While this technique is straightforward as long as current information on the  $X_{ij}$  is available for the borrower, its major weakness is that the estimated probabilities of default can often lie outside the interval 0 to 1. The logit model overcomes this weakness by restricting the estimated range of default probabilities from the linear regression model to lie between 0 and 1. Essentially this is done by plugging the estimated value of  $PD_i$  from the linear probability model into the following formula:

$$F(PD_i) = \frac{1}{1 + e^{-PD_i}}$$

Where  $e$  is exponential (equal to 2.718) and  $F(PD_i)$  is the logistically transformed value of  $PD_i$

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 11, pp. 316-319*

## 202. Describe loan portfolio diversification model using Modern Portfolio Theory

To the extent that an FI manager holds widely traded loans and bonds as assets or, alternatively, can calculate loan or bond returns, portfolio diversification models can be used to measure and control the FI's aggregate credit risk exposure. Suppose the manager can estimate the expected returns of each loan or bond ( $\overline{R}_i$ ) in the FI's portfolio.

After calculating the individual security return series, the FI manager can compute the expected return ( $\overline{R}_p$ ) on a portfolio of assets as:

$$\bar{R}_p = \sum_{i=1}^N X_i \bar{R}_i$$

In addition, the variance of returns or risk of the portfolio ( $\sigma_p^2$ ) can be calculated as:

$$\sigma_p^2 = \sum_{i=1}^N X_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ i \neq j}}^N X_i X_j \rho_{ij} \sigma_i \sigma_j$$

where

$R_p$  = Expected or mean return on the asset portfolio

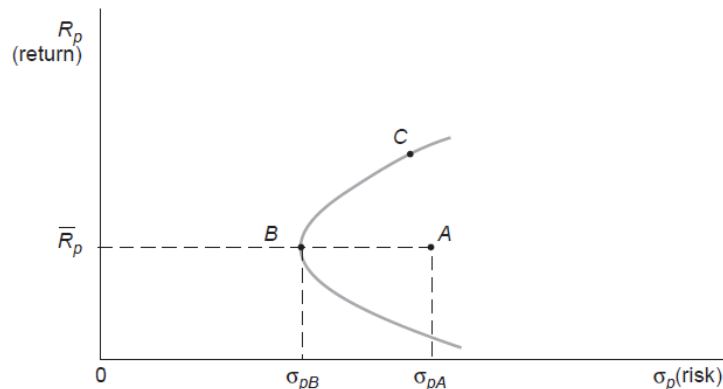
$\bar{R}_i$  = Mean return on the  $i$ th asset in the portfolio

$X_i$  = Proportion of the asset portfolio invested in the  $i$ th asset (the desired concentration amount)

$\sigma_i^2$  = Variance of returns on the  $i$ th asset

$\rho_{ij}$  = Correlation between the returns on the  $i^{\text{th}}$  and  $j^{\text{th}}$  assets

The fundamental lesson of modern portfolio theory (MPT) is that by taking advantage of its size, an FI can diversify considerable amounts of credit risk as long as the returns on different assets are imperfectly correlated with respect to their default risk adjusted returns. Consider the  $\sigma_p^2$  in equation. If many loans have negative covariances or correlations of returns ( $\rho_{ij}$  are negative)—that is, when one borrower's loans do badly and another's do well—the sum of the individual credit risks of loans viewed independently overestimates the risk of the whole portfolio.



Note that A is an undiversified portfolio with heavy investment concentration in just a few loans or bonds. By fully exploiting diversification potential with bonds or loans whose returns are negatively correlated or that have a low positive correlation with those in the existing portfolio, the FI manager can lower the credit risk on the portfolio from  $\sigma_{pA}$  to  $\sigma_{pB}$  while earning the same expected return. That is, portfolio B is the efficient (lowest-risk) portfolio associated with portfolio return level  $R_p$ . By varying the proportion of the asset portfolio invested in each asset (in other words, by varying the required portfolio return level  $R_p$  up and down), the manager can identify an entire frontier of efficient portfolio mixes (weights) of

loans and bonds. Each portfolio mix is efficient in the sense that it offers the lowest risk level to the FI manager at each possible level of portfolio returns. However, as you can see in Figure, of all possible efficient portfolios that can be generated, portfolio B produces the lowest possible risk.

Level for the FI manager. That is, it maximizes the gains from diversifying across all available loans and bonds so that the manager cannot reduce the risk of the portfolio below  $\sigma_{pB}$ . For this reason,  $\sigma_{pB}$  is usually labelled the minimum risk portfolio.

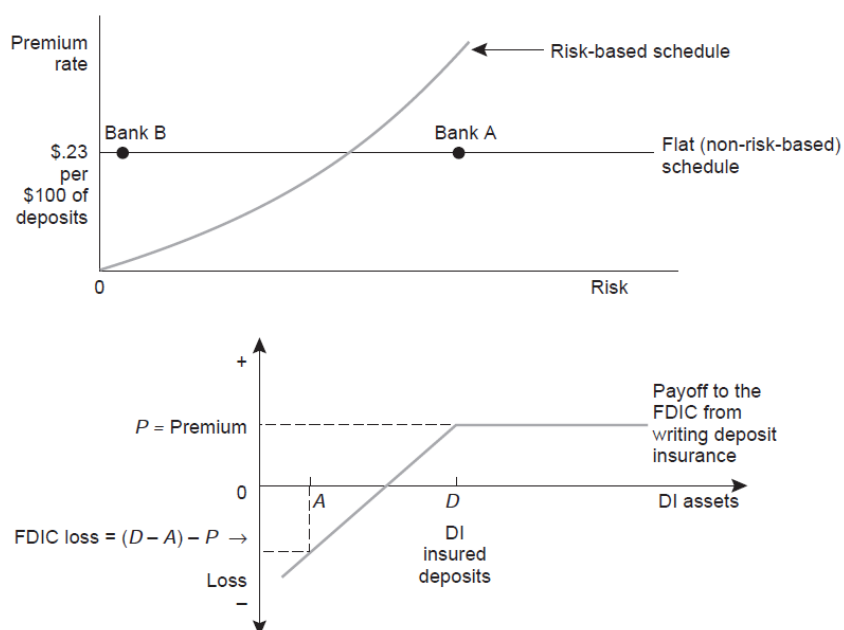
*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 12, pp. 350-353*

### **203. Describe Option Pricing Model for Deposit Institutions**

Economists have suggested a number of approaches for calculating the fair deposit insurance premium that a cost-minimizing insurer such as the FDIC (Federal Deposit Insurance Committee) should charge. One approach would be to set the premium equal to the expected severity of loss times the frequency of losses due to DI failure plus some load or mark-up factor. This would exactly mimic the approach toward premium setting in the property-casualty industry. However, the most common approach, the option pricing model of deposit insurance (OPM), has been to view the FDIC's provision of deposit insurance as virtually identical to the FDIC's writing a put option on the assets of the DI that buys the deposit insurance. We depict the conceptual idea underlying the option pricing model approach in Figure below.

In this framework, the FDIC charges the DI a premium  $P$  to insure the DI's deposits ( $D$ ). If the DI does well and the market value of the DI's assets is greater than  $D$ , its net worth is positive and it can continue in business. The FDIC would face no charge against its resources and would keep the premium paid to it by the DI ( $P$ ). If the DI is insolvent, possibly because of a bad or risky asset portfolio, such that the value of the DI's assets falls below  $D$  (say to  $A$ ), and its net worth is negative, the DI owners will "put the bank" back to the FDIC. If this happens, the FDIC will pay out to the insured depositors an amount  $D$  and will liquidate the DI's assets ( $A$ ). As a result, the FDIC bears the cost of the insolvency (or negative net worth) equal to  $(D - A)$  minus the insurance premiums paid by the DI ( $P$ ).





When valued in this fashion as a simple European put option, the FDIC's cost of providing deposit insurance increases with the level of asset risk ( $\sigma_A^2$ ) and with the DI's leverage ( $D/A$ ). That is, the actuarially fair premium ( $P$ ) is equivalent to the premium on a put option and as such should be positively related to both asset risk ( $\sigma_A^2$ ) and leverage risk ( $D/A$ ). The value of a deposit insurance guaranty is the same as the Black-Scholes model for a European put option of maturity  $T$  (where  $T$  is the time period until the next premium assessment):

$$P(T) = D e^{-rT} \phi(X_2) - A \phi(X_1)$$

Where

$$X_1 = \left[ \log\left(\frac{D}{A}\right) - \left(r + \frac{\sigma_A^2}{2}\right)T \right] / \sigma_A \sqrt{T}$$

$$X_2 = X_1 + \sigma_A \sqrt{T}$$

and  $\phi$  is the standard normal distribution.

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 19, pp. 557-561*

## 204. Describe three pillars that Basel II Accord consists of

The new Basel Accord or Agreement (called Basel II) consists of three mutually reinforcing pillars, which together contribute to the safety and soundness of the financial

system. Pillar 1 covers regulatory minimum capital requirements for credit, market, and operational risk. In the 2006 Accord, the BIS allows for a range of options for addressing both credit and operational risk. Two options are for the measurement of credit risk. The first is the Standardized Approach, and the second is an Internal Ratings–Based (IRB) Approach. Under the Standardized Approach, banks assets are weighted according to their riskiness and certain percentage (8% generally) is taken as required capital. Under the IRB Approach, banks are allowed to use their internal estimates of borrower creditworthiness to assess credit risk in their portfolios (using their own internal rating systems and credit scoring models) subject to strict methodological and disclosure standards, as well as explicit approval by the bank’s supervising regulator. Three different approaches are available to measure operational risk: the Basic Indicator, Standardized, and Advanced Measurement approaches.

In Pillar 2, the BIS stress the importance of the regulatory supervisory review process as a critical complement to minimum capital requirements. Specifically Basel II created procedures through which regulators ensure that each bank has sound internal processes in place to assess the adequacy of its capital and set targets for capital that are commensurate with the bank’s specific risk profile and control environment. In Pillar 3, the BIS sought to encourage market discipline by developing a set of requirements on the disclosure of capital structure, risk exposures, and capital adequacy. Such disclosure requirements allow market participants to assess critical information describing the risk profile and capital adequacy of banks.

Pillar 1	Pillar 2	Pillar 3
Calculation of regulatory minimum capital requirements	Regulatory supervisory review so as to complement and enforce minimum capital requirements calculated under Pillar 1	Requirements on rules for disclosure of capital structure, risk exposures, and capital adequacy so as to increase FI transparency and enhance market/investor discipline
1. Credit risk: on-balance-sheet and off-balance-sheet (Standardized vs. Internal Ratings–Based Approach)		
2. Market risk (Standardized vs. Internal Ratings–Based Approach)		
3. Operational risk (Basic Indicator vs. Standardized vs. Advanced Measurement Approach)		

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 20, pp. 598-599*

### 205. How do you calculate TIER I and TIER II Capital?

A bank's capital is divided into Tier I and Tier II. Tier I capital is primary or core capital; Tier II capital is supplementary capital. The total capital that the bank holds is defined as the sum of Tier I and Tier II capitals.

**Tier I Capital** Tier I capital is closely linked to a bank's book value of equity, reflecting the concept of the core capital contribution of a bank's owners.<sup>9</sup> Basically, it includes the book value of common equity plus an amount of perpetual (non-maturing) preferred stock plus minority equity interests held by the bank's subsidiaries minus goodwill. Goodwill is an accounting item that reflects the amount a bank pays above market value when it purchases or acquires other banks or subsidiaries.

**Tier II Capital** Tier II capital is a broad array of secondary capital resources. It includes a bank's loan loss reserves up to a maximum of 1.25 percent of risk adjusted assets plus various convertible and subordinated debt instruments with maximum caps.

To be adequately capitalized, a bank must hold a minimum ratio of total capital (Tier I core capital plus Tier II supplementary capital) to credit risk-adjusted assets of 8 percent; that is, its **total risk-based capital ratio** is calculated as:

$$\text{Total risk – based capital ratio} = \frac{\text{Total capital (Tier I + Tier II)}}{\text{Credit risk – adjusted assets}} \geq 8\%$$

In addition, the Tier I core capital component of total capital has its own minimum guideline. The **Tier I (core) capital ratio** is calculated as:

$$\text{Tier I (core) capital ratio} = \frac{\text{Core Capital (Tier I)}}{\text{Credit risk – adjusted assets}} \geq 4\%$$

That is, of the 8 percent total risk-based capital ratio, a minimum of 4 percent has to be held in core or primary capital.

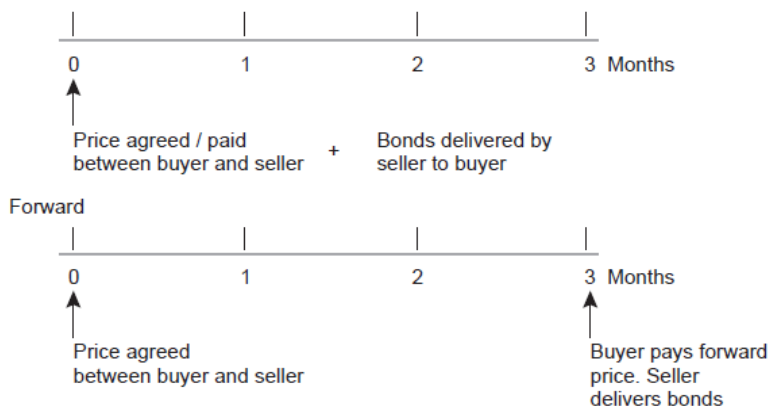
*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 20, p. 601*

### 206. Describe spot contracts and forward contracts

A **spot contract** is an agreement between a buyer and a seller at time 0, when the seller of the asset agrees to deliver it immediately and the buyer of the asset agrees to pay for that asset immediately.<sup>3</sup> Thus, the unique feature of a spot market contract is the immediate and simultaneous exchange of cash for securities, or what is often called delivery versus payment.

A **forward contract** is a contractual agreement between a buyer and a seller at time 0 to exchange a pre specified asset for cash at a later date. For example, in a three month forward contract to deliver 20-year bonds, the buyer and seller agree on a price and quantity today (time 0) but the delivery (or exchange) of the 20-year bond for cash does not occur until three months hence. If the forward price agreed to at time 0 was \$97 per \$100 of face value, in three months' time the seller delivers \$100 of 20-year bonds and receives \$97 from the buyer.

This is the price the buyer must pay and the seller must accept no matter what happened to the spot price of 20-year bonds during the three months between the time the contract was entered into and the time the bonds are delivered for payment.



*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 23, p. 693*

## 207. Describe Naïve Hedging strategy

To see the usefulness of forward contracts in hedging the interest rate risk of an FI, consider a simple example of a **naïve hedge** (the hedge of a cash asset on a direct dollar-for-dollar basis with a forward or futures contract). Suppose an FI portfolio manager holds a 20-year, \$1 million face value bond on the balance sheet. At time 0, these bonds are valued by the market at \$97 per \$100 face value, or \$970,000 in total. Assume the manager receives a forecast that interest rates are expected to rise by 2 percent from their current level of 8 to 10 percent over the next three months. Knowing that rising interest rates mean that bond prices will fall, the manager stands to make a capital loss on the bond portfolio. Having read Chapters 8 and 9, the manager is an expert in duration and has calculated the 20-year maturity bonds' duration to be exactly 9 years. Thus, the manager can predict a capital loss, or change in bond values ( $\Delta P$ ), from the duration equation:

$$\frac{\Delta P}{P} = -D * \frac{\Delta R}{1 + R}$$

where

$\Delta P$  - capital loss on bonds

$P$  - Initial value of bond position = 970 000

$D$  - Duration of the bonds = 9 Years

$\Delta R$  - change in forecast yield = .02

$$\frac{\Delta P}{970\,000} = -9 * \frac{0.02}{1.08}$$

$$\Delta P = -\$161\,666.67$$

As a result, the FI portfolio manager expects to incur a capital loss on the bond portfolio of \$161,666.67 (as a percentage loss  $(\Delta P/P)=16.67\%$ ) or as a drop in price from \$97 per \$100 face value to \$80.833 per \$100 face value. To offset this loss—in fact, to reduce the risk of capital loss to zero—the manager may hedge this position by taking an off-balance-sheet hedge, such as selling \$1 million face value of 20-year bonds for forward delivery in three months' time. 8 Suppose at time 0 the portfolio manager can find a buyer willing to pay \$97 for every \$100 of 20-year bonds delivered in three months' time. Now consider what happens to the FI portfolio manager if the gloomy forecast of a 2 percent rise in interest rates proves to be true. The portfolio manager's bond position has fallen in value by 16.67 percent, equal to a capital loss of \$161,667. After the rise in interest rates, the manager can buy \$1 million face value of

20-year bonds in the spot market at \$80.833 per \$100 of face value, a total cost of \$808,333, and deliver these bonds to the forward contract buyer. Remember that the forward contract buyer agreed to pay \$97 per \$100 of face value for the \$1 million of face value bonds delivered, or \$970,000. As a result, the portfolio manager makes a profit on the forward transaction of

$$970\,000 - 808\,333 = 161\,667$$

Price paid from forward buyer to forward seller	cost of purchasing Bonds in the spot market at $t = \text{month } 3$ for delivery to the forward buyer
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As you can see, the on-balance-sheet loss of \$161,667 is exactly offset by the off-balance-sheet gain of \$161,667 from selling the forward contract. In fact, for any change in interest rates, a loss (gain) on the balance sheet is offset by a gain (loss) on the forward contract. Indeed, the success of a hedge does not hinge on the manager's ability to accurately forecast interest rates. Rather, the reason for the hedge is the lack of ability to perfectly predict interest rate changes. The hedge allows the FI manager to protect against interest rate changes even if they are unpredictable. Thus, the FI's net interest rate exposure is zero; in the parlance of finance, it has immunized its assets against interest rate risk.

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett;  
McGraw-Hill, 2008; Chapter 23, pp. 696-697*

## 208. Describe MicroHedging strategy

An FI is **microhedging** when it employs a futures or a forward contract to hedge a particular asset or liability risk. For example, earlier we considered a simple example of microhedging asset-side portfolio risk, where an FI manager wanted to insulate the value of the institution's bond portfolio fully against a rise in interest rates. An example of microhedging on the liability side of the balance sheet occurs when an FI, attempting to lock in a cost of funds to protect itself against a possible rise in short-term interest rates, takes a short (sell) position in futures contracts on CDs or T-bills. In microhedging, the FI manager often tries to pick a futures or forward contract whose underlying deliverable asset is closely matched to the asset (or liability) position being hedged. The earlier example, where we had an exact matching of the asset in the portfolio with the deliverable security underlying the forward contract (20-year bonds) was unrealistic. Such exact matching cannot be achieved often, and this produces a residual unhedgable risk termed basis risk. We discuss basis risk in detail later in this chapter; it arises mainly because the prices of the assets or liabilities that an FI wishes to hedge are imperfectly correlated over time with the prices on the futures or forward contract used to hedge risk.

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 23, pp. 697*

## 209. Describe MacroHedging strategy with futures

**Macrohedging** occurs when an FI manager wishes to use futures or other derivative securities to hedge the entire balance sheet duration gap. This contrasts to microhedging, where an FI manager identifies specific assets and liabilities and seeks individual futures and other derivative contracts to hedge those individual risks. Note that macrohedging and microhedging can lead to quite different hedging strategies and results. In particular, a macrohedge takes a whole portfolio view and allows for individual asset and liability interest sensitivities or durations to net each other out. This can result in a very different aggregate futures position than when an FI manager disregards this netting or portfolio effect and hedges individual asset and liability positions on a one-to-one basis.

The number of futures contracts that an FI should buy or sell in a macrohedge depends on the size and direction of its interest rate risk exposure and the return risk trade-off from fully or selectively hedging that risk. FI's net worth exposure to interest rate shocks was directly related to its leverage adjusted duration gap as well as its asset size. Again, this is:

$$\Delta E = -[D_A - kD_L] * A * \frac{\Delta R}{1 + R}$$

Where

$\Delta E$  - Change in an FI's net worth

$D_A$ - Duration of its asset portfolio

$D_L$ - Duration of its liability portfolio

$k$  - Ratio of an FI's liabilities to assets ( $L/A$ )

$A$  - Size of an FI's asset portfolio

$\frac{\Delta R}{1+R}$  - Shock to interest rates

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 23, pp. 698-699*

## 210. What problem arises with Basis Risk

Because spot bonds and futures on bonds are traded in different markets, the shifting yields,  $\Delta R/(1+R)$ , affecting the values of the on-balance-sheet cash portfolio may differ from the shift in yields,  $\Delta R_F/(1+R_F)$ , affecting the value of the underlying bond in the futures contract; that is, changes in spot and futures prices or values are not perfectly correlated. This lack of perfect correlation is called basis risk. In the previous section, we assumed a simple world of no basis risk in which  $\Delta R/(1+R) = \Delta R_F/(1+R_F)$ .

Basis risk occurs for two reasons. First, the balance sheet asset or liability being hedged is not the same as the underlying security on the futures contract. For instance, in Example 23–2 we hedged interest rate changes on the FI's entire balance sheet with T-bond futures contracts written on 20-year maturity bonds with duration of 9.5 years. The interest rates on the various assets and liabilities on the FI's balance sheet and the interest rates on 20-year T-bonds do not move in a perfectly correlated (or one-to-one) manner. The second source of basis risk comes from the difference in movements in spot rates versus futures rates. Because spot securities (e.g., government bonds) and futures contracts (e.g., on the same bonds) are traded in different markets, the shift in spot rates may differ from the shift in futures rates (i.e., they are not perfectly correlated). To solve for the risk-minimizing number of futures contracts to buy or sell,  $N_F$ , while accounting for greater or less rate volatility and hence price volatility in the futures market relative to the spot or cash market, we look again at the FI's on-balance-sheet interest rate exposure:

$$\Delta E = -[D_A - kD_L] * A * \frac{\Delta R}{1 + R}$$

and its off-balance-sheet futures position:

$$\Delta F = -D_F [N_F - kP_F] * \frac{\Delta R_F}{1 + R_F}$$

Setting

$$\Delta E = \Delta F$$

and solving for  $N_F$ , we have:

$$N_F = \frac{[D_A - kD_L] * A * \frac{\Delta R}{1+R}}{D_F * P_F * \frac{\Delta R_F}{1+R_F}}$$

Let  $br$  reflect the relative sensitivity of rates underlying the bond in the futures market relative to interest rates on assets and liabilities in the spot market, that is,  $br = [\Delta R_F / (1 + R_F)] / [\Delta R / (1 + R)]$ . Then the number of futures contracts to buy or sell is:

$$N_F = \frac{[D_A - kD_L] * A}{D_F * P_F * br}$$

The only difference between this and the previous formula is an adjustment for basis risk ( $br$ ), which measures the degree to which the futures price (yield) moves more or less than spot bond price (yield).

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 23, p. 707*

## 211. Describe strategy of Credit Risk hedging with forwards

A credit forward is a forward agreement that hedges against an increase in default risk on a loan (a decline in the credit quality of a borrower) after the loan rate is determined and the loan is issued. Common buyers of credit forwards are insurance companies and common sellers are banks. The credit forward agreement specifies a credit spread (a risk premium above the risk-free rate to compensate for default risk) on a benchmark bond issued by an FI borrower. For example, suppose the benchmark bond of a bank borrower was rated BBB at the time a loan was originated. Further, at the time the loan was issued, the benchmark bonds had a 2 percent interest rate or credit spread (representing default risk on the BBB bonds) over a U.S. Treasury bond of the same maturity. To hedge against an increase in the credit risk of the borrower, the bank enters into (sells) a credit forward contract when the loan is issued. We define  $CS_F$  as the credit spread over the U.S. Treasury rate on which the credit forward contract is written (equals 2 percent in this example). Table below illustrates the payment pattern resulting from this credit forward. In Table below,  $CS_T$  is the actual credit spread on the bond when the credit forward matures, for example, one year after the loan was originated and the credit forward contract was entered into,  $MD$  is the modified duration on the benchmark BBB bond, and  $A$  is the principal amount of the forward agreement.

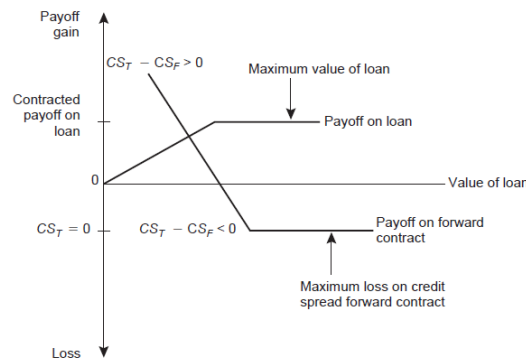
Credit Spread at End of Forward Agreement	Credit Spread Seller (Bank)	Credit Spread Buyer (Counterparty)
$CS_T > CS_F$	Receives $(CS_T - CS_F) \times MD \times A$	Pays $(CS_T - CS_F) \times MD \times A$
$CS_F > CS_T$	Pays $(CS_T - CS_F) \times MD \times A$	Receives $(CS_T - CS_F) \times MD \times A$

From the payment pattern established in the credit forward agreement, table shows that the credit forward buyer (an insurance company) bears the risk of an increase in default risk on the benchmark bond of the borrowing firm, while the credit forward seller (the bank lender) hedges itself against an increase in the borrower's default risk. That is, if the borrower's default risk increases so that when the forward agreement matures the market requires a higher credit spread on the borrower's benchmark bond,  $CS_T$ , than that originally agreed to in the forward contract,  $CS_F$  (i.e.,  $CS_T > CS_F$ ), the credit forward buyer pays the credit forward seller, which is the bank,  $(CS_T - CS_F) * MD * A$ . For example, suppose the credit spread between BBB bonds and U.S. Treasury bonds widened to 3 percent from 2



percent over the year, the modified duration ( MD ) of the benchmark BBB bond was five years, and the size of the forward contract A was \$10 million. Then the gain on the credit forward contract to the seller (the bank) would be \$500,000  $[(3\% - 2\%) * 5 * \$10,000,000]$ . This amount could be used to offset the loss in market value of the loan due to the rise in the borrower's default risk. However, if the borrower's default risk and credit spread decrease over the year, the credit forward seller pays the credit forward buyer  $(CS_F - CS_T) * MD * A$ . [However, the maximum loss on the forward contract (to the bank seller) is limited, as will be explained below.]

Figure 23–7 illustrates the impact on the bank from hedging the loan. If the default risk on the loan increases, the market or present value of the loan falls below its value at the beginning of the hedge period. However, the bank hedged the change in default risk by selling a credit forward contract. Assuming the credit spread on the borrower's benchmark bond also increases (so that  $CS_T > CS_F$ ), the bank receives  $(CS_T - CS_F) * MD * A$  on the forward contract. If the characteristics of the benchmark bond (i.e., change in credit spread, modified duration, and principal value) are the same as those of the bank's loan to the borrower, the loss on the balance sheet is offset completely by the gain (off the balance sheet) from the credit forward (i.e., in our example a \$500,000 market value loss in the loan would be offset by a \$500,000 gain from selling the credit forward contract).



*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 23, p. 716-718*

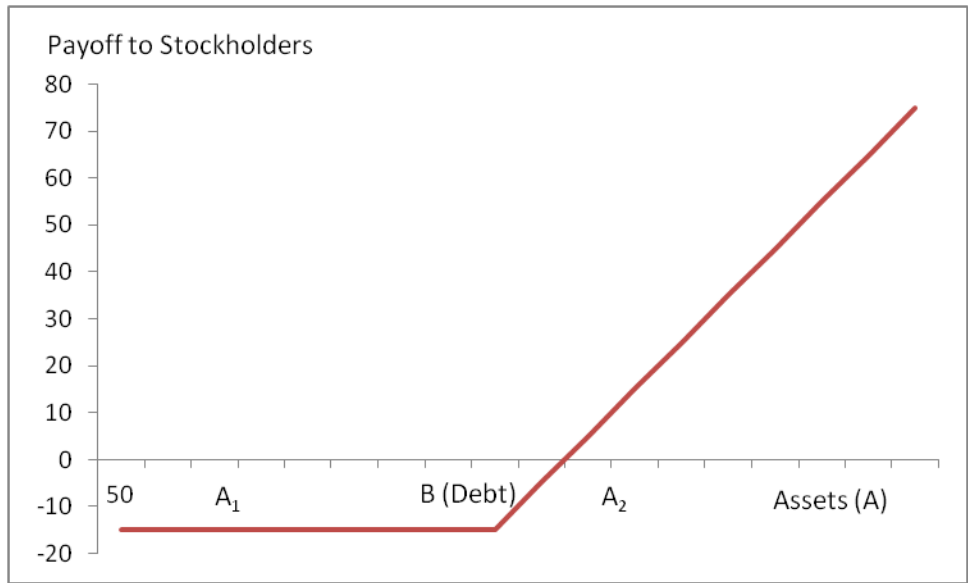
## 212. Show that payoffs to Bondholders and Stockholders may be described as options

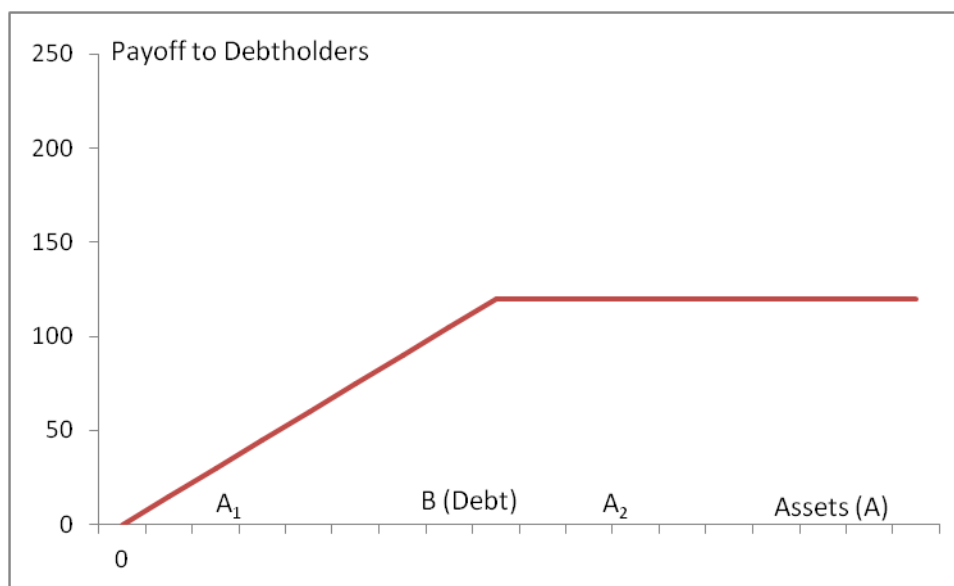
Look at the payoff function for the borrower, where  $S$  is the size of the initial equity investment in the firm,  $B$  is the value of outstanding bonds or loans (assumed for simplicity to be issued on a discount basis), and  $A$  is the market value of the assets of the firm.

If the investments in turn out badly such that the firm's assets are valued at point  $A_1$ , the limited-liability stockholder—owners of the firm will default on the firm's debt, turn its assets

(such as  $A_1$ ) over to the debt holders, and lose only their initial stake in the firm ( $S$ ). By contrast, if the firm does well and the assets of the firm are valued highly ( $A_2$ ), the firm's stockholders will pay off the firm's debt and keep the difference ( $A_2 - B$ ). Clearly, the higher  $A_2$  is relative to  $B$ , the better off are the firm's stockholders. Given that borrowers face only a limited downside risk of loss of their equity investment but a very large potential upside return if things turn out well, equity is analogous to buying a call option on the assets of the firm.

Consider the same loan or bond issue from the perspective of the FI or bondholder. The maximum amount the FI or bondholder can get back is  $B$ , the promised payment. However, the borrower who possesses the default or repayment option would rationally repay the loan only if  $A > B$ , that is, if the market value of assets exceeds the value of promised debt repayments. A borrower whose asset value falls below  $B$  would default and turn over any remaining assets to the debtholders. The payoff function to the debt holder is shown in figure below.





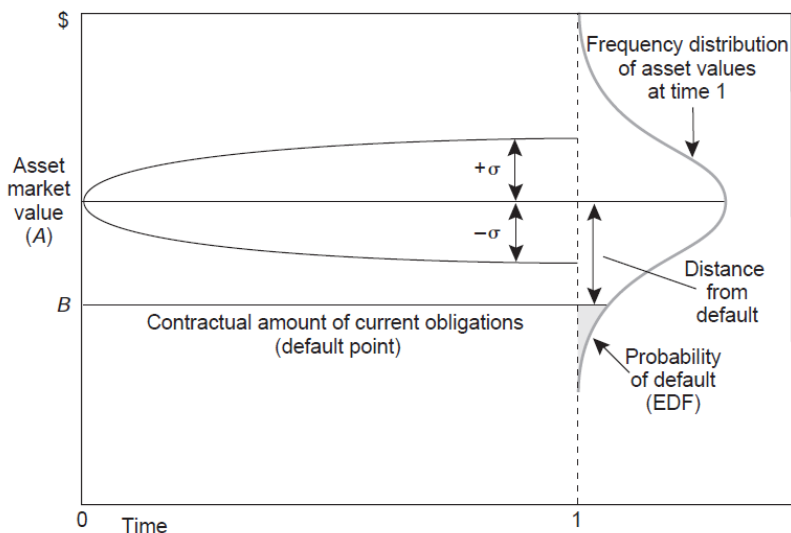
After investment of the borrowed funds has taken place, if the value of the firm's assets lies to the right of  $B$ , the face value of the debt—such as  $A_2$ —the debt holder or FI will be paid off in full and receive  $B$ . On the other hand, if asset values fall in the region to the left of  $B$ —such as  $A_1$ —the debt holder will receive back only those assets remaining as collateral, thereby losing  $B - A_1$ . Thus, the value of the loan from the perspective of the lender is always the minimum of  $B$  or  $A$ , or  $\min[B, A]$ . That is, the payoff function to the debt holder is similar to writing a put option on the value of the borrower's assets with  $B$ , the face value of debt, as the exercise price. If  $A > B$ , the loan is repaid and the debt holder earns a small fixed return (similar to the premium on a put option), which is the interest rate implicit in the discount bond. If  $A < B$ , the borrower defaults and the debt holder stands to lose both interest and principal. In the limit, default for a firm with no assets left results in debtholders' losing all their principal and interest. In actuality, if there are also costs of bankruptcy, the debt holder can potentially lose even more than this.

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 11, pp. 332-333*

### 213. KMV Option Model and expected default frequency

The KMV model uses the value of equity in a firm (from a stockholder's perspective) as equivalent to holding a call option on the assets of the firm (with the amount of debt borrowed acting similarly to the exercise price of the call option). From this approach, and the link between the volatility of the market value of the firm's equity and that of its assets, it is possible to derive the asset volatility (risk) of any given firm ( $\sigma_A$ ) and the market value of the firm's assets ( $A$ ). Using the implied value of  $\sigma_A$  for assets and  $A$ , the market value of assets, the likely distribution of possible asset values of the firm relative to its current

debt obligations can be calculated over the next year. The expected default frequency (EDF) that is calculated reflects the probability that the market value of the firm's assets (A) will fall below the promised repayments on its short-term debt liabilities (B) in one year. If the value of a firm's assets falls below its debt liabilities, it can be viewed as being economically insolvent.



*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 11, pp. 335-336*

## 214. Describe mechanics of hedging bond portfolio with options

Suppose that an FI manager has purchased a \$100 zero-coupon bond with exactly two years to maturity. A zero-coupon bond, if held to maturity, pays its face value of \$100 on maturity in two years. Assume that the FI manager pays \$80.45 per \$100 of face value for this zero-coupon bond. This means that if held to maturity, the FI's annual yield to maturity ( $R_2$ ) from this investment would be:

$$BP_2 = \frac{100}{(1 + R_2)^2} \rightarrow R_2 = 11.5\%$$

Suppose also that, at the end of the first year, interest rates rise unexpectedly. As a result, depositors, seeking higher returns on their funds, withdraw deposits. To meet these unexpected deposit withdrawals, the FI manager is forced to liquidate (sell) the two-year bond before maturity, at the end of year 1. Treasury securities are important liquidity sources for an FI. Because of the unexpected rise in interest rates at the end of year 1, the FI manager must sell the bond at a low price.

Assume when the bond is purchased, the current yield on one-year discount bonds ( $R_1$ ) is  $R_1 = 10$  percent. Also, assume that at the end of year one, the one-year interest rate ( $r_1$ ) is forecasted to rise to either 13.82 percent or 12.18 percent. If one-year interest rates rise from  $R_1 = 10$  percent when the bond is purchased to  $r_1 = 13.82$  percent at the end of year 1, the FI

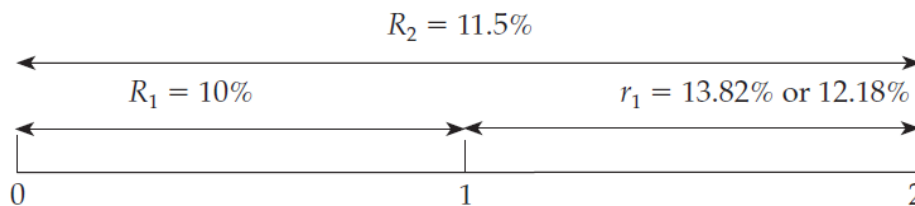
manager will be able to sell the zero-coupon bond with one year remaining to maturity for a bond price,  $BP_1$ , of:

$$BP_1 = \frac{100}{(1 + r_1)} = \frac{100}{(1.1382)} = \$87.86$$

If, on the other hand, one-year interest rates rise to 12.18 percent, the manager can sell the bond with one year remaining to maturity for:

$$BP_1 = \frac{100}{(1 + r_1)} = \frac{100}{(1.1218)} = \$89.14$$

In these equations,  $r_1$  stands for the two possible one-year rates that might arise one year into the future. That is:



Assume the manager believes that one-year rates ( $r_1$ ) one year from today will be 13.82 percent or 12.18 percent with an equal probability. This means that the expected one-year rate one year from today would be:

$$[E(r_1)] = 0.5 * (0.1382) + 0.5 * (0.1218) = 0.13 = 13\%$$

Thus, the expected price if the bond has to be sold at the end of the first year is:

$$E(P_1) = \frac{100}{(1.13)} = \$88.50$$

Assume that the FI manager wants to ensure that the bond sale produces at least \$88.50 per \$100; otherwise, the FI has to find alternative and very costly sources of liquidity (for example, the FI might have to borrow from the central bank's discount window and incur the direct and indirect penalty costs involved). One way for the FI to ensure that it receives at least \$88.50 on selling the bond at the end of the year is to buy a put option on the bond at time 0 with an exercise price of \$88.50 at time (year) 1. If the bond is trading below \$88.50 at the end of the year—say, at \$87.86—the FI can exercise its option and put the bond back to the writer of the option, who will have to pay the FI \$88.50. If, however, the bond is trading above \$88.50—say, at \$89.14—the FI does not have to exercise its option and instead can sell the bond in the open market for \$89.14. The FI manager will want to recalculate the fair premium to pay for buying this put option or bond insurance at time 0.

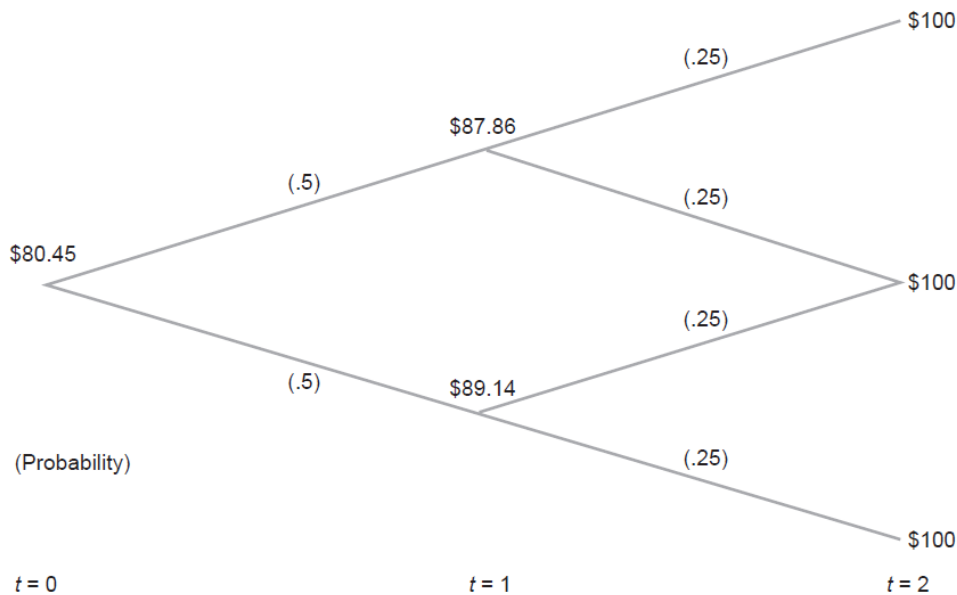


Figure shows the possible paths (i.e., the binomial tree or lattice) of the zero-coupon bond's price from purchase to maturity over the two-year period. The FI manager purchased the bond at \$80.45 with two years to maturity. Given expectations of rising rates, there is a 50 percent probability that the bond with one year left to maturity will trade at \$87.86 and a

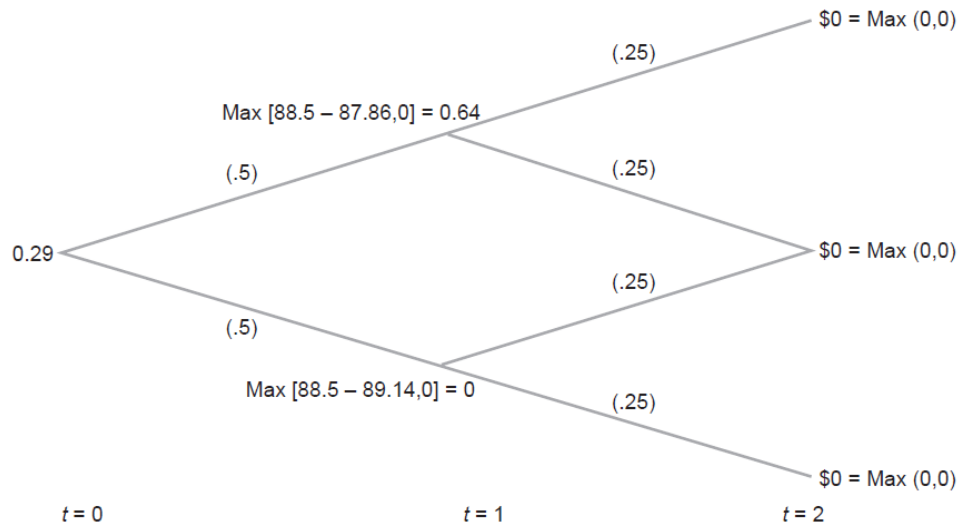
50 percent probability that it will trade at \$89.14. Note that between t = 1, or one year left to maturity, and maturity (t = 2), there must be a pull to par on the bond; that is, all paths must lead to a price of \$100 on maturity. The value of the option is shown in figure below. The option can be exercised only at the end of year 1 (t = 1). If the zero-coupon bond with one year left to maturity trades at \$87.86, the option is worth \$88.50 - \$87.86 in time 1 dollars, or \$0.64. If the bond trades at \$89.14, the option has no value since the bond could be sold at a higher value than the exercise price of \$88.50 on the open market. This suggests that in time 1 dollars, the option is worth:

$$0.5 * 0.64 + 0.5 * 0 = \$0.32$$

However, the FI is evaluating the option and paying the put premium at time t = 0, that is, one year before the date when the option might be exercised. Thus, the fair value of the put premium (P) the FI manager should be willing to pay is the discounted present value of the expected payoff from buying the option. Since one-year interest rates (R<sub>1</sub>) are currently 10 percent, this implies:

$$P = \frac{\$0.32}{(1 + R_1)} = \frac{\$0.32}{(1.1)} = \$0.29$$

or a premium, P, of approximately 29 cents per \$100 bond option purchased.



*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 24, pp. 737-740*

## 215. Describe mechanics of hedging interest rate risk using options

Our previous simple example showed how a bond option could hedge the interest rate risk on an underlying bond position in the asset portfolio. Next, we determine the put option position that can hedge the interest rate risk of the overall balance sheet; that is, we analyze macrohedging rather than microhedging.

FI's net worth exposure to an interest rate shock could be represented as:

$$\Delta E = -(D_A - kD_L) * A * \frac{\Delta R}{1 + R}$$

Suppose the FI manager wishes to determine the optimal number of put options to buy to insulate the FI against rising rates. An FI with a positive duration gap would lose on-balance-sheet net worth when interest rates rise. In this case, the FI manager would buy put options. That is, the FI manager wants to adopt a put option position to generate profits that just offset the loss in net worth due to an interest rate shock (where  $E_0$  is the FI's initial equity (net worth) position).

Let  $\Delta P$  be the total change in the value of the put option position in T-bonds.

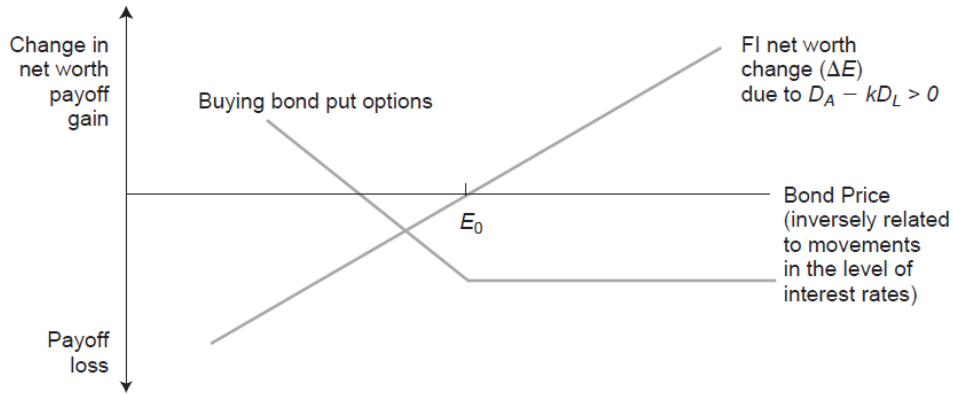
This can be decomposed into:

$$\Delta P = N_p * \Delta p$$

where  $N_p$  is the number of \$100,000 put options on T-bond contracts to be purchased (the number for which we are solving) and  $\Delta p$  is the change in the dollar value for each \$100,000 face value T-bond put option contract.

The change in the dollar value of each contract ( $\Delta p$ ) can be further decomposed into:

$$\Delta p = \frac{dp}{dB} * \frac{dB}{dR} * \Delta R$$



This decomposition needs some explanation. The first term ( $dp / dB$ ) shows the change in the value of a put option for each \$1 change in the underlying bond. This is called the delta of an option ( $\delta$ ), and its absolute value lies between 0 and 1. For put options, the delta has a negative sign since the value of the put option falls when bond prices rise. The second term ( $dB / dR$ ) shows how the market value of a bond changes if interest rates rise by one basis point. This value of one basis point term can be linked to duration. Specifically, we know from that:

$$\frac{dB}{B} = -MD * dR$$

That is, the percentage change in the bond's price for a small change in interest rates is proportional to the bond's modified duration (MD). Equation (3) can be rearranged by cross multiplying as:

$$\frac{dB}{dR} = -MD * B$$

Thus, the term  $dB / dR$  is equal to minus the modified duration on the bond (MD) times the current market value of the T-bond (B) underlying the put option contract.

As a result, we can rewrite equation (2) as:

$$\Delta p = [(-\delta) * (-MD) * B * \Delta R]$$

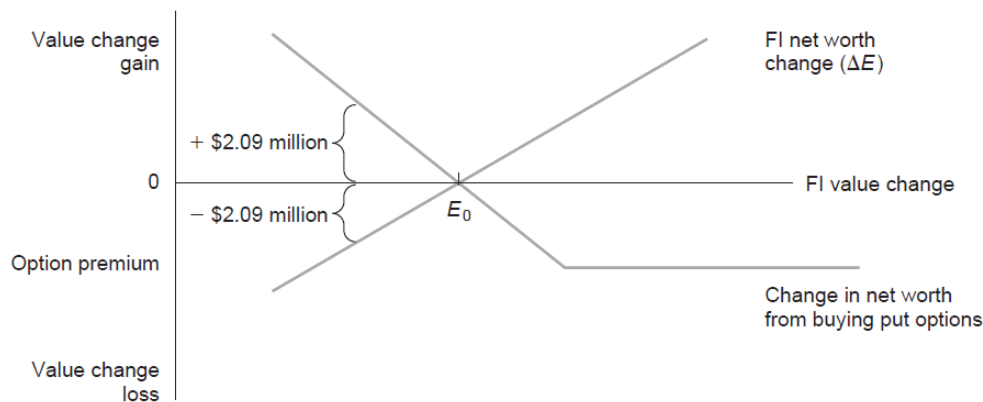
Where  $\Delta R$  is the shock to interest rates (i.e., the number of basis points by which rates change). Since we know that  $MD = D / (1 + R)$ , we can rewrite last equation as:

$$\Delta p = \left[ (-\delta) * (-D) * B * \frac{\Delta R}{1 + R} \right]$$

Thus, the change in the total value of a put position ( $\Delta P$ ) is:



$$\Delta P = N_p * \left[ \delta * D * B * \frac{\Delta R}{1 + R} \right]$$



The term in brackets is the change in the value of one \$100,000 face-value T-bondput option as rates change, and  $N_p$  is the number of put option contracts. To hedge net worth exposure, we require the profit on the off-balance-sheet put options ( $\Delta P$ ) to just offset the loss of on-balance-sheet net worth ( $\Delta E$ ) when interest rates rise (and thus, bond prices fall). That is:

$$\Delta P = -\Delta E$$

$$N_p * \left[ \delta * D * B * \frac{\Delta R}{1 + R} \right] = (D_A - kD_L) * A * \frac{\Delta R}{1 + R}$$

Canceling  $\Delta R / (1 + R)$  on both sides, we get:

$$N_p * [\delta * D * B *] = (D_A - kD_L) * A$$

Solving for  $N_p$ —the number of put options to buy—we have:

$$N_p = \frac{(D_A - kD_L) * A}{[\delta * D * B *]}$$

*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 24, pp. 743-746*

## 216. Describe mechanics of hedging credit risk using options

Options also have a potential use in hedging the credit risk of an FI. Relative to their use in hedging interest rate risk, option use to hedge credit risk is a relatively new phenomenon. Although FIs are always likely to be willing to bear some credit risk as part of the

intermediation process (i.e., exploit their comparative advantage to bear such risk), options may allow them to modify that level of exposure selectively. FI can seek an appropriate credit risk hedge by selling credit forward contracts. Rather than using credit forwards to hedge, an FI has at least two alternative credit option derivatives with which it can hedge its on-balance-sheet credit risk.

A credit spread call option is a call option whose payoff increases as the (default) risk premium or yield spread on a specified benchmark bond of the borrower increases above some exercise spread,  $S$ . An FI concerned that the risk on a loan to that borrower will increase can purchase a credit spread call option to hedge the increased credit risk.

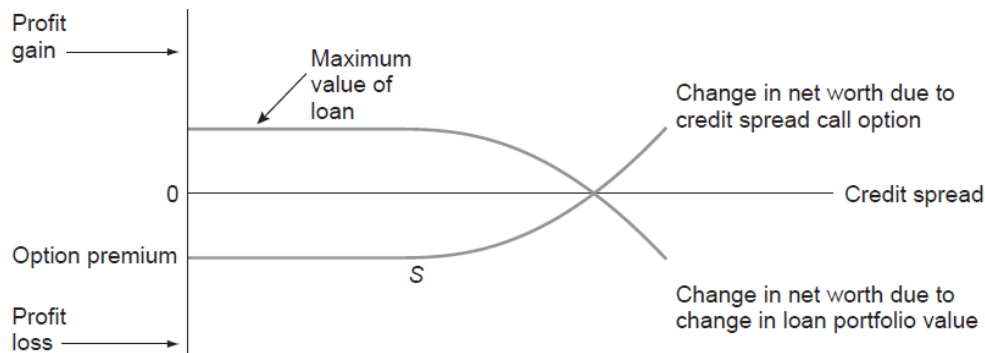
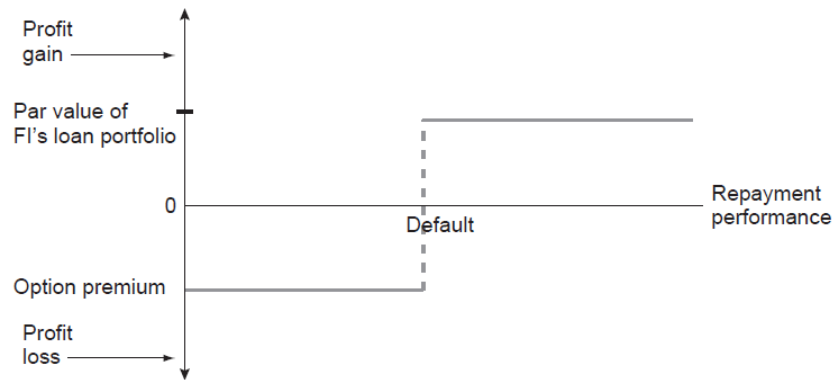


Figure illustrates the change in the FI's capital value and its payoffs from the credit spread call option as a function of the credit spread. As the credit spread increases on an FI's loan to a borrower, the value of the loan, and consequently the FI's net worth, decreases. However, if the credit risk characteristics of the benchmark bond (i.e., change in credit spread) are the same as those on the FI's loan, the loss of net worth on the balance sheet is offset with a gain from the credit spread call option. If the required credit spread on the FI's loan decreases (perhaps because the credit quality of the borrower improves over the loan period), the value of the FI's loan and net worth increases (up to some maximum value), but the credit spread call option will expire out of the money. As a result, the FI will suffer a maximum loss equal to the required (call) premium on the credit option, which will be offset by the market value gain of the loan in the portfolio (which is reflected in a positive increase in the FI's net worth).

A digital default option is an option that pays a stated amount in the event of a loan default (the extreme case of increased credit risk). As shown in below, the FI can purchase a default option covering the par value of a loan (or loans) in its portfolio. In the event of a loan default, the option writer pays the FI the par value of the defaulted loans. If the loans are paid off in accordance with the loan agreement, however, the default option expires unexercised. As a result, the FI will suffer a maximum loss on the option equal to the premium (cost) of buying the default option from the writer (seller).



*Financial Institutions Management, 6th ed., Anthony Saunders, Marcia Millon Cornett; McGraw-Hill, 2008; Chapter 24, pp. 749-751*

## REAL OPTIONS

*Course Book: Real Option Analysis, JohnathanMunn; Wiley Finance, 2002*

*Course Book: DerivativesMarkets, 2nd ed., Robert L. McDonald; Pearson, 2006*

*Course Book: Project Valuationusing Real Options, Dr. Prasad Kodukula, PMP  
Chandra Papudesu; J. Ross Publishing, 2006*

*Course Book: StrategicInvestment, Real Options and Games, Han T. J. Smit,  
LenosTrigeorgis,PrinstonUniversityPress, 2006*

### 217. Define Real Options. Illustrate how an investment project is a call option

In finance, an option is a contract which gives the buyer (the owner) the right, but not the obligation, to buy or sell an underlying asset or instrument at a specified strike price on or before a specified date. Real Options are options written on Real Assets instead of Financial Assets.

A call option gives the holder the right to buy an asset at a certain price within a specific period of time. If price rises, option is executed and option holder receives underlying asset for pre specified price. Notice similarity between Call Option and Investment Project

Investment Project		Call Option
Initial Investment	=	Strike Price
Present Value of future Cash Flows	=	Underlying Asset

Investment Project is a Call Option with Initial Investment as Strike Price and Present Value of Future Cash Flows as Underlying Asset.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 17, p. 547*

### 218. Explain the correct use of NPV

The NPV rule for making investmentdecisions entails two steps:

- Compute NPV by discounting expected cash flows at the opportunity cost of capital.
- Accept a project if and only if its NPV is positive and it exceeds the NPV of all mutually exclusive alternative projects.<sup>2</sup>

When we compute NPV, we neglected totake into account the NPV of alternative mutually exclusive projects, namely investingin the project tomorrow or at some other future date. Static NPV - NPV if we acceptthe project today-ignores project delay. Because static

NPV measures the value of an action we could take, namely investing today, it at least provides a lower bound on the value of the project.

It would be correct to invest in the project today if not activating the project today meant that we would lose it forever: Under this assumption, the mutually exclusive alternative (never taking the project) has a value of 0, so taking it today would be correct.

To decide whether or not and when to invest in an arbitrary project, we need to be able to compute the value of delaying investment. As suggested at the start of the chapter, option pricing theory can help us to value delay.

*Derivatives Markets, 2<sup>nd</sup> ed., Robert L. McDonald; Pearson, 2006; Chapter 17, pp. 549-550*

### 219. What is DCF problem?

In reality, there are several issues that an analyst should be aware of prior to using discounted cash flow models. The most important aspects include the business reality that risks and uncertainty abound when decisions have to be made and that management has the strategic flexibility to make and change decisions as these uncertainties become known over time. In such a stochastic world, using deterministic models like the discounted cash flow may potentially grossly underestimate the value of a particular project. A deterministic discounted cash flow model assumes at the outset that all future outcomes are fixed. If this is the case, then the discounted cash flow model is correctly specified as there would be no fluctuations in business conditions that would change the value of a particular project. In essence, there would be no value in flexibility. However, the actual business environment is highly fluid, and if management has the flexibility to make appropriate changes when conditions differ, then there is indeed value in flexibility, a value that will be grossly underestimated using a discounted cash flow model. Here are some assumptions that are used by DCF and real life complications associated with them:

DCF Assumption	Reality
Decisions are made now, and cash flow streams are fixed for the future.	Uncertainty and variability in future outcomes. Not all decisions are made today, as some may be deferred to the future, when uncertainty becomes resolved.
Once launched, all projects are passively managed.	Projects are usually actively managed through project life cycle, including checkpoints, decision options, budget constraints, and so forth.
All risks are completely accounted for by the discount rate.	Firm and project risk can change during the course of a project.
Future free cash flow streams are all highly predictable and deterministic.	It may be difficult to estimate future cash flows as they are usually stochastic and risky in nature.

All factors that could affect the outcome of the project and value to the investors are reflected in the DCF model through the NPV or IRR.	Because of project complexity and so-called externalities, it may be difficult or impossible to quantify all factors in terms of incremental cash flows. Distributed, unplanned outcomes (e.g., strategic vision and entrepreneurial activity) can be significant and strategically important.
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*Real Option Analysis, Johnathan Munn; Wiley Finance, 2002; Chapter 2, pp. 58-59*

## 220. Give examples Real Options uses in practice

Below are some examples of how real options have been or should be used in different industries.

**Automobile and Manufacturing Industry** In automobile manufacturing, General Motors (GM) applies real options to create switching options in producing its new series of autos. This is essentially the option to use a cheaper resource over a given period of time. GM holds excess raw materials and has multiple global vendors for similar materials with excess contractual obligations above what it projects as necessary. The excess contractual cost is outweighed by the significant savings of switching vendors when a certain raw material becomes too expensive in a particular region of the world. By spending the additional money in contracting with vendors as well as meeting their minimum purchase requirements, GM has essentially paid the premium on purchasing a switching option. This is important especially when the price of raw materials fluctuates significantly in different regions around the world. Having an option here provides the holder a hedging vehicle against pricing risks.

**Oil and Gas Industry** In the oil and gas industry, companies spend millions of dollars to refurbish their refineries and add new technology to create an option to switch their mix of outputs among heating oil, diesel, and other petrochemicals as a final product, using real options as a means of making capital and investment decisions. This option allows the refinery to switch its final output to one that is more profitable based on prevailing market prices, to capture the demand and price cyclicity in the market.

**Real Estate Industry** In the real estate arena, leaving land undeveloped creates an option to develop at a later date at a more lucrative profit level. However, what is the optimal wait time? In theory, one can wait for an infinite amount of time, and real options provide the solution for the optimal timing option.

**High-Tech and e-Business Industry** In e-business strategies, real options can be used to prioritize different e-commerce initiatives and to justify those large initial investments that have an uncertain future. Real options can be used in e-commerce to create incremental

investment stages, options to abandon, and other future growth options, compared to a large one-time investment (invest a little now, wait and see before investing more).

All these cases where the high cost of implementation with no apparent payback in the near future seems foolish and incomprehensible in the traditional discounted cash flow sense are fully justified in the real options sense when taking into account the strategic options the practice creates for the future, the uncertainty of the future operating environment, and management's flexibility in making the right choices at the appropriate time.

*Real Option Analysis, Johnathan Munn; Wiley Finance, 2002; Chapter 1, pp. 26-27*

## **221. Why Are Real Options Important?**

An important point is that the traditional discounted cash flow approach assumes a single decision pathway with fixed outcomes, and all decisions are made in the beginning without the ability to change and develop over time. The real options approach considers multiple decision pathways as a consequence of high uncertainty coupled with management's flexibility in choosing the optimal strategies or options along the way when new information becomes available. That is, management has the flexibility to make midcourse strategy corrections when there is uncertainty involved in the future. As information becomes available and uncertainty becomes resolved, management can choose the best strategies to implement. Traditional discounted cash flow assumes a single static decision, while real options assume a multidimensional dynamic series of decisions, where management has the flexibility to adapt given a change in the business environment.

Another way to view the problem is that there are two points to consider, one, the initial investment starting point where strategic investment decisions have to be made; and two, the ultimate goal, the optimal decision that can ever be made to maximize the firm's return on investment and shareholder's wealth. In the traditional discounted cash flow approach, joining these two points is a straight line, whereas the real options approach looks like a map with multiple routes to get to the ultimate goal, where each route is conjoint with others. The former implies a one-time decision-making process, while the latter implies a dynamic decision-making process wherein the investor learns over time and makes different updated decisions as time passes and events unfold.

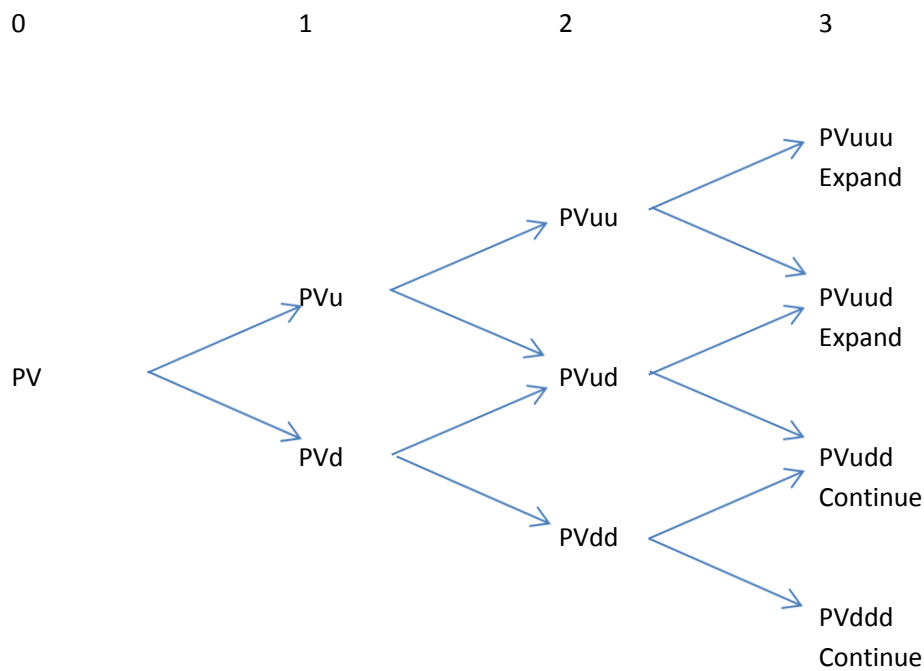
*Real Option Analysis, Johnathan Munn; Wiley Finance, 2002; Chapter 3, pp. 82-83*

## 222. Define option to expand

An expansion option provides management the right and ability to expand into different markets, products and strategies or to expand its current operations under the right conditions.

i.e. exercise the option – should conditions turn out to be favorable. A project with the option to expand will cost more to establish, the excess being the option premium, but is worth more than the same without the possibility of expansion. This is equivalent to a call option with underlying asset as increase in cash flow and expansion cost as strike price.

See illustration below



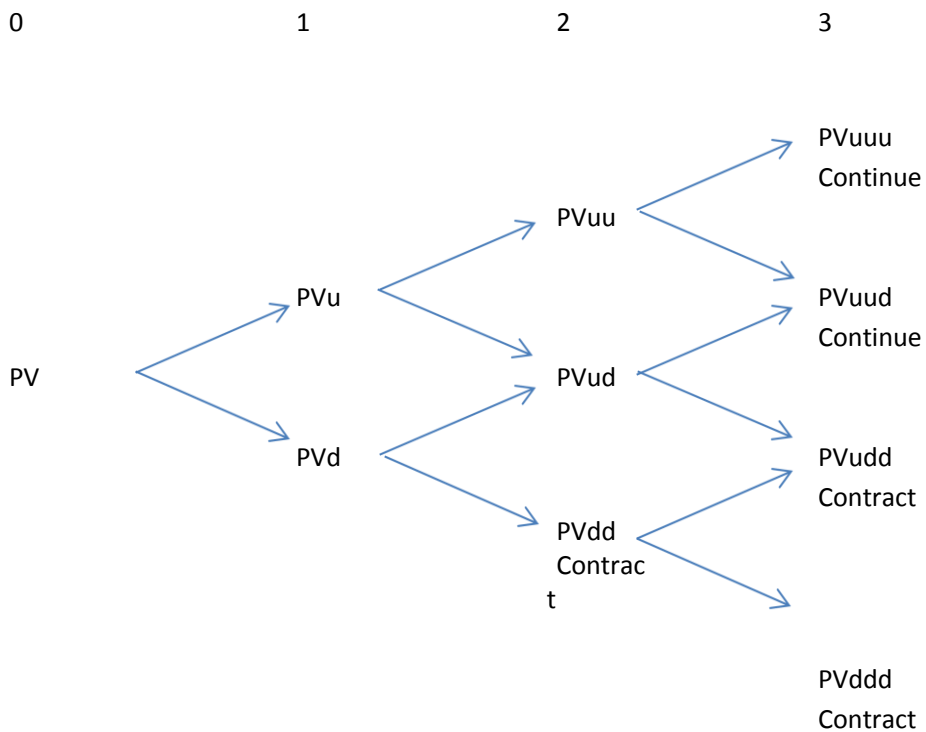
*Real Option Analysis, Johnathan Munn; Wiley Finance, 2002; Chapter 7, pp. 175-178*



### 223. Define option to contract

A contract option evaluates the flexibility value to reduce production output or to contract scale and scope when conditions are not amenable, therefore reducing value of an asset or project by contraction factor, but at the same time creating some cost saving. A closed-form approximation of an American Put option can be used, because the option to contract the firm's operations can be exercised at any time upto the expiration date and additional value is created while value of a project falling.

See illustration below

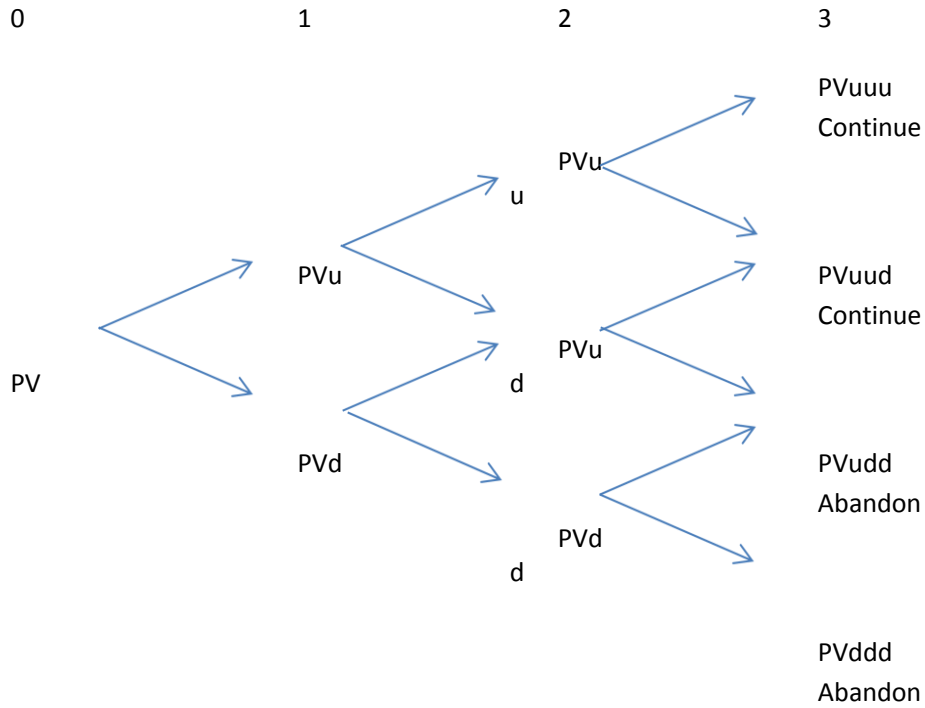


*Real Option Analysis, Johnathan Munn; Wiley Finance, 2002; Chapter 7, pp. 178-181*

### 224. Define option to abandon

When investing in new projects, firms worry about the risk that the investment will not pay off, and that actual cash flows will not measure up to expectations. Having the option to abandon a project that does not pay off can be valuable, especially on projects with a significant potential for losses. A closed-form approximation of an American Put option can be used, because the option to contract the firm's operations can be exercised at any time up to the expiration date and additional value is created while value of a project falling.

See illustration below

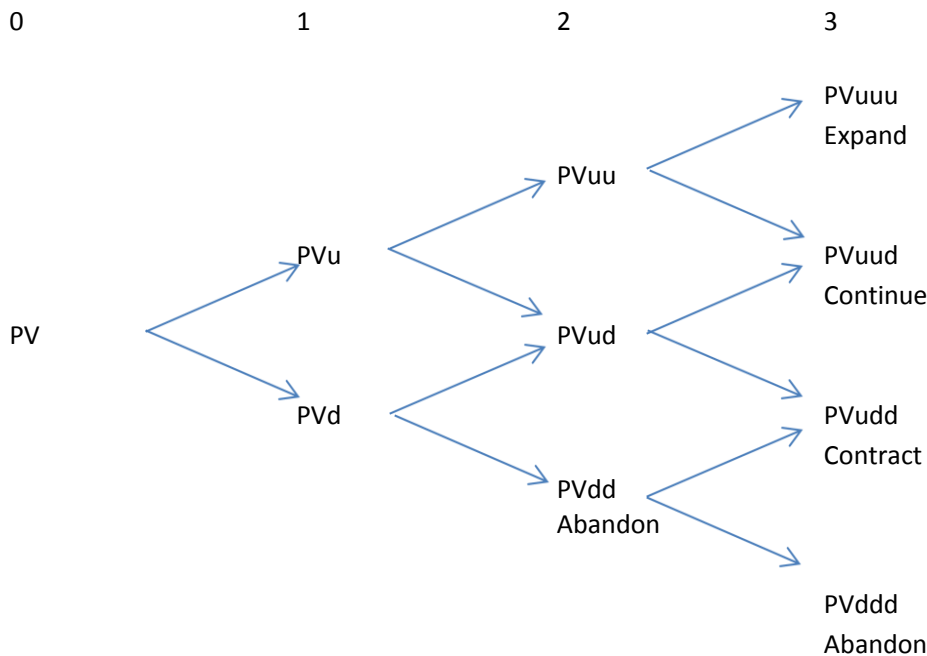


*Real Option Analysis, Johnathan Munn; Wiley Finance, 2002; Chapter 7, pp. 171-175*

**225. Define option to choose. Does chooser option value equal to the sum of option values included in it?**

The option to choose consists of multiple options combined as a single option. The multiple options are abandonment, expansion, and contraction. The reason it is called a chooser option is that you can choose to keep the option open and continue with the project or choose to exercise any one of the options to expand, contract, or abandon. The main advantage with this option is the choice. This is a unique option in the sense that, depending upon the choice to be made, it can be considered a put (abandonment or contraction) or call (expansion) option.

See illustration below



*Real Option Analysis, Johnathan Munn; Wiley Finance, 2002; Chapter 7, pp. 181-185*

## 226. Define Sequential and Parallel Compound options

Many project initiatives (research and development, capacity expansion, launching of new services, etc.) are multistage project investments where management can decide to expand, scale back, maintain the status quo, or abandon the project after gaining new information to resolve uncertainty. For example, a capital investment project divided into multiple phases, including permitting, design, engineering, and construction, can either be terminated or continued into the next phase depending upon the market conditions at the end of each phase.

These are compound options where exercising one option generates another, thereby making the value of one option contingent upon the value of another option. A compound option derives its value from another option — not from the underlying asset. The first

investment creates the right but not the obligation to make a second investment, which in turn gives you the option to make a third investment, and so on. You have the option to abandon, contract, or scale up the project at any time during its life.

A compound option can either be sequential or parallel, also known as simultaneous. If you must exercise an option in order to create another one, it is considered a sequential option. For example, you must complete the design phase of a factory before you can start building it. In a parallel option, however, both options are available at the same time. The life of the independent option is longer than or equal to the dependent option. A television broadcaster may be building the infrastructure for digital transmission and acquiring the required broadcast spectrum at the same time, but cannot complete testing of the infrastructure without the spectrum license. Acquiring the spectrum — an option itself — gives the broadcaster the option to complete the infrastructure and launch the digital broadcast service. For both sequential and compound options, valuation calculations are essentially the same except for minor differences.

*Real Option Analysis, Johnathan Munn; Wiley Finance, 2002; Chapter 7, pp. 185-187*  
*Project Valuation using Real Options, Dr. Prasad Kodukula, PMP Chandra Papudesu;*  
*J. Ross Publishing, 2006, Chapter 8, pp. 146-162*

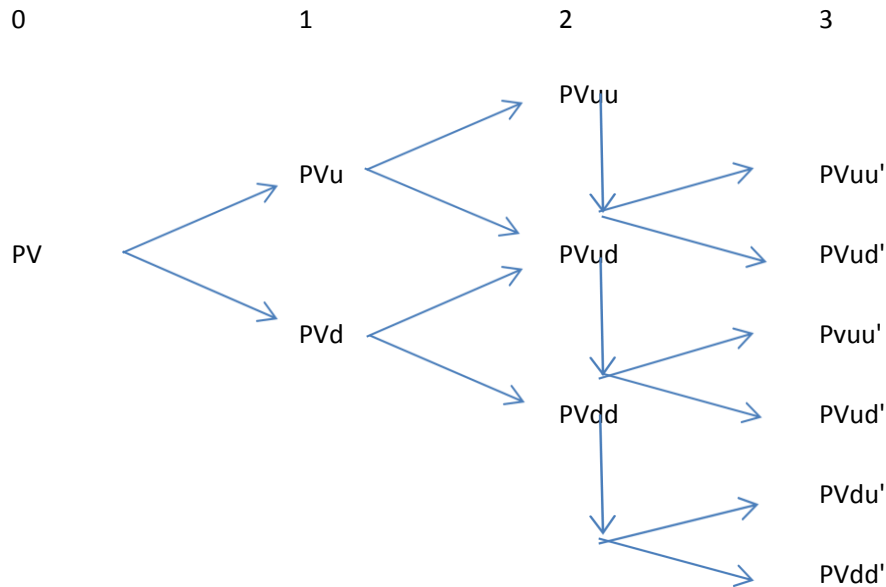
## **227. Define switching options**

A switching option refers to the flexibility in a project to switch from one mode of operation to another. For example, a dual-fuel heater offers the option of switching from oil to natural gas and vice versa, depending on the relative market costs of these fuels. This flexibility has value and accounts for the price premium for dual-fuel heaters compared to single fuel heaters.

*Real Option Analysis, Johnathan Munn; Wiley Finance, 2002; Chapter 8, pp. 252-256*  
*Project Valuation using Real Options, Dr. Prasad Kodukula, PMP Chandra Papudesu;*  
*J. Ross Publishing, 2006, Chapter 8, p. 187*

## **228. Define barrier options**

A barrier option is an option where your decision to exercise it depends not only on the strike and asset prices but also on a predefined “barrier” price. This type of option can be either a call or a put option, such as an option to wait or an option to abandon, respectively. A traditional call option (Figure 7-12A) is in the money when the asset value is above the strike price, whereas a barrier call option (Figure 7-12B) is in the money when the asset value is above the barrier price, which is predefined at a value higher than the strike price. As a rational investor, you exercise the barrier call only when the asset value is above the barrier price, irrespective of the strike price. Similarly, you exercise a barrier put when the asset value is below the barrier price, which is set at a value lower than the strike price.



*Project Valuation using Real Options, Dr. Prasad Kodukula, PMP Chandra Papudesu; J. Ross Publishing, 2006, Chapter 7, pp. 130-131*

## 229. Define rainbow options

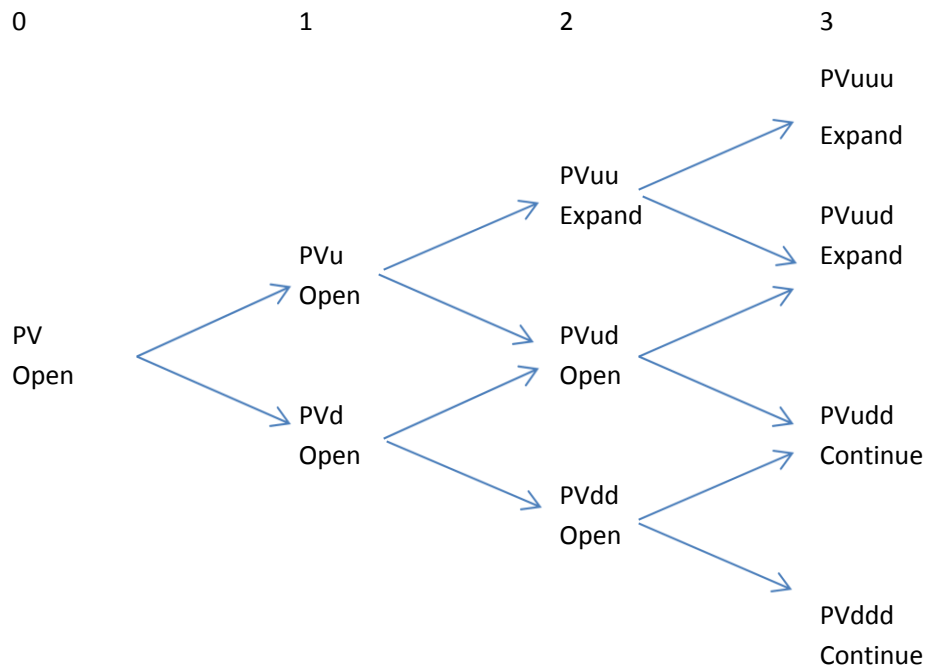
A key input parameter of any ROA problem is the volatility factor that represents the uncertainty associated with the underlying asset value. Typically, it is calculated as an aggregate factor built from many of the uncertainties that contribute to it. For example, the aggregate volatility used in the ROA of a product development project is representative of and a function of multiple uncertainties, including the unit price, number of units sold unit variable cost, etc. If one of the sources of uncertainty has a significant impact on the options value compared to the others or if management decisions are to be tied to a particular source of uncertainty, you may want to keep the uncertainties separate in the options calculations. For instance, if you own a lease on an undeveloped oil reserve, you face two separate uncertainties: the price of oil and the quantity of oil in the reserve. You may want to treat them separately in evaluating the ROV.

When multiple sources of uncertainty are considered, the options are called rainbow options, and this warrants the use of different volatility factors — one for each source of uncertainty — in the options calculations. The options solution method is basically the same as for a single volatility factor except that it involves a quadrinomial tree instead of a binomial.

*Project Valuation using Real Options, Dr. Prasad Kodukula, PMP Chandra Papudesu; J. Ross Publishing, 2006, Chapter 8, pp. 164-168*

### 230. Define options with changing strikes

A modification to the option types we have thus far been discussing is the idea of changing strikes—that is, implementation costs for projects may change over time. Putting off a project for a particular period may mean a higher cost, thus it may be the case when early exercise is reasonable. (see figure below) Keep in mind that changing strikes can be applied to any previous option types as well; in other words, one can mix and match different option types.

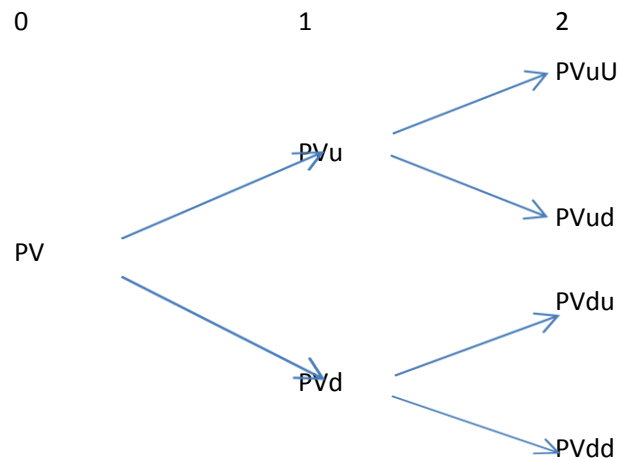


*Real Option Analysis, Johnathan Munn; Wiley Finance, 2002; Chapter 7, pp. 188-189*

### 231. Define options with changing volatility

In most ROA calculations, the volatility of the project payoff is assumed to be relatively constant over the option life and is represented by a single aggregate factor. Therefore, a single volatility factor is used across the binomial tree to represent the option life. However, if the volatility is expected to change during the option life and is significant, it can be accounted for by modifying the binomial method. Start with the initial volatility factor, build the binomial tree, and calculate the asset values at each node of the tree using the corresponding up and down factors up to the point where the volatility changes. From that point on, calculate the asset values using the new up and down factors related to the new

volatility factor, which will result in a non-recombining lattice. The option value calculation method using backward induction will be the same for the entire tree.



*Real Option Analysis, Johnathan Munn; Wiley Finance, 2002; Chapter 7, pp. 189-191*  
*Project Valuation using Real Options, Dr. Prasad Kodukula, PMP Chandra Papudesu;*  
*J. Ross Publishing, 2006, Chapter 8, pp. 168-174*

### 232. Describe real options analysis assumptions

One important assumption behind the options pricing models is that no “arbitrage” opportunity exists. This means that in efficient financial markets, you cannot buy an asset at one price and simultaneously sell it at a higher price. Professional investors supposedly will buy and sell assets quickly, closing any price gaps, thereby making arbitrage opportunities rare. Critics argue that a “no arbitrage” condition is impossible with real assets because they are not as liquid as financial assets, and therefore option pricing models are inappropriate for real options valuation.

Another big assumption regarding use of Real Options is that competition will not have a significant impact on cash flows of the company. Problem occurs when there is more than one company, developing same product and first one to complete will obtain strategic advantage. Thus Real Option Analysis is fair for Monopolistic market, whereas in real life case, there are plenty of competitors.

*Project Valuation using Real Options, Dr. Prasad Kodukula, PMP Chandra Papudesu;*  
*J. Ross Publishing, 2006, Chapter 8, pp. 83-85*

### 233. How game theory can be used in real options analysis?

Game theory is a study of strategic decision making. More formally, it is "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers". To be fully defined, a game must specify the following elements: the players of the game (there must be at least two players), the information and actions available to each player at each decision point (list of strategies that each player is able to choose), and the payoffs for each outcome. These elements are used, along with a solution concept of their choosing, to deduce a set of equilibrium strategies for each player such that, when these strategies are employed, no player can profit by unilaterally deviating from their strategy. These equilibrium strategies determine equilibrium to the game—a stable state in which either one outcome occurs or a set of outcomes occurs with known probability.

An investment decision in competitive markets can be seen, in its essence, as a “game” among firms, since in their investment decisions firms implicitly take into account what they think will be the other firms’ reactions to their own actions, and they know that their competitors think the same way. Consequently, as game theory aims to provide an abstract framework for modeling situations involving interdependent choices, so a merger between these two theories appears to be a logic step.

*Strategic Investment, Real Options and Games, Han T. J. Smit, Lenos Trigeorgis, Princeton University Press, 2006, Chapter 1, pp. 13-31*

### **234. Case Study: Investment Projects Valuation and Real Options and Games Theory**

Consider an investor willing to build a hotel with initial investment \$25 mln. Given the negative relationship between occupancy of the hotel and the price per night per room, it was estimated that maximum profits were obtained at price \$140, which would produce 35% occupancy level. Given these data, future revenues were estimated using Monte Carlo simulation. NPV was estimated to be \$-3.75 mln. As this number is close to zero, it is not clear to decide whether to invest or not. Additional analysis needs to be conducted in order to better estimate fair value of the hotel business.

While operating a hotel, manager might face several options such as Expansion, Contract, and Abandonment. Expansion involves building one more, similar class hotel. This decision may double value of the company, but requires additional investment of \$20mln. This sort of decision may be made at time when there is high demand and value of business increases. Contract option involves renting part of the hotel, thus value of the company is decreased by contraction rate, however this decision generates some cost saving. This sort of decision may be made at time when the demand has slightly decreased. Abandonment option involves sale of company’s assets and in this manner obtaining higher value than by operating a business. This sort of decision may be made when demand has significantly decreased and investor is willing to leave market. Investor would choose option that generates maximum value of these three, i.e. Chooser Option. However remember that starting business is itself a call option, thus we are having Sequential Option. Sequential Option gives the right to start the business which automatically gives us chooser option.



Taking into consideration these opportunities that investor may use, increased NPV to the level of \$36.77 mln. However, as stated before, real option analysis doesn't take into consideration competition and competitive analysis. In order to incorporate competition into real options, Games Theory was used. To describe Game, Cournot competition model was used, which states that, if number of market participants increase, market output goes to competitive level and prices converges to marginal cost. Game can be illustrated by the following matrix

Value / Occ. rate	31%	25.61%
NPV	-3.75	-25
NPV + options	36.77	<b><u>-18.25</u></b>

Matrix shows the value of the company with and without options at given level of occupancy level and competitive occupancy level. Equilibrium of this game is \$-18.25 mln, stating that it is not good time for the investor to enter market.

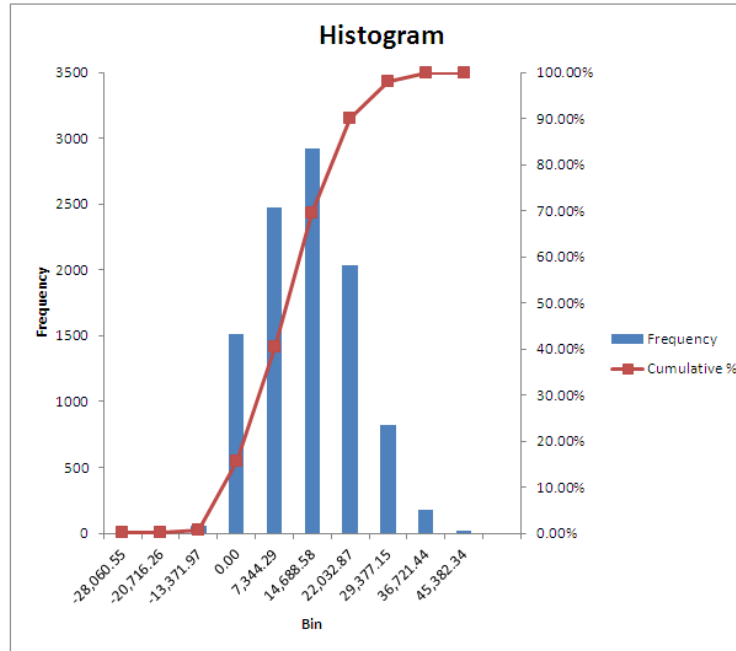
*Investment Project Valuation with Real Options and Game Theory,  
Bachelor Thesis, Levan Gachechiladze, Irakli Chelidze, 2010*

### 235. Case Study: Real Option Valuation of the Project

In order to introduce the project valuation technique we are considering the case with gasoline station which require import of gasoline.

Traditional valuation technique - NPV (net present value)- with (I) investment \$80,000 monthly cash flows ( $C_n$ ) \$8,153 during the 12 month ( $N=12$ ), and discount rate recalculated for monthly data 0.8 %, shows us that  $NPV = 12,943$ . This is slightly positive NPV. But cash flows in the project are random variable with some volatility. Adjusting the project for randomness of cash flows can be done via Monte Carlo simulation. Monte Carlo simulation in the thesis is done based on uniform distribution - uniformly simulating monthly quantity of gasoline sales between 9,510 and 19,020 gallons. Constructing the histogram on simulated data (10,000 of iterations) we can find that up to 15.71% of the iterations the project gives negative NPV.

Bin	Frequency	Cumulative %
-28,060.55	1	0.01%
-20,716.26	3	0.04%
-13,371.97	59	0.63%
0.00	1508	15.71%
7,344.29	2471	40.42%
14,688.58	2918	69.60%
22,032.87	2032	89.92%
29,377.15	818	98.10%
36,721.44	175	99.85%
45,382.34	15	100.00%



Therefore we need to adjust the project to accommodate associated risk with uncertain sales of gasoline. It could be done by introducing real option analysis of project. In our case we are presenting the project as simple expansion real option - American call option (or if we will be more precise - Bermuda call option - combination of European and American options). Calculations of this option can be done via Black-Scholes formula or using recombining binomial lattice which gives us good visualization of the project valuation and reaches the value provided by Black-Scholes formula as number of nodes of the binomial lattice increase and go to infinity.

Using call option formula  $(S_n - I)^+$  and backward induction we can calculate option price, which in our case is \$7,291. Therefore our new project value is  $NPV + Option Value = \$12,943 + \$7,291 = \$20,234$ .

*Real Option Valuation of the Project with Robust Mean-variance Hedging, PHD Thesis, Tamaz Uzunashvili, 2013*

# MASTERING FINANCIAL CALCULATIONS

*Course book: Mastering Financial Calculations, Robert Steiner, Prentis Hall*

*Course book: Financial Mathematics, E.M. Chetirkin, 2004*

## 236. What is meant under term “Simple Interest” and how is it calculated for fractional time periods? Show graphically

On short-term financial instruments, interest is usually “simple” rather than “compound”. Simple interest is called simple because it ignores the effects of compounding. The interest charge is always based on the original principal, so interest on interest is not included. This method may be used to find the interest charge for short-term loans, where ignoring compounding is less of an issue.

Simple interest is calculated on the original principal only. Accumulated interest from prior periods is not used in calculations for the following periods. Simple interest is normally used for a single period of less than a year, such as 30 or 60 days.

$$\text{Simple Interest} = p \times i \times n$$

Where

$p$  = principal (original amount borrowed or loaned)

$i$  = interest rate for one period

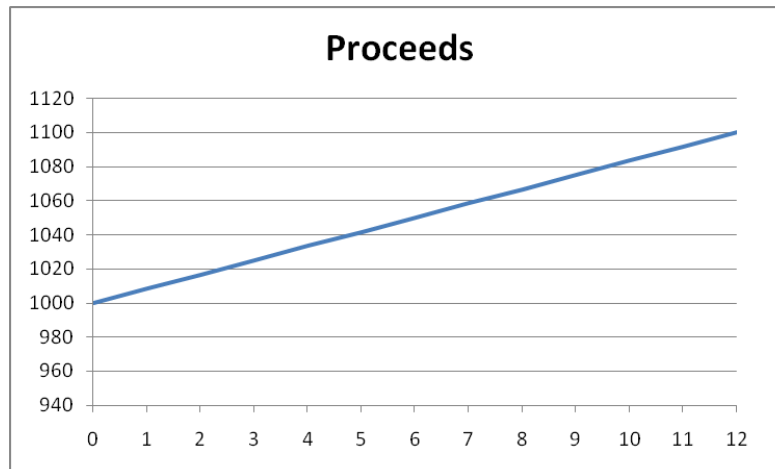
$n$  = number of periods

Therefore,

*Total proceeds of short – term investment*

$$= \text{principal} \times \left( 1 + \text{interest rate} \times \frac{\text{Days}}{\text{Year}} \right)$$

By "Days/Year" we mean "Number of days in Period / Number of days in a Year"



*Mastering Financial Calculations, Robert Steiner, Prentis Hall, Chapter 1, p. 5*  
*Financial Mathematics, E.M. Chetirkin, 2004; Chapter 2, pp. 19-20*

**237. How to convert interest rate for different day/year conventions?**

As a general rule in the money markets, the calculation of interest takes account of the exact numbers of days in the period in question, as a proportion of a year. Thus:

**Interest paid = interest rate quoted x days in period/days in year**

A variation between different money markets arises, however, in the conventions used for the number of days assumed to be in the base “year.”

In order to convert an interest rate  $i$  quoted on a 360-day basis to an interest rate  $i^*$  quoted on a 365-day basis:

$$\text{Interest rate on 360 day basis (i)} = \text{interest rate on 365 daybasis}(i^*) * \frac{360}{365}$$

Similarly

$$\text{Interest rate on 365 day basis (i}^*) = \text{interest rate on 360 daybasis}(i) * \frac{365}{360}$$

*Mastering Financial Calculations, Robert Steiner, Prentis Hall, Chapter 2, p. 48-49*

**238. What happens when interest rate changes over time with and without assumption of reinvestment?**

Credit agreements sometimes are using the changing interest rates. Consider an example where deposit/loan considers  $i_1$  for  $n_1$  fraction of the year and  $i_2$  for  $n_2$  fraction of the year. Total interest proceeds for period of  $n_1+n_2$  will equal to:

$$I = \text{Principal} * i_1 * n_1 + \text{Principal} * i_2 * n_2$$

Thus formula for calculating total proceeds from deposit/loan in  $m$  period can be written as:

$$S = P(1 + i_1 * n_1 + i_2 * n_2 + \dots + i_m * n_m)$$

*Financial Mathematics, E.M. Chetirkin, 2004; Chapter 2, pp. 23-24*

**239. How is loan repayment schedules formed?**

A straightforward bond has a “bullet” maturity, whereby all the bond is redeemed at maturity. An alternative is for the principal amount to be repaid in stages – “amortized” – over the bond’s life.

Amortization refers to the process of paying off a debt (often from a loan or mortgage) over time through regular payments. A portion of each payment is for interest while the remaining amount is applied towards the principal balance. The percentage of interest versus principal in each payment is determined in an amortization schedule. The schedule

differentiates out the portion of payment that belongs to interest expense and the portion used to close the gap of a discount or premium from the principal after each payment.

While a portion of every payment is applied towards both the interest and the principal balance of the loan, the exact amount applied to principal each time varies (with the remainder going to interest). An amortization schedule reveals the specific monetary amount put towards interest, as well as the specific amount put towards the principal balance, with each payment. Initially, a large portion of each payment is devoted to interest. As the loan matures, larger portions go towards paying down the principal.

*Example:* Consider one year loan of \$1000, interest rate 12%. amortizations schedule will look like:

Months	Loan Balance	Principal Payment	Interest Payment	Total Payment	Remaining Amount
1	1 000	83	10	93	917
2	917	83	9	93	833
3	833	83	8	92	750
...	...	...	...	...	...
12	83	83	1	84	0

*Financial Mathematics, E.M. Chetirkin, 2004; Chapter 9, pp. 184-195*

#### **240. What is compounding interest? Given the interest rate, which is preferable in short run time period? Long run time period?**

Consider an investment of 1 made for two years at 10 percent per annum. At the end of the first year, the investor receives interest of 0.10. At the end of the second year he receives interest of 0.10, plus the principal of 1. The total received is  $0.10 + 0.10 + 1 = 1.20$ . However, the investor would in practice reinvest the 0.10 received at the end of the first year, for a further year. If he could do this at 10 percent, he would receive an extra 0.01 ( $= 10 \text{ percent} \times 0.10$ ) at the end of the second year, for a total of 1.21. In effect, this is the same as investing 1 for one year at 10 percent to receive  $1 + 0.10$  at the end of the first year and then reinvesting this whole  $(1 + 0.10)$  for a further year, also at 10 percent, to give  $(1 + 0.10) \times (1 + 0.10) = 1.21$ . The same idea can be extended for any number of years, so that the total return after  $N$  years, including principal, is:

$$\text{Principal} * (1 + r)^n$$

This is “compounding” the interest, and assumes that all interim cashflows can be reinvested at the same original interest rate. “Simple” interest is when the interest is not reinvested.

In order to see which interest is preferable for different terms, we need to compare growth rates for each of them.

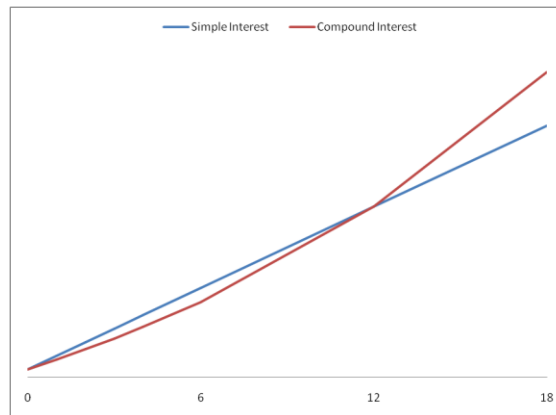
in case of short term investment

$$(1 + ni) > (1 + i)^n$$

in case of long term investment

$$(1 + ni) < (1 + i)^n$$

We can see results graphically



*Financial Mathematics, E.M. Chetirkin, 2004; Chapter 3, pp. 48-49*

### 241. What is interim year compounding? Write formula for nonstandard time periods

From the formula for calculating future value of an investment, it is easily seen that increasing compounding per year will raise FV of investment.

$$FV = P \left( 1 + \frac{R}{m} \right)^{N \cdot m}$$

Where

$FV$  = future value

$P$  = initial deposit (principal)

$R$  = annual rate of interest

$N$  = number of years

$m$  = number of compounding periods

Thus interim-year compounding serves as additional source of income.

*Finance, 5th ed; Groppelli, Ehsan Nikbakht; Barron's Inc 2006, Chapter 3, pp. 8-9*

**242. Write formula for discount factors using Simple Interest, Compound Interest. Show graphically results of discounting using each of them.**

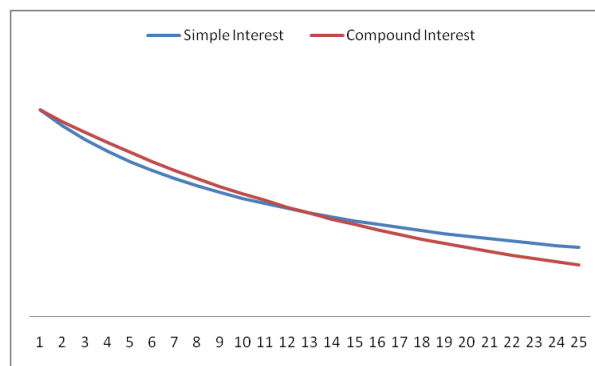
Discount factor for Simple interest is calculated as following:

$$DF = \frac{1}{1 + i * n}$$

Discount factor for compound interest is calculated as following:

$$DF = \frac{1}{(1 + i)^n}$$

Effect is shown graphically below:



*Financial Mathematics, E.M. Chetirkin, 2004; Chapter 3, pp. 55-57*

**243. How compounding interest works with nonstandard number of years?**

In case of nonstandard number of years formulae of interim year compounding will be rewritten as:

$$FV = P \left( 1 + R * \frac{\text{Days}}{\text{Years}} \right)^{N * \frac{\text{Years}}{\text{Days}}}$$

Where

$FV$  = future value

$P$  = initial deposit (principal)

$R$  = annual rate of interest

$N$  = number of years

*Financial Mathematics, E.M. Chetirkin, 2004; Chapter 3*

#### 244. What are discount instruments? Why are they called “Discounts”?

The pure discount bond is perhaps the simplest kind of bond. It promises a single payment, say \$1, at a fixed future date. If the payment is one year from now, it is called a one-year discount bond; if it is two years from now, it is called a two-year discount bond, and so on. The date when the issuer of the bond makes the last payment is called the maturity date of the bond, or just its maturity for short. The bond is said to mature or expire on the date of its final payment. The payment at maturity (\$1 in this example) is termed the bond’s face value. Pure discount bonds are often called zero-coupon bonds or “zeros” to emphasize the fact that the holder receives no cash payments until maturity.

Consider a pure discount bond that pays a face value of  $F$  in  $n$  years, where the interest rate is  $r$  in each of the  $n$  years. (We also refer to this rate as the market interest rate.) Because the face value is the only cash flow that the bond pays, the present value of this face amount is Value of a Pure Discount Bond:

$$PV = \frac{F}{(1 + R)^n}$$

An instrument which does not carry a coupon is a “discount” instrument. Because there is no interest paid on the principal, a buyer will only ever buy it for less than its face value – that is “at a discount” (unless yields are negative!). For example, all treasury bills are discount instruments.

*Mastering Financial Calculations, Robert Steiner, Prentis Hall, Chapter 2, p. 40*

#### 245. Write discount/true yield relationship.

Sometimes discount rate is referred as the amount of discount expressed as an annualized percentage of the face value, rather than as a percentage of the original amount paid.

The discount rate is always less than the corresponding yield. If the discount rate on an instrument is  $D$ , then the amount of discount is:

$$F * D * \frac{Days}{Years}$$

where  $F$  is the face value of the instrument.

The price  $P$  to be paid is the face value less the discount:

$$P = F * \left(1 - D * \frac{Days}{Years}\right)$$

If we expressed the price in terms of the equivalent yield rather than the discount rate, we would still have the same formula as earlier:

$$P = \frac{F}{\left(1 + i * \frac{Days}{Years}\right)}$$

Combining these two relationships, we get



$$D = \frac{i}{\left(1 + i * \frac{Days}{Years}\right)}$$

where  $i$  is the equivalent yield (often referred to as the “true yield”). This can perhaps be understood intuitively, by considering that because the discount is received at the beginning of the period whereas the equivalent yield is received at the end, the discount rate should be the “present value of the yield”. Reversing this relationship:

$$i = \frac{D}{\left(1 - D * \frac{Days}{Years}\right)}$$

*Mastering Financial Calculations, Robert Steiner, Prentis Hall, Chapter 2, p. 58-59*

#### **246. How do you calculate FV, PV and Interest Rates of short term investment**

Future value and present values clearly depend on both the interest rate used and length of time involved. Similarly, the future value after 98 days of \$100, at 10% per annum would be  $FV = \$100 \left(1 + 0.10 \times \frac{98}{365}\right) = 102.68$

The expression above can be turned upside down, so that present value of 102.68 in 98 days’ time, using 10% interest rate per annum, is:

$$PV = \frac{102.68}{\left(1 + 0.10 \times \frac{98}{365}\right)} = \$100$$

If we know how much money we invest at the beginning (PV) and we know the total amount at the end (FV), then we can calculate rate of return or interest rate ( $r$ )

$$r = \left(\frac{102.68}{100} - 1\right) \times \frac{365}{98} = 10\%$$

This gives the rate as normally expressed—that is, the rate for the period of the investment. This can of course then be converted to an effective annual rate (EAR) by using:

$$EAR = \left(\frac{102.68}{100}\right)^{\frac{365}{98}} - 1 = 10.35\%$$

The calculation of present value is sometimes known as “discounting” a future value to a present value and the interest rate used is sometimes known as the “rate of discount”.

In general, these calculations demonstrate the fundamental principles behind market calculation, the “Time value of money”. As long as interest rates are not negative, any given amount of money is worth more sooner than it is later because if you have it sooner, you can place it on deposit to earn interest. The extent to which it is worthwhile having the money sooner depends on the interest rate and the time period involved.

For short-term investment calculations:

$$FV = PV \left( 1 + r \times \frac{\text{days}}{\text{year}} \right)$$

$$PV = \frac{FV}{\left( 1 + r \times \frac{\text{days}}{\text{year}} \right)}$$

$$r = \left( \frac{FV}{PV} - 1 \right) \times \frac{\text{year}}{\text{days}}$$

$$EAR = \left( \frac{FV}{PV} \right)^{\frac{\text{year}}{\text{days}}} - 1$$

*Mastering Financial Calculations, Robert Steiner, Prentis Hall, Chapter 2, p. 12-14*

### 247. What is Time deposit / loan? List it's characteristics

A time deposit or “clean” deposit is a deposit placed with a bank. This is not a security which can be bought or sold (that is, it is not “negotiable”), and it must normally be held to maturity.

*Term:* from one day to several years, hut usually less than one year

*Interest:* usually all paid on maturity, but deposits of over a year (and sometimes those of less than a year) pay interest more frequently – commonly each six months or each year. A sterling 18-month deposit, for example, generally pays interest at the end of one year and then at maturity

*Quotation:* As an interest rate

*Currency:* any domestic or international currency

*Settlement:* Generally same day for domestic, two working days for international

*Registration:* there is no registration

Negotiable: no

*Mastering Financial Calculations, Robert Steiner, Prentis Hall, Chapter 2, p. 50*

### 248. What is Certificate of deposit (CD)? List it's characteristics

A CD is a security issued to a depositor by a bank or building society, to raise money in the same way as a time deposit.

*Term:* generally up to one year, although longer-term CDs are issued.

*Interest:* usually pay a coupon, although occasionally sold as a discount instrument. Interest usually all paid on maturity, but CDs of over a year (and sometimes those of less than a year) pay interest more frequently – commonly each six months or each year. Some CDs pay a “floating” rate (FRCD), which is paid and prefixed at regular intervals

*Quotation:* As a yield

*Currency:* any domestic or international currency

*Settlement:* Generally same day for domestic, two working days for international

*Registration:* usually in bearer form

*Negotiable:* yes

*Calculation CD*

*Maturity proceeds*

$$= \text{Face value} \times \left(1 + \text{Coupon Rate} \times \frac{\text{days from issue to maturity}}{\text{year}}\right)$$

$$\text{Secondary market price} = \frac{\text{maturity proceeds}}{\left(1 + \text{market yield} \times \frac{\text{days left to maturity}}{\text{year}}\right)}$$

*Return on holding a CD*

$$= \left[ \frac{\left(1 + \text{purchase yield} \times \frac{\text{days from purchase to maturity}}{\text{year}}\right)}{\left(1 + \text{sale yield} \times \frac{\text{days from sales to maturity}}{\text{year}}\right)} - 1 \right] \times \frac{\text{year}}{\text{days}}$$

*Mastering Financial Calculations, Robert Steiner, Prentis Hall, Chapter 2, p. 57*

## 249. Derive formula for price of CD paying more than one coupon

Most CDs are short-term instruments paying interest on maturity only. Some CDs however are issued with a maturity of several years. In this case, interest is paid periodically – generally every six months or every year. The price for a CD paying more than one coupon will therefore depend on all the intervening coupons before maturity, valued at the current yield. Suppose that a CD has three more coupons yet to be paid, one of which will be paid on maturity together with the face value  $F$  of the CD. The amount of this last coupon will be:

$$F * R * \frac{d_{23}}{\text{Years}}$$

Where:  $R$  = the coupon rate on the CD

$d_{23}$  = the number of days between the second and third (last) coupon

The total amount paid on maturity will therefore be:

$$F * \left(1 + R * \frac{d_{23}}{\text{Years}}\right)$$

The value of this amount discounted to the date of the second coupon payment, at the current yield  $i$ , is:

$$\frac{F * \left(1 + R * \frac{d_{23}}{\text{Years}}\right)}{\left(1 + i * \frac{d_{23}}{\text{Years}}\right)}$$

To this can be added the actual cashflow received on the same date – that is, the second coupon, which is:

$$F * R * \frac{d_{12}}{Years}$$

The total of these two amounts is

$$F * \left( \frac{\left(1 + R * \frac{d_{23}}{Years}\right)}{\left(1 + i * \frac{d_{23}}{Years}\right)} + R * \frac{d_{12}}{Years} \right)$$

Again, this amount can be discounted to the date of the first coupon payment at the current yield  $i$  and added to the actual cashflow received then, to give:

$$F * \left( \frac{\left(1 + R * \frac{d_{23}}{Years}\right)}{\left(1 + i * \frac{d_{23}}{Years}\right) \left(1 + i * \frac{d_{13}}{Years}\right)} + \frac{R * \frac{d_{12}}{Years}}{\left(1 + i * \frac{d_{13}}{Years}\right)} + R * \frac{d_{01}}{Years} \right)$$

The result will be the present value of all the cashflows, which should be the price  $P$  to be paid. This can be written as:

$$P = F * \left( \frac{\left(1 + R * \frac{d_{23}}{Years}\right)}{\left(1 + i * \frac{d_{23}}{Years}\right) \left(1 + i * \frac{d_{13}}{Years}\right) \left(1 + i * \frac{d_{01}}{Years}\right)} + \frac{R * \frac{d_{12}}{Years}}{\left(1 + i * \frac{d_{13}}{Years}\right) \left(1 + i * \frac{d_{01}}{Years}\right)} + \frac{R * \frac{d_{01}}{Years}}{\left(1 + i * \frac{d_{01}}{Years}\right)} \right)$$

*Mastering Financial Calculations, Robert Steiner, Prentis Hall, Chapter 2, p. 61-63*

## 250. What is Treasury bill (T-bill)? List its characteristics

Treasury bills are domestic instruments issued by governments to raise short-term finance.

*Term: generally 13, 26 or 52 weeks; in France also 4 to 7 weeks; in the UK generally 13 weeks*

*Interest: in most countries non-coupon bearing, issued at a discount*

*Quotation: in USA and UK, quoted on a "discount rate" basis, but in most places on a true yield basis*

*Currency: usually the currency of the country; however the UK, for example, also issues T-bills in euros*

*Registration: bearer security*

*Negotiable: yes*

*Mastering Financial Calculations, Robert Steiner, Prentis Hall, Chapter 2, p. 52*

### 251. What is Commercial paper (CP)? List it's characteristics

CP is issued usually by a company (although some banks also issue CP) in the same way that a CD is issued by a bank. CP is usually, however, not interest-bearing. A company generally needs to have a rating from a credit agency for its CP to be widely acceptable. Details vary between markets.

*Term:* from one day to 270 days; usually very short-term

*Interest:* non-interest bearing, issued at a discount

*Quotation:* on a “discount rate” basis

*Currency:* domestic US\$

*Settlement:* same day

*Registration:* in bearer form

*Negotiable:* yes

*Mastering Financial Calculations, Robert Steiner, Prentis Hall, Chapter 2, p. 52-53*

### 252. What is bill of exchange? Write it's characteristics

A bill of exchange is used by a company essentially for trade purposes. The party owing the money is the “drawer” of the bill. If a bank stands as guarantor to the drawer, it is said to “accept” the bill by endorsing it appropriately, and is the “acceptor”. A bill accepted in this way is a “banker’s acceptance” (BA).

*Term:* from one week to one year but usually less than six months

*Interest:* non-interest bearing, issued at a discount

*Quotation:* in USA and UK, quoted on a “discount rate” basis, but elsewhere on a true yield basis

*Currency:* mostly domestic, although it is possible to draw foreign currency bills

*Settlement:* available for discount immediately on being drawn

*Registration:* none

*Negotiable:* yes, although in practice banks often tend to hold the bills they have discounted until maturity

*Mastering Financial Calculations, Robert Steiner, Prentis Hall, Chapter 2, p. 53-54*

### 253. What is Repurchase agreement (repo)? Write it's characteristics

A repo is an arrangement whereby one party sells a security to another party and simultaneously agrees to repurchase the same security at a subsequent date at an agreed price. This is equivalent to the first party borrowing from the second party against collateral, and the interest rate reflects this – that is, it is slightly lower than an unsecured loan. The security

involved will often be of high credit quality, such as a government bond. A reverse repurchase agreement (reverse repo) is the same arrangement viewed from the other party's perspective. The deal is generally a "repo" if it is initiated by the party borrowing money and lending the security and a "reverse repo" if it is initiated by the party borrowing the security and lending the money.

*Term: usually very short-term, although in principle can be for any term*

*Interest: usually implied in the difference between the purchase and repurchase prices*

*Quotation: as a yield*

*Currency: any currency*

*Settlement: generally cash against delivery of the security (DVP)*

*Registration: n/a*

*Negotiable: no*

*Mastering Financial Calculations, Chapter 2, pp. 1-3*

*Mastering Financial Calculations, Robert Steiner, Prentis Hall, Chapter 2, pp.54*

## **254. What is Spot Exchange Rate? Cross Rate? How do you calculate Cross Rate**

A "spot" transaction is an outright purchase or sale of one currency for another currency, for delivery two working days after the dealing date (the date on which the contract is made).

The currency exchange rate between two currencies, both of which are not the official currencies of the country in which the exchange rate quote is given in.

The idea of cross rates implies two exchange rates with a common currency, which enables you to calculate the exchange rate between the remaining two currencies.

For example, you can easily find, say, the euro-dollar or the yen-dollar exchange rates in financial media. However, the euro-yen exchange rate may not be listed. Because the dollar is the common currency in this example, you can calculate the euro-yen (and also the yen-euro) exchange rate.

For example if you know exchange rate for EUR/USD and YEN/USD and want to find out Euro/YEN exchange rate. as USD is common currency in both, we need to cancel it.

$$\frac{EUR}{USD} \cdot \frac{YEN}{USD} = \frac{EUR}{USD} * \frac{USD}{YEN} = \frac{EUR}{YEN}$$

*Mastering Financial Calculations, Robert Steiner, Prentis Hall, Chapter 7, pp.153-155*

## **255. How do you calculate forward exchange rate?**

Although "spot" is settled two working days in the future, it is not considered in the foreign exchange market as "future" or "forward", but as the baseline from which all other dates (earlier or later) are considered.

A “forward outright” is an outright purchase or sale of one currency in exchange for another currency for settlement on a fixed date in the future other than the spot value date. Rates are quoted in a similar way to those in the spot market, with the bank buying the base currency “low” (on the left side) and selling it “high” (on the right side). In some emerging markets, forward outright are non-deliverable and are settled in cash against the spot rate at maturity as a contract for differences.

The forward outright rate may be seen both as the market’s assessment of where the spot rate will be in the future and as a reflection of current interest rates in the two currencies concerned. Consider, for example, the following “round trip” transactions, all undertaken simultaneously:

- (i) Borrow Deutschmarks for 3 months starting from spot value date.**
- (ii) Sell Deutschmarks and buy US dollars for value spot.**
- (iii) Deposit the purchased dollars for 3 months starting from spot value date.**
- (iv) Sell forward now the dollar principal and interest which mature in 3 months’ time, into Deutschmarks.**

In general, the market will adjust the forward price for (iv) so that these simultaneous transactions generate neither a profit nor a loss.

Forward exchange rate is calculated using following formula:

$$F = S * \frac{1 + r_{Domestic}}{1 + r_{Foreign}}$$

*Mastering Financial Calculations, Robert Steiner, Prentis Hall, Chapter 7, pp.156-157*

## 256. What is Interpolation and Extrapolation

In the money market, prices are generally quoted for standard periods such as 1 month, 2 month, etc. If a dealer needs quote a price for an “odd date” between these periods, need to “interpolate”. For example, that the 1-month rate (30 days) is 8.0% and that 2-month rate (61) is 8.5%. The rate 1 month and 9 days (39 days) assumes that interest rate increase steadily from the 1-month rate to 2-month rate – a straight line interpolation. The increase from 30 days to 39 days will therefore be  $\frac{9}{31}$  proportion of the increase from 30 days to 61 days. The 39 day rate is:

$$8.0\% + (8.5\% - 8.0\%) \times \frac{9}{31} = 8.15\%$$

The same process can be used for interpolating exchange rates.

An alternative is “exponential” or “logarithmic” interpolation in which straightline interpolation is performed on the logarithms of the date rather than on the date themselves. This can provide a smoother interpolation and, although the difference between the two methods can be rather small it can be significant for capital market calculations. The logarithmic interpolation is:

$$\log_e 8.0 + (\log_e 8.5 - \log_e 8.0) \times \frac{9}{31} = 2.09704$$

$$e^{2.09704} \approx 8.15\%$$

The formulas of both interpolations are:

*Straight line interpolation:*

$$i = i_1 + (i_2 - i_1) \times \left( \frac{d - d_1}{d_2 - d_1} \right)$$

*Exponential interpolation:*

$$\log(i) = \log(i_1) + (\log(i_2) - \log(i_1)) \times \left( \frac{d - d_1}{d_2 - d_1} \right) \text{ or equivalently:}$$

$$i = i_1 + \left( \frac{i_2}{i_1} \right)^{\left( \frac{d - d_1}{d_2 - d_1} \right)}$$

Where:

*i* is the rate required for *d* days  
*i*<sub>1</sub> is the rate required for *d*<sub>1</sub> days  
*i*<sub>2</sub> is the rate required for *d*<sub>2</sub> days

*Mastering Financial Calculations, Robert Steiner, Prentis Hall, Chapter 1, p. 25-27*

## 257. What are Forward-Forward and FRA?

A **forward-forward** is a cash borrowing or deposit which starts on one forward date and ends on another forward date. The term, amount and interest rate are all fixed in advance. Someone who expects to borrow or deposit cash in the future can use this to remove any uncertainty relating to what interest rates will be when the time arrives.

An **FRA** is an off-balance sheet instrument which can achieve the same economic effect as a forward-forward. Someone who expects to borrow cash in the future can buy an FRA to fix in advance the interest rate on the borrowing. When the time to borrow arrives, he borrows the cash in the usual way. Under the FRA, which remains quite separate, he receives or pays the difference between the cash borrowing rate and the FRA rate.

Amount paid at maturity of FRA is computed as following:

$$\text{Payment amount} = \frac{(\text{FRA Rate} - \text{Reference Rate}) * \frac{\text{Days}}{\text{Year}}}{1 + \text{Reference Rate} * \frac{\text{Days}}{\text{Year}}}$$

*Mastering Financial Calculations, Robert Steiner, Prentis Hall, Chapter 3, pp. 51-58;*



## 258. What is Convexity?

For small yield changes, the approximation above is fairly accurate. For larger changes, it becomes less accurate. The reason that the modified duration did not produce exactly the correct price change in the last example is that the slope of the curve changes as you move along the curve. The equation:

$$\text{Change in Price} = \text{Change in Yield} * \frac{dP}{di}$$

in fact calculates the change along the straight line rather than along the curve.

As a result, using modified duration to calculate the change in price due to a particular change in yield will generally underestimate the price. When the yield rises, the price does not actually fall as far as the straight line suggests; when the yield falls, the price rises more than the straight line suggests. The difference between the actual price and the estimate depends on how curved the curve is. This amount of curvature is known as “convexity” – the more curved it is, the higher the convexity.

Mathematically, curvature is defined as:

$$\text{Convexity} = \frac{\frac{d^2P}{di^2}}{\text{Price}}$$

Using convexity, it is possible to make a better approximation of the change in price due to a change in yield:

$$\begin{aligned} \text{Change in Price} = & -\text{Price} * \text{Modified Duration} * \text{Change in yield} + 0.5 * \text{price} \\ & * \text{Convexity} * \text{Change in Yield}^2 \end{aligned}$$

*Mastering Financial Calculations, Robert Steiner, Prentis Hall, Chapter 3, pp. 111-113;*

## 259. Consider Portfolio Duration

As mentioned above, the concept of duration – and also modified duration and convexity – can be applied to any series of cashflows, and hence to a whole portfolio of investments rather than to a single bond. To calculate precisely a portfolio’s duration, modified duration or convexity, the same concept should be used as for a single bond, using all the portfolio’s cashflows and the portfolio’s yield (calculated in the same way as for a single bond). In practice, a good approximation is achieved by taking a weighted average of the duration etc. of each investment:

$$\text{Duration} = \frac{\sum(\text{Duration of each investment} * \text{Value of each investment})}{\text{Value of portfolio}}$$

$$\text{ModifiedDuration} = \frac{\sum(\text{Mod. Dur. of each investment} * \text{Value of each investment})}{\text{Value of portfolio}}$$

$$\text{Convexity} = \frac{\sum(\text{Convexity of each investment} * \text{Value of each investment})}{\text{Value of portfolio}}$$

*Mastering Financial Calculations, Robert Steiner, Prentis Hall, Chapter 3, pp. 113-114;*

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